A LABORATORY MANUAL

OF

PHYSICS AND APPLIED ELECTRICITY

ARRANGED AND EDITED

BY

EDWARD L. NICHOLS

PROFESSOR OF PHYSICS IN CORNELL UNIVERSITY

IN TWO VOLUMES

VOL. II

SENIOR COURSES AND OUTLINES OF ADVANCED WORK

BY

GEORGE S. MOLER, FREDERICK BEDELL, HOMER J. HOTCHKISS
CHARLES P. MATTHEWS, AND THE EDITOR

New York

MACMILLAN AND CO.

AND LONDON

1894

Norwood Press:
J. S. Cushing & Co. — Berwick & Smith.
Boston, Mass., U.S.A.

PREFACE.

LITTLE need be added, in bringing this second volume of the Manual before the public, to what has already been said in the preface to the first volume. The first volume consists of a laboratory course in general physics for beginners, in the performance of which the student is guided by explicit directions as to the method of manipulation and the arrangement of results. The present volume is intended for students who have completed such a course, and who are prepared to take up special work.

The needs of those who are in training to become electrical engineers have been especially considered, and it is for them particularly that Parts I., II., and III., which treat of Applied Electricity, Heat, and Photometry, have been written. To add to the usefulness of the volume to the student of general physics, Part IV. has been prepared.

The editor desires to express his obligations to his co-laborers, to whose efforts the various parts of this Manual are due, and in particular to Dr. Frederick Bedell for his assistance in carrying the work through the press.

EDWARD L. NICHOLS.

CORNELL UNIVERSITY, June, 1894.

CONTENTS.

PART I.

PART II.

PART III.

PART IV.

A LABORATORY MANUAL OF PHYSICS AND APPLIED ELECTRICITY.

Volume II. — Part I.

EXPERIMENTS WITH DIRECT CURRENT APPARATUS.

By G. S. Moler, H. J. Hotchkiss, and C. P. Matthews.

INTRODUCTORY TO CONTINUOUS CURRENT EXPERIMENTS.

The student entering upon a course of experiments in applied electricity is supposed to be familiar with the general principles of electrical measurement, and to have had such experience in adjusting instruments and practice in manipulation that he is ready at the start to investigate the operation of electrical apparatus. This preliminary training can only be acquired by a course of laboratory practice, such as that given in the first volume of this manual. The student should, furthermore, be sufficiently familiar with the design and construction of dynamo-electric machinery to enable him to understand the principles of operation. Such questions can only be touched upon here, and reference is therefore necessary to standard works on the subject.

Although the general subject of electrical measurements is not within the scope of this portion of the manual, the use of such instruments as are ordinarily employed in an engineering laboratory necessarily forms an important part of the practice work. The student should in all cases be familiar with the methods of adjusting and testing the instruments he uses, and in written reports upon experiments performed should fully describe the apparatus employed.

The experiments first performed would ordinarily be those with small dynamos and motors, inasmuch as the student can work quicker and with less liability to accident with such apparatus, and can proceed to the operation of larger machinery as he gains experience. The experiments are not arranged in a

fixed order, which must be followed in the laboratory, for with many experiments the order is of no consequence. The student is not to limit his experiment to the ground covered in the manual, but is to extend it at his discretion, so as to include such points as are associated with the investigation at hand. The manual is simply a guide to call attention to the principal phenomena to be studied. After a sufficient general practice, the student is to take up extended investigations which introduce new problems, in which originality in methods of experiment should be encouraged.

In all cases the student should carefully study and interpret the results of his experiment. These results are usually best shown by curves, which are in general more significant than a mass of tabulated data. The value of experimental work is not measured by the number of phenomena observed, or amount of data obtained; but depends more upon the ability acquired to account for the phenomena by tracing cause and effect in accordance with the underlying principles, and upon the interpretation of the data furnished by experiment. The experimenter thus not only obtains a knowledge of facts, but soon becomes familiar with the cause and meaning of phenomena, and can draw correct conclusions from them, so that he is prepared to deal with problems that reach outside of past experience or instruction.

The details of an experiment and the conditions under which it is performed should be carefully noted, especially where such are not constant. Valuable experience may be gained in this way which will help the experimenter in further work to avoid or overcome obstacles which usually make it difficult to obtain accurate and reliable results. Much depends upon selecting from the means which are available for reaching a desired end those best adapted to the existing or obtainable conditions. For this no general rule can be laid down.

In the performance of the experiment the student should show such originality and independence as his experiment

allows. The written reports should include, besides the general description of the apparatus used, the method employed and the results obtained, a brief statement of the principles involved (together with references to standard works where necessary), so as to show the full significance of the work. The difficulties encountered should be stated, and the methods for overcoming them fully explained. The conventional symbols given in Fig. 1 will be found useful in designating electrical apparatus.

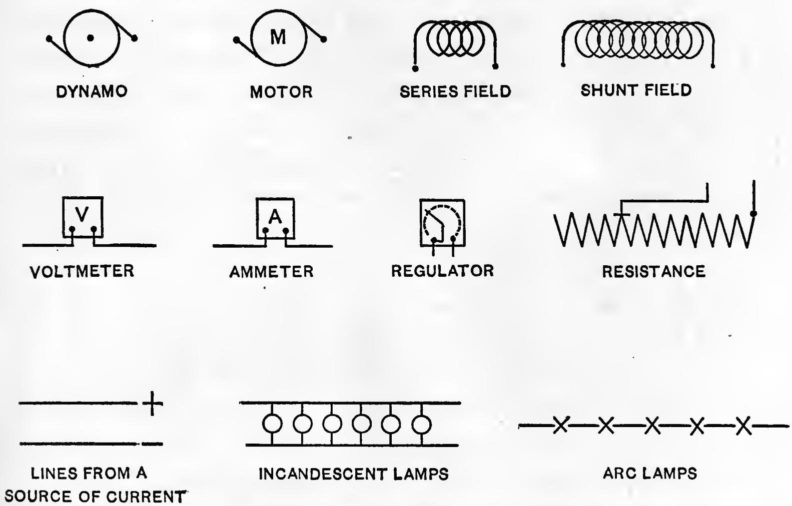


Fig. 1.—A Set of Conventional Signs to be used in designating Electrical Apparatus.

It is impossible to include here a description of the apparatus necessary for the following experiments, many of which are quite general, and will apply to any apparatus whatsoever, while others can only be performed with the specific apparatus for which they are intended. A description of such specific apparatus is usually incorporated in the directions for the separate experiments, or a reference given to a description of the apparatus in some standard work. The determination of the fluctuation in speed is usually of considerable importance, and

for such determinations the following self-registering speed-counter will be particularly valuable.

The printer * is a speed-counter which prints at the end of each minute, upon a strip of paper, the speed of the main shaft to which it is connected. It has a type wheel, which is revolved by means of a worm gear, the worm being driven by sprocket wheels and chains, which connect it to the main shaft so that it will revolve at exactly the same rate. An electromagnet, operated by a clock, releases the printing hammer once a minute, after which the paper is moved along, and the type wheel is slipped upon its shaft to make it read the same as if it had started from zero at the beginning of the minute. The numbers move past a stationary point, which prints its position upon the paper when the hammer makes its blow. If the number printed is exactly opposite the point, it shows that there has been a whole number of revolutions that minute, but if the number is above or below it, the fraction indicated by its position can easily be determined. Figure 2 shows how the fraction of a revolution appears on the record. Usually enough of either the preceding or following number is printed to easily make it out, and to estimate the fraction, which can be read to a quarter of a revolution.

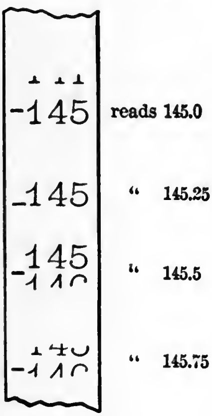


Fig. 2.

To use the printer in connection with an experiment, first determine the ratio of its speed to that of the dynamo as follows: Take the average of six or eight one-minute observations of the speed of the dynamo, and every time a set of readings are made also read the printer, and so calculate the desired ratio. This will only answer when the belt is able to drive the dynamo without slipping.

* Moler, "An Automatic Printing Speed-Counter for Dynamo Shafting." Transactions of the American Institute of Electrical Engineers, Vol. 10.

A spindle which is tight-geared to the worm shaft runs exactly ten times faster, and an electric bell or an electric light, operated each minute by the printer, gives the time, so by these a student may test his ability to take a speed by means of an ordinary speed-counter. He should make a number of such trials.

One of the most important adjuncts of a dynamo laboratory is a convenient adjustable resistance. One of the handiest forms for such a resistance is that described in Exp. 40; another is that described in Exp. 57, Part II. The following form of German silver resistance possesses many advantages: The resistance consists of 102 spiral coils of German silver wire, which are divided up into three sections. In two of the sections the coils are approximately 30 ohms each; in the

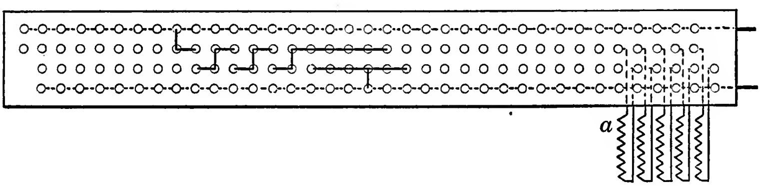


Fig. 3.—Switchboard for German Silver Resistance.

third, about 3 ohms each. In all of these the wires are of the same size, being 1.37 mm. in diameter, and for ordinary work each coil may carry 4 amperes. The wires are strung upon porcelain knobs, and are enclosed in a fixed case about 18 feet high. The ends of the wires are connected to mercury cups, consisting of copper-lined holes, arranged in rows, in three pieces of paraffined timber placed end to end so as to form a shelf. In each section there are four rows of holes, the holes in each of the two outer rows being connected, and together forming the two terminals of that section; the holes in the two inner rows are the terminals of the individual resistance coils, as shown at *a* in Fig. 3. The mercury cups can be joined by wire staples in multiple, or in series, or in any desired combination. Take, for example, the arrangement shown in the figure, in

which the heavy lines represent staples. To the left are three sets of coils, each set containing two coils in parallel. Each pair of coils has a resistance of 15 ohms. These are connected in series with a set of six coils in parallel (which has a resistance of 5 ohms), so that the whole combination has a resistance of 50 ohms, and will carry 8 amperes. Other combinations may be similarly made. If necessary, the sections may be joined together by copper connectors. (In the set of coils which is here described, the above values for the resistances are only approximate ; the coils are not to be taken as standards.)

For convenience in laboratory work, it is well to have the apparatus connected to a suitable switchboard, upon which all the connections may be made ; such a switchboard will be constructed to meet the requirements of each laboratory. It should be left to the student to arrange the apparatus and to make the necessary connections.

Experiment I. **Study of a dynamo.**

A careful study of the design and construction of the dynamo should be made without running it, including an investigation of such points as the following: *

Kind of *dynamo*, — direct or alternating current. Whether designed for constant potential, constant current, or otherwise. Normal speed, and E. M. F. or current for which it is constructed. External resistance, current, and watts at full load. Horse power necessary to drive it at full load, assuming an efficiency of 80 per cent. Peripheral velocity of the armature at normal speed ; or, if that cannot be obtained, the speed for a peripheral velocity of 3000 feet per minute. To what particular uses it is adapted, and why.

---

* A full description of the physical theory of the dynamo may be found in Chapter III. of S. P. Thompson's Dynamo-Electrical Machinery; also in parts of Chapters IV. and V., on Actions and Reactions in the Armature, and Mechanical Actions and Reactions. For armature windings and connections, see Chapter XII. of the same book.

Style of *field,* — form, material, construction. Winding: series, shunt, or compound. Resistance of field coils. Size of wire. Field current at normal potential, assuming half of the regulator resistance cut out if one is used. Circular mils * per ampere in the field coils. Watts lost in heating the field coils at normal excitation.

Type of *armature,* — closed or open coil. Drum, ring, disk, or other form. Number of coils (obtained from the commutator). Resistance of the armature circuit between brushes. Watts lost in the armature resistance at full load. Resistance of a single coil of the armature. Size of armature wire. Circular mils per ampere in the armature wire. Length of wire of one coil, assuming the resistance of a "mil-foot" † of copper wire to be 10.8 ohms. Number of turns per coil. Number of conductors on the armature, and average force tangential to the armature on each at full load. Percentage of armature covered by pole pieces.

Connection of coils to the commutator bars.

Kind and proper position of the brushes.

Means of regulation, if any.

EXPERIMENT 2. **Running small dynamos.**

This experiment may be performed with one compound and two shunt Edison dynamos, designed for 110 volts at a speed of 2200 r. p. m. and 2.3 amperes at full load. Six 100-volt incandescent lamps, connected in parallel on a support, are used for a variable external resistance.

Compound dynamo. See that the brushes are on opposite segments of the commutator and make good contact. Find the position of the neutral points, or diameter of commutation for

---

* The cross-section of a wire having a diameter of one mil (*i.e.* .001 of an inch) is said to be one circular mil. The square of the diameter of a wire, in thousandths of an inch, gives the number of circular mils in its cross-section.

† Mil-foot is used for a wire one foot long and having a cross-section of one circular mil.

maximum potential, on open circuit by use of a voltmeter, and give the brushes a little lead. Explain why the diameter of maximum commutation is in the direction found, and why a little lead is given.

Make connections as shown in Fig. 4, to run it as a long-shunt compound dynamo with open belt. (If the shunt field coils were connected at *b* instead of *a*, the field winding would be called short-shunt compound.) Vary the number of lamps in circuit, and prove by experiment that the series coils are connected compound instead of differentially; that is, that they increase the magnetization instead of decreasing it as current increases.

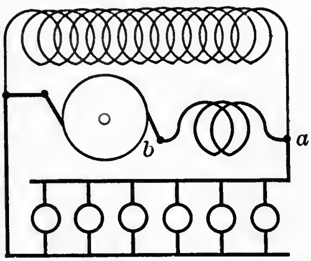


Fig. 4.—Connections for the Small Edison Compound Dynamo.

What changes must be made to run it with the belt crossed as a compound dynamo at the same potential as before? Prove by experiment, and explain why.

Shunt dynamo run as a motor by the compound dynamo. — Set the brushes at what is considered the right position. Connect preparatory to starting the motor.

Why is it usually unsafe or impracticable to try to start a *shunt* motor without a "starting box" or resistance in series with the armature?

Start the motor (with no load), and determine the effect on speed and current when the brushes are moved away from the neutral points. Account for the phenomena observed, and show how they may be used to determine the best position.

Make, and give reason for, the changes necessary to run the motor in the opposite direction.

Shunt dynamo run by the shunt motor. — Set the brushes and make connections, with the lamps in the external circuit. Note the effect on speed and the brightness of the lamps when the number of lamps in circuit is increased; also the effect on the

brightness of one or two lamps when the brushes of the motor, or of either dynamo are shifted. Adjust the brushes on each to the best position. Explain the causes for the phenomena observed.

INTRODUCTORY TO CHARACTERISTICS OF DYNAMOS.

If the resistance (R) of a dynamo circuit be varied, there will always be a corresponding change of the current (I) and the electromotive force (E) such that $\frac{E}{I}=R$ (from Ohm's law).

For a given resistance, with speed of dynamo constant, difference of potential and current may be measured, and their values taken as the co-ordinates of a point. Values of current should be plotted as abscissas, and values of potential difference as ordinates. As the resistance is suitably varied, the measurements of current and difference of potential give data for a series of points through which a curve may be drawn, showing for each current the corresponding difference of potential. Such a curve is commonly called a characteristic of the dynamo.

For a *series* dynamo, the resistance of the circuit is that of the armature, the field coils, and the external resistance; the last being larger than the internal. When current flows in the circuit, a voltmeter connected to the points where the external circuit is connected to the dynamo measures only the fall of potential (e) through the external resistance. The E. M. F. (E), or the fall of potential in the entire circuit, is obtained by adding to e the fall of the potential through the armature and field coil resistances (R_a+R_f). Hence $E=e+I\,(R_a+R_f)$. Plotting I and e gives the *external* series characteristic. Plotting I and E gives the *total* characteristic.

If the field be separately excited, and the armature be left on open circuit, field current and electromotive force at the brushes when plotted, give the *magnetization* curve, or open circuit characteristic, which will differ somewhat from the total characteristic, on account of there being current through the armature in latter case.

If, for a *shunt* dynamo, field current and brush electromotive force, with the external circuit open, be plotted as co-ordinates, the curve is called the *internal* shunt characteristic. The co-ordinates of the *external* shunt characteristic are current in the external circuit, and difference of potential at the brushes, with the resistance of the field circuit constant. The *total* shunt characteristic has total current and electromotive force as co-ordinates. Sometimes field current and external current are both varied so as to maintain a constant difference of potential at the brushes. These currents, when plotted, give what is called an *armature* characteristic.

If the series coils of a *compound* dynamo are cut out, it is then a plain shunt machine having characteristics as already given. If the series coils alone are used, any of the characteristics of an ordinary series machine may be taken. When the series coils are connected so that they add to the magnetization due to the shunt winding, thus maintaining the potential difference more nearly constant as the current increases, the curve for difference of potential at the terminals of the dynamo and current in the external circuit, is called the *compound* characteristic; but if the terminals of the series coils are reversed so that the magnetization is decreased by them, a *differential* characteristic is obtained instead of a compound.

The following curves are the more important ones for which data are taken in experiments on dynamos of the classes named: — Series dynamo: external series; total series; magnetization. Shunt dynamo: external shunt; internal shunt; total shunt; armature. Compound dynamo: compound; differential; series; external shunt; internal shunt.

General Directions for Experimental Work.

Much depends upon having the brushes in good condition and properly adjusted. For an ordinary two-pole machine they should be 180° apart. Secure contact of the entire bearing surface of the brush, and pressure sufficient to maintain good

contact when running. Adjust the brushes, by rocking the holders, to give maximum difference of potential. They will then be at the neutral points. A little lead may be given if necessary.

Make a diagram or outline of the connections and apparatus needed for each characteristic or part of the experiment. Find for which part the apparatus is most readily available and connect up for that first.

Sometimes a dynamo will not pick up when connected and started. This may be due to one or more of various causes, some of which are:

(*a*) Wrong position, or poor contact of brushes.

(*b*) A break or defective contact in the connections.

(*c*) Lack of residual magnetization on which to pick up.

(*d*) Current in the wrong direction through the field coils.

(*e*) Field coils working against each other.

(*f*) Too much resistance in the circuit containing the field coils.

If the dynamo is *series* wound, break the external circuit and note the voltmeter reading. If it indicates zero E. M. F., the difficulty may be due to (*a*), (*b*), or (*c*). The E. M. F. due to residual magnetism varies from 1 volt for one dynamo, with wrought-iron field cores and wound for 60 or 70 volts, to about 70 volts for another, with cast-iron cores, constructed to give 1000 or 1200 volts. The percentage of magnetization retained is greater when the magnetizing current is gradually decreased from a maximum to zero than when the circuit is suddenly broken. If there are indications of sufficient residual magnetism, test for (*d*) by closing the external circuit. If the voltmeter gives a lower reading than when the external circuit was open, it shows that the direction of current in the field coils must be reversed by changing the connection between brush and field from one brush to the other, or from one end of the coils to the other end. Reversing the polarity will reverse the current, but will not remedy the difficulty. If the external resistance is near

the minimum, found approximately by dividing the maximum E. M. F. of the dynamo by the maximum safe current, (*f*) need not be considered.

If the dynamo is *shunt* wound, the external circuit should be left open till all causes preventing picking up are removed. The remaining circuit is connected like that of a series dynamo; hence to find the particular cause of the difficulty one may proceed as indicated above. For the shunt dynamo however, the voltmeter is connected to both brushes instead of one brush and the end of the field joining the external resistance. The resistance of the shunt field coils is generally large enough to keep the field current down to a safe value when other resistance in series is all cut out.

EXPERIMENT 3. **Characteristics of a series dynamo.**

In this experiment the external series characteristic, the total characteristic, and the magnetization curve are obtained; and their forms, relations, and uses studied.

For the external series characteristic, connections may be made as shown in Fig. 5, putting into the circuit enough resist-

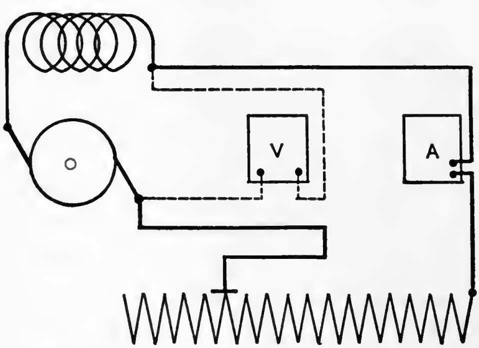


Fig. 5.—Connections for Series Characteristic.

ance to prevent a current greater than the safe carrying capacity of the armature. Start the dynamo, and see that it picks up and works properly. (See directions, page 13.)

To begin at the lower end of the characteristic, add resistance

till the current and E. M. F. are quite small. For some machines the current may be reduced to zero. Then decrease the resistance by such amounts successively as will give data for points at desired intervals along the characteristic up to the safe current capacity of the dynamo. Take corresponding values of current, difference of potential, and speed. Also E. M. F. and speed for zero current.

If desired, data may be taken going from maximum current to no current, for the purpose of showing the difference between curves for increasing and decreasing magnetization.

For any given value of current a small change of speed produces an approximately proportional change of E.M.F. Hence, if the speed varies, the voltmeter readings may be corrected to correspond to some convenient constant speed near the mean. Plot current for abscissas and corrected potential difference for ordinates of the series characteristics. Explain why the proportionality is more nearly correct for the total characteristic than for the external; also what other cause makes the E. M. F. corrections smaller than they should be.

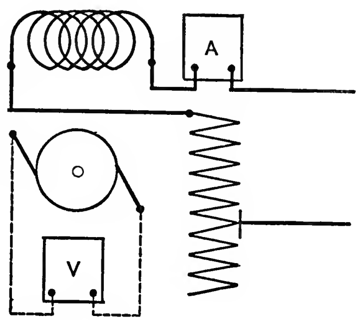


Fig. 6.—Connections for the Magnetization Curve.

Take the necessary data and plot a line showing the fall of potential through the resistance of field and armature for different values of current. The total characteristic may be obtained from the external characteristic, and this line for fall of potential in the machine.

For the magnetization curve, connect as shown in Fig. 6. Measure the field current, E. M. F. at the brushes, and the speed. Correct the E. M. F. readings to the same speed as for the other curves. Plot all the curves on the same sheet and to the same scale. In Fig. 7 is shown a set of curves for a series dynamo

with wrought-iron field cores, and stationary brushes having several degrees of lead, which accounts for the magnetization curve falling below the total characteristic for part of its length.

The product of the co-ordinates of any point on the sheet

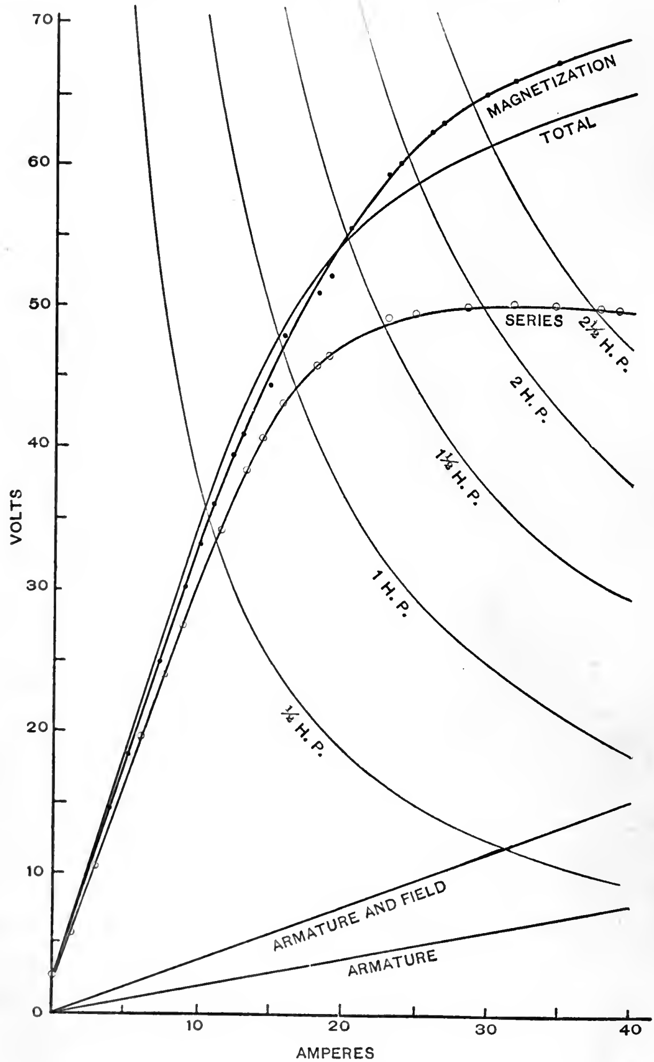


Fig. 7.—Characteristics of a Series Dynamo.

gives the power in watts (joules per second) represented by that point. Curves called horse-power curves may be drawn showing the loci of points representing the same rate of doing work. For any one curve the product of the co-ordinates of one point is the same as the product of the co-ordinates of any other point. Since 746 watts equal one horse-power, the product is usually made some multiple of 746.

The external resistance corresponding to any point on the series characteristic is given by the equation $R=\frac{e}{I}$. Hence R is proportional to the tangent of the angle between the I axis and a line drawn from the origin to the point.* This line is sometimes called a resistance line.

EXPERIMENT 4. **Characteristics of a shunt dynamo.**

The internal, external, and total shunt characteristics may be included in this experiment, leaving the armature characteristic for a separate experiment (Exp. 5) if preferred.

The connections for the *internal* characteristic are shown in Fig. 8. The external circuit is left open, and resistance in addition to that of the regulator is placed in series with it, so that points may be obtained for lower values of E. M. F. and current than the total regulator resistance alone would give. The E. M. F. readings may be corrected for variations of speed the same as for a series or magnetization curve. The field current is so small that there is no appreciable arma-

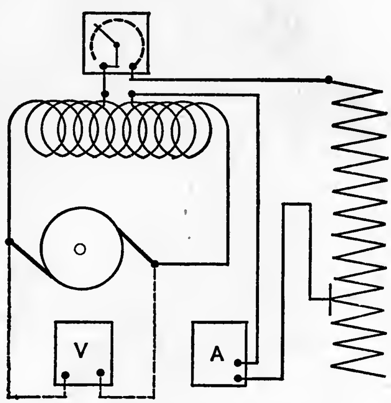


Fig. 8.—Connections for the Internal Shunt Characteristic.

* For method of representing resistance graphically, see S. P. Thompson's Dynamo-Electric Machinery, p. 258.

ture reaction, and scarcely any fall of potential in the armature, so the internal shunt characteristic is about the same as a magnetization curve obtained by separate excitation. The field current being small, the current per scale division for the inter-

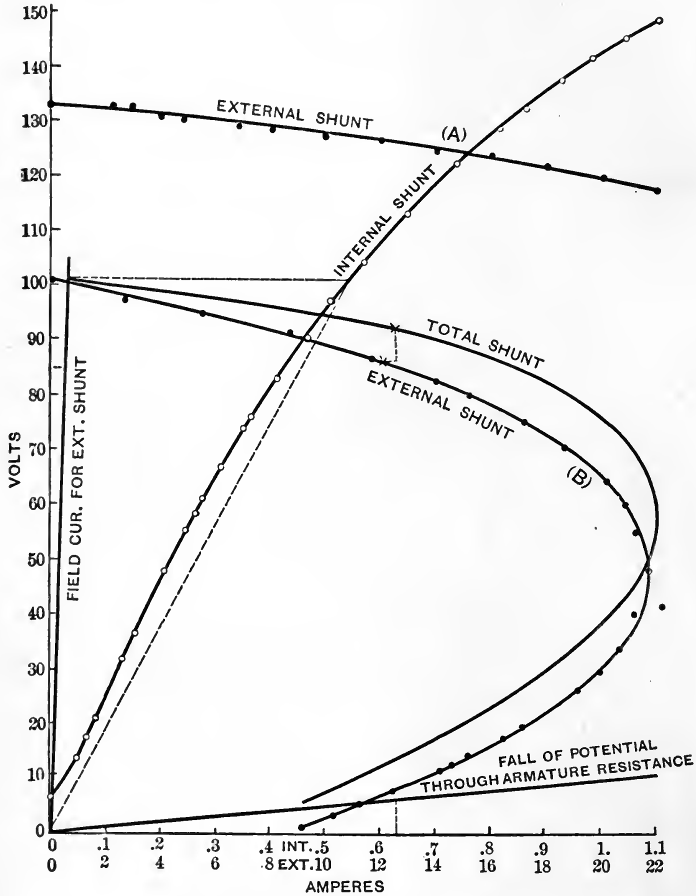


Fig. 9.—Characteristics of a Shunt Dynamo.

nal characteristic should be much smaller than for the external characteristic. The general form of the curve is shown in Fig. 9. However, for some dynamos the lower portion will be straighter, and the upper part more curved, showing a decided

"knee" as the magnets approach saturation. The internal characteristic in Fig. 9 is for a dynamo having cast-iron field cores of rather large cross-section, so that the magnetic density is much below saturation, even with the regulator resistance all cut out. It is taken with increasing magnetization produced by decreasing the resistance of the field circuit from infinity (or open circuit) to that of the coils alone.

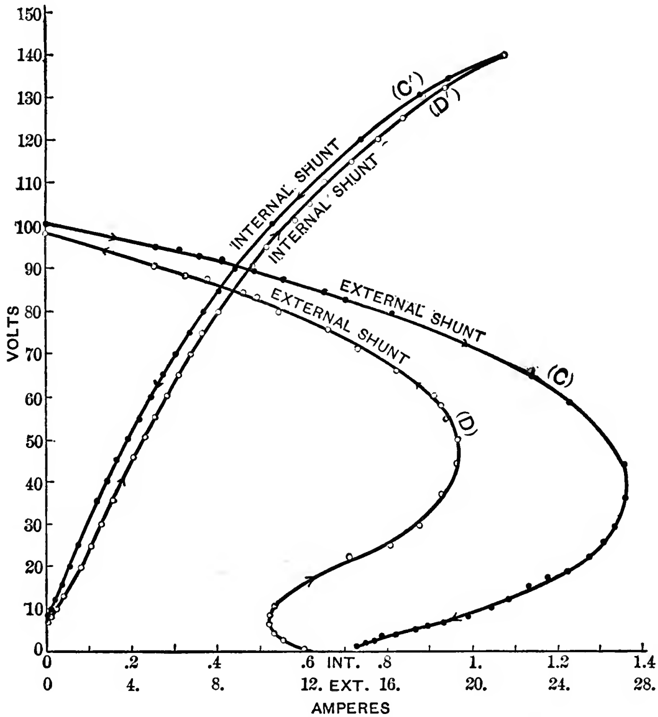


Fig. 10.—Shunt Characteristics with Increasing and Decreasing Magnetization.

If desired, readings may begin at the upper end instead of the lower. In that case, however, a different curve would be obtained, as shown in Fig. 10, where curve D' is given for increasing, and curve C' for decreasing magnetization. The corresponding external characteristics are also shown. The forms of these external characteristics depend, to a large extent, upon

the forms of their corresponding internal characteristics; so, for the purpose of studying their forms, relations, etc., it is generally better to take the two characteristics with the change of magnetization in the same direction, or to take the characteristics for both increasing and decreasing magnetization. Beginners generally find less difficulty with decreasing magnetization. Advanced students will probably find the curves, both up and down, more instructive, giving material for valuable study upon the interpretation of curves, or telling much to the one already familiar with their interpretation. Note should be taken of all conditions affecting the form of the curves. Ordinarily they should be the conditions of common use of the dynamo. Some of them that should be considered may be found in the remarks on the external characteristics. For the significance of ascending and descending curves of magnetization, read Exp. 62, Part II.

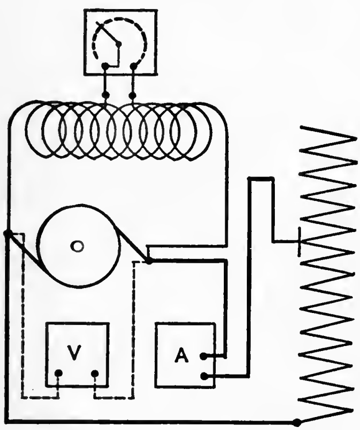


Fig. 11.—Connections for the External Shunt Characteristic.

For the *external shunt characteristic* an arrangement of connections is shown in Fig. 11. At first leave the outside circuit open, and adjust the regulator in the field circuit, so that the dynamo will give about its normal E. M. F. This gives the reading for potential for zero external current, and fixes the point where the curve leaves the axis of ordinates. Leave the resistance of the field circuit constant while taking the external characteristic.

Close the external circuit through enough resistance to give a point as near as desired to the one for zero current; then gradually decrease the resistance so as to give points suitably spaced along the curve, which will have the general form of *B*

(Fig. 9). It is generally best to make observations of speed at convenient intervals, so that if there are irregularities in the points for the curve, it can be determined whether they are probably due to fluctuations of speed, or to some other cause. Corrections of E. M. F. for change of speed cannot be made for the external characteristic in the same way as for the internal, or for a series or magnetization curve. Explain why.

If the maximum current for the complete curve, starting near normal potential, should exceed the safe current capacity of the dynamo, it may be carried to the safe current limit, as shown by curve *A* (Fig. 9); then start another curve at a lower E. M. F., such that the maximum current will fall within the safe limit. Thus the complete characteristic can be taken, as shown by curve *B* (Fig. 9).

To obtain good results, care should be taken not to break the circuit, or make too large steps in changing the resistance. A slow uniform rate of change, if it could be obtained, would be better than a step-wise change.

If at any time a new point is further than desired from the last one taken, it is generally better not to go back to get a nearer point, for the curves for decreasing and increasing magnetization are quite different, both for external and internal characteristics, as shown in Fig. 10.

If the external circuit should be accidentally broken while working on the lower part of the curve, the magnetization would run up to its first value, and if the circuit were closed again through the same resistance, there would be danger of getting an excessive current before the magnetization would fall to the low value it had before the break. Hence, under such circumstances, the circuit should be closed through all the resistance, which may then be gradually decreased till the conditions previous to the break are again obtained. When the external resistance is all cut out, bringing the lower end of the curve as near as possible to the current axis; those desiring the curve for increasing magnetization may then take it by simply

increasing the resistance without any break of circuit or change of any kind. If, however, the external circuit is broken, or the up curve is to be taken first, the field circuit should be open before short-circuiting the brushes so that the external circuit contains only the ammeter (the resistance of which is usually a very small fraction of an ohm, *e.g.* 0.00033 ohm in a 100-ampere Weston ammeter). When the circuit through the ammeter is closed, the field circuit may be closed also; for then the fall of potential from brush to brush and the current in the field circuit are almost zero. The value of current for zero difference of potential at the brushes, depends upon the resistance of the armature circuit, and the E. M. F. due to residual magnetism, which will be called E_r for convenience. If the field excitation is gradually decreased from a high value to zero, E_r will have a higher value than when the circuit is suddenly broken. The difference may be as great as 25 per cent of the latter; hence the point where the up curve leaves the axis of abscissas depends upon whether E_r is that due to a sudden or to a gradual decrease of field current, and whether the current is reduced from a maximum or a mean value to zero. The lower end of curve *D* (Fig. 10) would probably have been much nearer that of curve *C* if the magnetization had not been allowed to vary between completing *C* and starting *D*; or if, after increase, it had been gradually reduced from a maximum to its residual value.

For the higher values of current in field and armature, the temperature and resistance of the coils may be increased (especially if the machine has not been previously warmed by running at its normal load), so as to give the curve a different form from that which it would have if the resistances remained constant. Self-induction and armature reaction also affect the curves. The slight change of speed would account for only a part of the difference between the open-circuit E. M. F. of the curve *D* and that of *C*, the balance probably being due principally to increase of field circuit resistance. The first adjustment of

open-circuit E. M. F. may have been from a higher value instead of a lower. The brushes remained in the same position for all of the curves in Fig. 10, otherwise the form of the curves would have been affected.

The points for the *total characteristic* are obtained from points on the external characteristic and the corresponding values of field current and fall of potential through the armature resistance.

The abscissas of a line drawn from the origin to a point on the internal characteristic having an ordinate equal to the open-circuit E. M. F. of the external characteristic, represent field currents. (See the dotted line in Fig. 9.) The full line near the vertical axis is the same, drawn to the scale for external current. To get data for the fall of potential in the armature, proceed as in the measurement of resistance by fall of potential. To get a point on the total characteristic corresponding to any point on the external, measure to the right of the point on the external a distance equal to the field current for that difference of potential, then upward a distance equal to the fall of potential in the armature, as shown by the dotted lines in Fig. 9, starting with the point for 85 volts on the external characteristic. Interpret the curves obtained by explaining the causes which determine their form; indicate the uses to which the curves may be put, and the information derived from them.

EXPERIMENT 5. **Armature characteristic.**

This experiment may be taken alone or in conjunction with either the preceding one on shunt characteristics, or Exp. 49 on compounding a dynamo, in which the armature characteristic is used.

Connections may be made as indicated in Fig. 12. The change of resistance in moving the rheostat contact from one block to the next is too great for close adjustments of potential difference to a constant value; hence it is better to have an additional resistance which may be varied by small amounts, or

the rheostat ordinarily used may be replaced by such a resistance, if large enough.

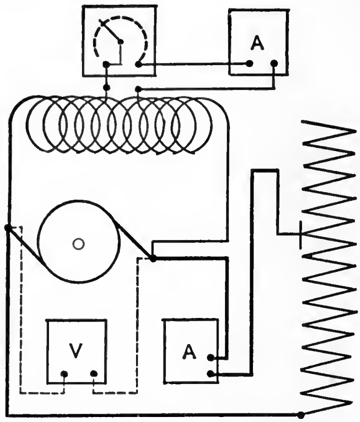


Fig. 12. — Connections for Armature Characteristic.

Adjust the open-circuit E. M. F. to the value at which the difference of potential at the brushes is to be kept constant; then close the external circuit through the maximum resistance used, and decrease it step-wise to give points at

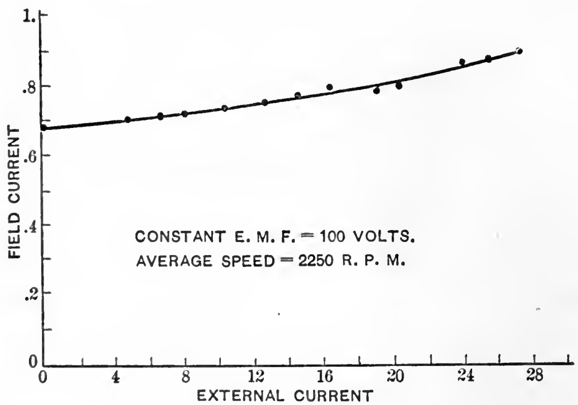


Fig. 13. — Armature Characteristic.

desired intervals, each time reading field current and external current when the difference of potential is adjusted to the constant value. Occasional readings for speed should also be taken.

The curve has the general form shown in Fig. 13. Find the percentage of increase of field current for full load. Interpret the curve.

EXPERIMENT 6. **Characteristics of a compound dynamo.**

The most important curves are: the external and internal shunt, series, compound, and differential.

The compound dynamo, being simply a shunt dynamo with the addition of a series field winding of a few turns, may be used as a shunt dynamo by leaving out the series coil. The connections are then made, and the external and internal

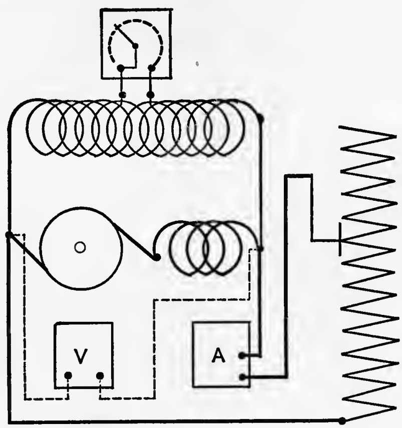


Fig. 14.—Connections for the Compound Characteristic.

characteristics obtained in the manner indicated in the experiment on characteristics of a shunt dynamo. (See p. 17.)

To obtain the series characteristic, the shunt field coils are left out, and the series coils are used alone. Connect and proceed as in taking the characteristic of an ordinary series machine. (See Exp. on characteristics of a series dynamo, p. 14.)

For the compound characteristic, the connections may be made as shown in Fig. 14, if long-shunt compound connections are desired. For short-shunt, change the shunt field terminal

from the outer to the inner end of the series coil. Before closing the external circuit, adjust the E. M. F. to the same value as that for the external shunt characteristic, so that the two curves will proceed from the same point on the axis of ordinates. When the regulator is properly adjusted, leave it without change during the run.

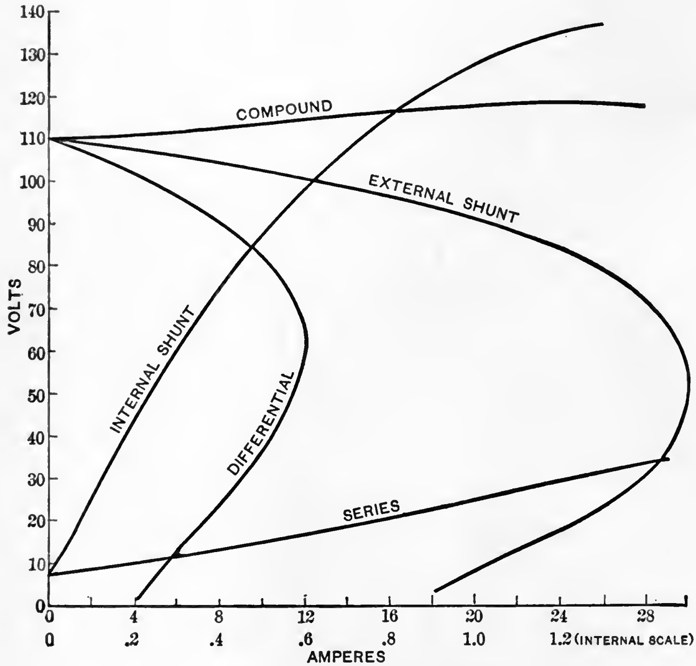


Fig. 15.—Characteristics of a Compound Dynamo.

As the external current is increased, the difference of potential at the terminals should increase slightly if the machine is designed to give a constant difference of potential at some distant point. In that case the increase for any particular current should equal the fall of potential due to that current through the resistance of the line. (See curve in Fig. 15.)

Short circuits are much more dangerous with a compound than with a shunt dynamo, and should be carefully avoided,

since, for very small resistances, the current of the compound increases far beyond the maximum current for the shunt.

The differential curve is taken with connections the same as for the compound, except that the direction of current through the series coils is changed. The open-circuit E. M. F. should be the same as for the external and compound. Since the field is weakened by increase of current in the series coils, which now oppose the shunt, the curve drops more rapidly than the external shunt, and the maximum current is much smaller; otherwise their forms are similar.

Discuss and interpret the curves as fully as possible.

EXPERIMENT 7. **Comparison of magnetization curves of dynamos.**

Magnetization curves may be plotted with ampere turns producing the field as abscissas, and lines of force N, through the armature as ordinates, if the field and armature windings are known in addition to the E. M. F., field current, and speed; observed in taking the characteristic of a dynamo separately excited, and with open circuit. N may be obtained from the fundamental equation* for the dynamo, $E = nCN10^{-8}$. This method of plotting the curves is useful in comparing the magnetization of different dynamos or motors, both series and shunt wound.

In this experiment two or more machines of different types, whose windings are known, may be selected and data taken for their open-circuit characteristics. The brushes should be adjusted to the neutral points, and the machines should run at or near their normal speeds. Plot the curves on the same sheet and to the same scale. Discuss the results, and study as many points relating to the subject as may be considered profitable.

* S. P. Thompson's Dynamo-Electric Machinery, p. 210.

EXPERIMENT 8. **Characteristics of the Waterhouse dynamo, and study of the third brush method of constant current regulation.***

The dynamo used is designed for running arc-lamps at a constant current, between seven and nine amperes. By a proper adjustment of the regulator, it can be made to give a constant current through a wide range of E. M. F. and resistance.

One way of making the connections is shown in Fig. 16. Any of the various kinds of resistance available may be used in place of the arc-lamps.

The proportion of total current that is used for field excitation is governed principally by the position of the movable contact *c* on the regulator resistance *R*. It is also affected by the position of the brushes relative to the neutral points.

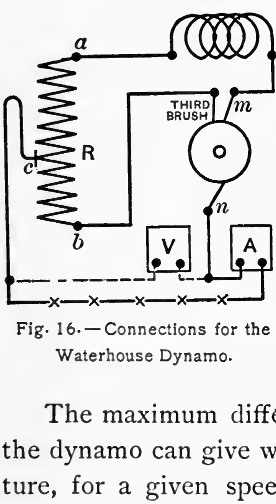


Fig. 16.—Connections for the Waterhouse Dynamo.

The range of E. M. F. and resistance for which the current may be kept constant depends largely on the resistance of *R*. Explain what would be the effect of making *R* quite large or quite small, and what should be taken into account in selecting the best resistance.

The maximum difference of potential at the terminals that the dynamo can give with say eight amperes through the armature, for a given speed, may be found by taking the series characteristic when connected as an ordinary two-brush series machine. This may be done by moving *c* to *a*, and breaking the line between the third brush and *R*.

A characteristic should be taken showing the range of potential difference through which the current may be kept constant

* See Chapter XV. of Desmond's Electricity for Engineers; also S. P. Thompson's Dynamo-Electric Machinery, p. 773.

at eight amperes, or some value near that. Note and account for the direction of variation from the eight amperes at the limits of regulation, and the difference between the upper limit and the maximum difference of potential found as above.

Seek to find out what part the third brush has in this method of regulation; also whether regulation could be accomplished if *b* were connected to the upper main brush, *m*, instead of the third, and if so, show what advantages would be possessed by the third brush connection.

Opinions or conclusions based on theory should be experimentally verified when they admit of it. The consideration of the many points that might be studied with profit is left to the option of the student; likewise the details of the phenomena observed in the experimental work.

EXPERIMENT 9. **Characteristics of the Edison arc-dynamo, and study of constant current regulation produced by the automatic shifting of a single pair of brushes.**

The Edison arc-dynamo regulates for a constant current, the one here referred to being designed to operate twelve arc-lamps with a current of 6.5 amperes.

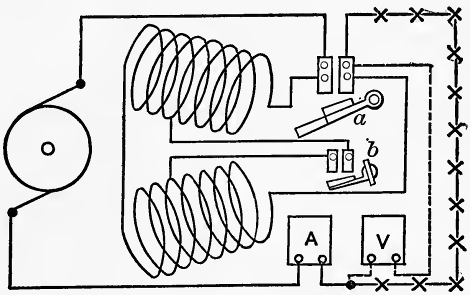


Fig. 17.—Connections for the Edison Arc-Dynamo.

In Fig. 17 is shown a diagram of the connections which are so arranged that part of the field-coils may be cut out by closing the switch *b*. Closing *a* cuts out all the field, and lets

the magnetization fall to its residual value which will give on open circuit, perhaps 60 or 70 volts at the brushes, if they are at the neutral points. The brushes may be rocked forward to reduce their difference of potential.

In starting up, one way to proceed is, to place the brush-holder as far forward as it will go, and have the large switch *a* closed. Then, if the external resistance is suitable, open the switch *a*, and pull the brush-holder back until the ammeter reads 6.5 amperes. When the circuit is to be broken to make a change, the large switch *a* should be closed.

The following are some of the curves that are useful in obtaining a knowledge of what characterizes the dynamo, of its operation, and reliability and range of automatic regulation: a magnetization curve, and a total characteristic with the brushes at the neutral points and the switch *b* open; also one of the above with the switch *b* closed, thus reducing the field. A characteristic showing the range of potential difference through which current may be kept constant by hand regulation at 6.5 amperes; and another showing how closely and through what range the automatic regulation maintains the current at 6.5 amperes.

The mode of operation and details of the regulating apparatus should be studied; also note whether the regulation is affected by cutting out part of the field turns.

EXPERIMENT 10. **Characteristic of the Thomson-Houston arc-dynamo, and study of the Thomson-Houston system of automatic current regulation.***

The dynamo is a series-wound open-coil machine intended to operate arc-lamps at a constant current. A diagram of connections is shown in Fig. 18. *R* is the regulator which operates the brush lever; *C* is the box containing the controller magnet; *S* is the switch for opening and closing the circuit.

---

* See S. P. Thompson's Dynamo-Electric Machinery, pp. 464–474; Charles Desmond's Electricity for Engineers, Chapter XIV.; Slingo and Brooker's Electrical Engineering, pp. 392–397.

See that the brushes are properly set by comparing the length from the clamp to the end of the brush with a marked length on a gauge provided for the purpose; also the point on the brush in contact with the commutator should be far enough from the end so that the end will not catch in the gaps between the segments of the commutator.

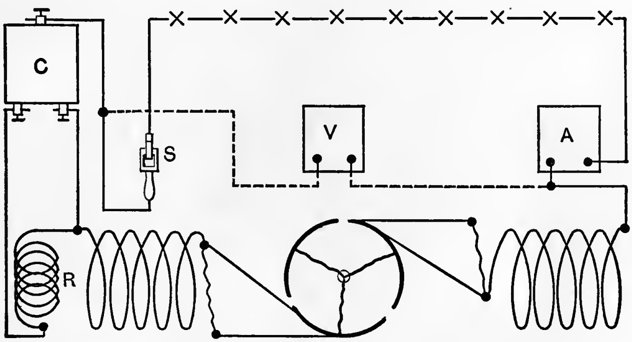


Fig. 18.—Connections for the Thomson-Houston Arc-Dynamo.

Since the dash pot prevents the regulator from raising the brush lever quickly, it is better to raise it by hand before closing the circuit when the dynamo is running, or while starting the dynamo under load.

The curve has the form of an ordinary series characteristic up to a point, beyond the knee of the curve, where the regulator begins to act. Beyond this point the current is maintained constant as the resistance is gradually decreased nearly to short circuit.

Discuss the curve obtained, and study in detail all that has a part in producing the regulation, and explain how it depends upon the different parts of the regulating system.

Experiment 11. **Characteristics of the Ball dynamo.**

The Ball dynamo is a series machine* used for arc lighting, and has two Gramme-ring armatures on the same shaft, and a sin-

* See experiment on series characteristics.

gle pole piece for each armature. It is so constructed, however, that the upper portion of the field circuit may be taken off and turned around so that both pole pieces will cover one armature like an ordinary two-pole machine.

Various modes of connecting may be used to secure different results, which will be shown by the corresponding characteristics.

Interpret the curves and study the advantages or disadvantages of this form of dynamo as compared with the ordinary two-pole type with a single armature.*

EXPERIMENT 12. **Study of Brackett cradle dynamometers.**

The object of the experiment is to familiarize the student with the theory, method of adjustment, and calibration of the dynamometers preparatory to using them in efficiency tests.

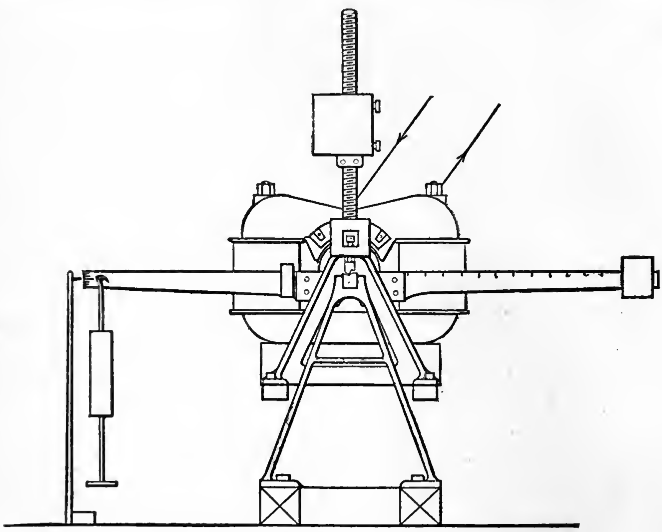


Fig. 19.—Brackett Cradle Dynamometer.

Explain the general principles upon which the dynamometer is based. Describe the arrangement and use of parts.

* See S. P. Thompson's Dynamo-Electric Machinery, p. 476. The Electrical Engineer (New York), Nov. 30, 1892, contains an article on the Ball dynamo by Leslie W. Collins.

Explain what adjustments are necessary in mounting a machine on the cradle ready for making a test; also why, and how the adjustments are made.

Two of the dynamometers, one of which is shown in Fig. 19, have their scale-beams graduated to read watts per 1000 revolutions; hence if, before starting, equilibrium is obtained with the weights at the zeros,

$$\text{watts} = \frac{\text{r.p.m.}}{1000} \times [\text{scale reading}]$$
$$= \frac{\text{r.p.m.}}{1000} \times \left[\frac{2\pi \times 1000 \times l \times G \times 746}{12 \times 33000}\right],$$

in which l is the number of inches the weight of G pounds on the arm is moved to produce equilibrium when the machine is running. If l' is taken in centimeters and G' in grams,

$$\text{watts} = \frac{\text{r.p.m.}}{1000}\left[\frac{2\pi \times 1000 \times l' \times G' \times 981}{60 \times 10^7}\right].$$

Show how these equations, or equivalent ones, are derived, and use them to determine whether the dynamometer is properly graduated for the weights used, or whether the weights are right to make the graduations correct. If l is found by measurement, G may be computed and the result checked with the value found by weighing; or G may be taken as known, and l computed and compared with measurement. Find what weight on the scale-pan at the end of the arm is equivalent to one large division of the scale.

Show that, if the system supported upon the knife-edges is in equilibrium when the weights are at any given points on the scale-arms, the measurement of power depends upon the distance and direction the weights are moved to balance the moment of the belt-pull; and not on the position of the points from which they are moved. Explain why the zeros are placed where they are.

Assume some power at a speed greater than 1000, and explain what weights must be used, and where placed to measure

it. Assume, also, a position of the weights with the torque reversed, and a speed less than 1000, and find the corresponding power. What is the capacity of each dynamometer at 1000 r.p.m., with the weights supplied?

Explain why the journal friction of a motor does not affect the dynamometer reading, while that of a dynamo does. When a motor runs with the belt off, why does the dynamometer tip while speed is changing, although balanced for constant speed?

One of the dynamometers is graduated to read kilogram-meters per revolution. The equation for it may be derived and applied in much the same manner as above. Note in what respects each dynamometer differs from the other.

Experiment 13. **Efficiency of double transformation with a small motor and dynamo.**

For this experiment take two small machines that are alike. Run one as a motor from a suitable source, and use it to drive the other as a dynamo. Vary the output, and measure the number of watts delivered from the dynamo; also the corresponding number of watts supplied to the motor. From these data find the efficiency of the double transformation.

Assume the efficiency of the motor to equal that of the dynamo, and find the efficiency of each at full load from the efficiency of double transformation at full load. Represent the results graphically, and discuss them. Note whether the efficiency is affected by changing the position of the brushes.

Experiment 14. **Efficiency of a small dynamo.**

A small dynamo is mounted on a small cradle dynamometer similar to the large Brackett dynamometers,* except that weights in a scale-pan at the end of the beam are used instead of the sliding weights on the beam.

---

* See Experiment 12, Study of cradle dynamometers.

See that the dynamo is properly adjusted on the dynamometer, and in working order.* Balance the dynamometer with the belt off; then put on the belt, and start the dynamo with both the field and the external circuits open. Balance, and find the number of watts required to overcome friction. Close the field circuit, and see that the dynamo picks up to about its normal potential. The number of watts will thus be increased because of field current, hysteresis, and foucault currents. Then close the external circuit through the set of six incandescent lamps in parallel provided for use with the small dynamos, or through any other suitable resistance. Take observations of current, difference of potential, speed, and dynamometer weights for different currents up to the safe current-capacity. Plot one curve for watts delivered and watts supplied to drive the dynamo, and another for efficiency and watts supplied, unless other graphical representations of the results are preferred. Interpret the results.

EXPERIMENT 15. **Efficiency of a small motor by use of a Raffard dynamometer.**

For this experiment a small Edison dynamo may be used as a motor, the power from which is absorbed and measured by a small Raffard dynamometer, the arrangement of which is shown in Fig. 20.

The large belt wheel turns the shaft on which are three brass wheels about $\frac{1}{3}$ m. in circumference. The end of one is shown at *a*. The middle one, about 4 cm. wide, is fastened rigidly to the shaft. The other two, about half as wide, are loose on the shaft. Over these wheels a bar, *b*, is supported by the shaft, and is free to rotate about it, near, and parallel to the face of the wheels. The moment of the bar about the shaft is balanced by a counterweight, not shown in the figure. To the bar are attached three canvas straps. The middle one, *c*,

* See Experiment 18, Efficiency test of a motor.

lies over the middle wheel, from which it hangs down and supports the weight *g*. The other two, *d*, pass under the two loose wheels, then upward to where they are attached to the lever arm *e*. Tension is applied to them by the weight *G*, hanging from the other end of the lever. The friction between the middle wheel and the strap over it, is increased by increasing the weight *g*, and tends to carry the bar, *b*, in the direction of rotation. The weight *G* is increased until its pull on

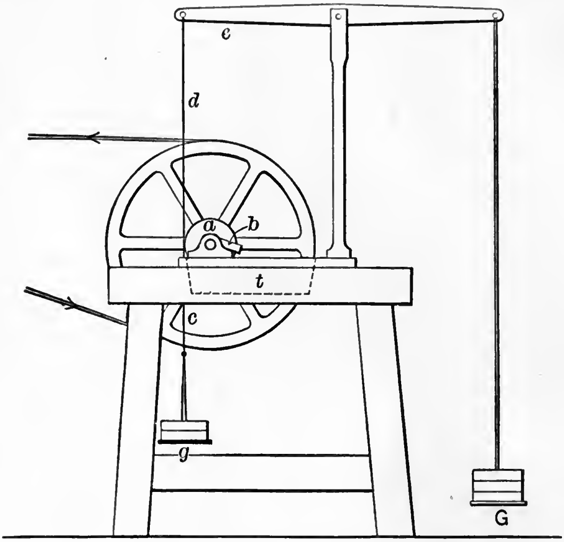


Fig. 20. — Raffard Dynamometer.

the bar is just equal and opposite to the pull due to the weight *g*, and the friction. When equilibrium is thus obtained, the energy absorbed per revolution is equal to the difference between the weights *G* and *g* multiplied by the circumference of the friction wheel, which is very nearly $\frac{1}{3}$ m. If the weights *G* and *g* are in kilograms, the number of kilogrammeters absorbed in a given time equals $\frac{1}{3} \times \text{Revolutions} \times (G-g)$. A trough, *t*, below the friction wheel, is filled with water high enough to be in contact with the under side of the wheel. Show why the belt should run as indicated by the arrows.

Measure the watts supplied to the motor when running without load; also measure the power supplied to the motor, and that absorbed by the dynamometer for several different loads up to full load.

The commercial efficiency of the motor is the ratio of the power delivered from it to the power supplied to it when both are expressed in the same units. Note what is the effect when the position of the brushes is changed.

Plot the results with efficiency and power delivered as ordinates, and power supplied as abscissas. Interpret the curves, and draw conclusions from them.

EXPERIMENT 16. **Efficiency test of a motor without a dynamometer.**

This experiment is closely related to the next, on the efficiency of a dynamo without a dynamometer; hence the following introductory remarks are made to apply to both.

The commercial efficiency of a machine is the ratio of the useful energy delivered from it to the total energy supplied to it. The electrical power delivered from a dynamo, or supplied to a motor, is obtainable in watts from the ammeter and voltmeter readings, and the mechanical power supplied to a dynamo, or delivered from a motor, may be measured by a dynamometer when a suitable one can be obtained. When a dynamometer is not available, another method may be used, which depends upon the fact that the power supplied equals the power delivered plus that expended in other ways than useful work, commonly spoken of as the loss of power. Let the number of watts in these three quantities be represented by W_s, W_d, and W_l, respectively; then $W_s = W_d + W_l$, and efficiency $= \frac{W_d}{W_s}$. From these, the efficiency of a motor may be expressed by the equation $\epsilon_M = \frac{W_s - W_l}{W_s}$, and that of a dynamo by $\epsilon_D = \frac{W_d}{W_d + W_l}$, in which the quantities W

for the motor, and W_s for the dynamo, requiring a dynamometer for their measurement, have been replaced by their equivalents, which may be obtained without a dynamometer. The total loss, W_l, may be divided into three parts: (1) Loss in heating the conductors of the machine by the current (w_c); (2) loss in the iron due to hysteresis and foucault currents (w_i); (3) loss due to friction (w_f). The first depends upon the current flowing, and the second and third upon the speed. Since $W_l = w_c + w_i + w_f$, the formulas above may be written as follows:—

$$\epsilon_M = \frac{W_s - w_c - (w_i + w_f)}{W_s}, \text{ and } \epsilon_D = \frac{W_d}{W_d + w_c + (w_i + w_f)}.$$

The method of applying these general formulas for efficiency to particular cases is indicated in the following directions for this experiment, and in those given for the next.

The values of the quantities in the formula

$$\epsilon_M = \frac{W_s - w_c - (w_i + w_f)}{W_s}$$

may be obtained as follows: $W_s = IE$, in which I is the total current through the motor, and E is the difference of potential at the terminals. To obtain w_c, it is necessary to have the armature resistance, R_a, and the resistance of the field coils, R_f. These may be readily found by the fall of potential method. If the armature and field currents are represented by I_a and I_f respectively, $w_c = I_a^2 R_a + I_f^2 R_f$; which, for a series motor, may be changed to $w_c = I^2(R_a + R_f)$. For a shunt motor, $\left[I - \frac{E}{R_f}\right]$ may be substituted for I_a, and $\frac{E}{R_f}$ for I_f; then

$$w_c = \left[I - \frac{E}{R_f}\right]^2 R_a + \frac{E^2}{R_f}.$$

This formula may also be used for a differentially wound long-shunt motor if the resistance of the series coils is added to R_a.

When the motor is run with no load, the efficiency is zero, and $W_s' - w_c' = w_i + w_f$. Hence $(w_i + w_f)$ may be obtained from the values of I and E observed for zero load. These may be represented by I' and E' to distinguish them from values when the motor is loaded. Since $(w_i + w_f)$ depends upon speed, values should be obtained for different speeds between those for no load and full load. Plot a curve with $(w_i + w_f)$ as ordinates, and speeds as abscissas. From it the sum of these two losses for any speed within the ordinary range may be obtained for use in determining the efficiency from the formula.

From the equations above it follows that the data to be observed include R_a, R_f, I', E', and the corresponding speeds S', from which the curve for losses in iron and friction is obtained; also I, E, and speed S, for several different loads up to full load. W_s and w_c are computed for different values of I, E, and S, and the corresponding values of $w_i + w_f$ are taken from the curve. These substituted in the formula give the efficiencies for different loads on the motor.

The results may be plotted with w_c, S, $(w_i + w_f)$, W_d, and ϵ_M as ordinates, and W_s as abscissas; or other ways of representing them may be used if preferred. Interpret and discuss the curves.

Give an opinion, stating reasons, as to whether any part of the method of determining the efficiency is objectionable or involves error.

EXPERIMENT 17. **Efficiency test of a dynamo without a dynamometer.**

The method used in performing this experiment is similar to that in the preceding one on testing a motor. Wherein they are alike, and in what respects they differ may be seen by comparing the explanations and directions given for the motor test with the following summary in which the same notation is used.

The dynamo is run at or near normal speed; and current, difference of potential, and speed are observed as in taking data for an

external characteristic. From these data are obtained W_d and w_c in the formula for efficiency as given in the introductory remarks. $\epsilon_D = \dfrac{W_d}{W_d + w_c + (w_i + w_f)}$; in which $W_d = IE$, and $w_c = I(R_a + R_f)$ for a series dynamo, or $w_c = \left[I + \dfrac{E}{R_f}\right]^2 R_a + \dfrac{E^2}{R_f}$ for a shunt dynamo. For a long-shunt compound dynamo, use the latter with the resistance of the series field added to R_a.

The dynamo is also run as a motor without load at the same speed that it ran as a dynamo. Since no useful work is done by the motor, the readings for difference of potential and current give the total loss for that speed (W_l). Subtracting the loss in the conductors (w_c) due to the current leaves the loss ($w_i + w_f$) due to hysteresis, foucault currents, and friction. If the speed varied when running as a dynamo, values of ($w_i + w_f$) for different speeds should be obtained and plotted in a curve to be used the same as in testing a motor.

Substitute in the formulas, and compute the values of efficiency corresponding to the different values of I and E observed; or, if the speed of the dynamo is constant, the characteristic may be plotted, and the efficiency computed for different values of output corresponding to points at desired intervals along the characteristic.

The results may be represented graphically by plotting curves with w_c, $(w_i + w_f)$, W_s, and ϵ_D as ordinates, and W_d as abscissas, unless other ways are preferred. Interpret and discuss the curves.

EXPERIMENT 18. **Efficiency test of a motor with a cradle dynamometer.**

In this experiment much depends upon the proper adjustment of the motor on the dynamometer. See that the axis of rotation of the armature is in line with the knife-edges, and that the center of gravity of the system supported on the knife-edges is below the axis, and near enough to it to make the dynamometer sufficiently sensitive.

The diagram of apparatus and connections shown in Fig. 21 represents a differentially wound motor, *M*, to which current is supplied from a compound dynamo, D_1, the electromotive force of which may be maintained constant. Another dynamo, D_2, is used as a variable load on the motor.

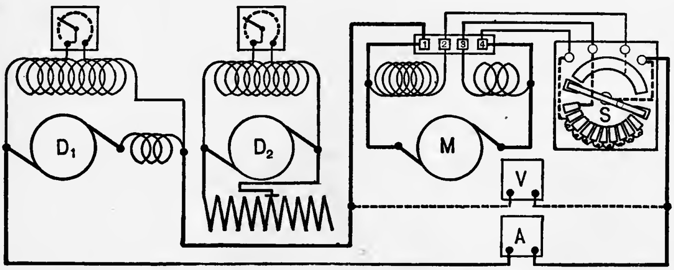


Fig. 21.—Apparatus and Connections for Efficiency Test of a Motor.

Note that the series field coils of the motor are not thrown into the circuit until the sliding contact of the starting box, *S*, is on the last block. Explain why; also explain how regulation for nearly constant speed is obtained by use of the series coils.

In testing a compound or shunt-wound motor, the difference of potential at the terminals should be kept constant, at the normal value for which it is constructed, by turning the regulator in the field circuit of the generator. See that the brushes are properly set, and adjust them so as to give the slowest speed and the least sparking. Leave them in the same position throughout the test.

The dynamometer should be carefully balanced with the belt off, and the motor at rest; then start the motor without the belt, and find whether the dynamometer is still balanced or not. If not, seek for the cause. Friction of bearings, hysteresis, or foucault currents cannot affect the dynamometer reading in testing a motor, for the energy consumed by them enters the machine as electrical energy, which does not tend to tip the system supported on the knife-edges. In testing a dynamo, mechanical energy to overcome them is supplied through the belt, and is measured by the dynamometer.

If a counter-shaft is used, that may be taken alone as the first load, and the dynamo D_2, on open circuit, as the second. Then for larger loads, close the external circuit of D_2 through suitable resistance, and vary the load, thus increasing the current in each of the machines till the safe current limit in one of them is reached. Arrange the resistance so that the part not cut out as the current increases will not get too warm.

The load may be varied by turning the regulator in the field circuit of D_2, as well as by varying the external resistance, since it varies as the product of current and potential difference.

The readings to be taken for each different load are speed, difference of potential, and current for the motor, and the dynamometer reading. The product of potential difference and current gives the number of watts supplied to the motor. The dynamometer reading corrected for speed gives the number of watts delivered from the motor. The ratio of the latter to the former is the commercial efficiency.

Plot a curve with efficiency as ordinates and current as abscissas; and as many others as may be desired, or profitable, to show the relations between the different quantities involved, or that may be derived from the data.

Interpret the curves expressing the results, and indicate their uses.

Experiment 19. **Efficiency test of a dynamo with a dynamometer.**

This experiment is similar to a shop test of a dynamo, but of shorter duration, lasting only an hour or two instead of several hours. A dynamo that will run safely at a given current for a short time may heat excessively by a long run with the same current. If the dynamo runs without undue heating for a long time at the load for which it was designed or constructed, that is taken as the normal load of the machine in rating its capacity. A determination of efficiency is made only for normal load and conditions of running, unless, for some special cases, a different

load, or other conditions, are preferable. The output and speed are maintained as constant as possible throughout the test.

Before beginning the run, measure the cold resistances of the armature and field circuit. If the fall of potential method is used, avoid heating, by not using too large currents, and by letting the current flow only long enough to take good readings. The dynamo should be carefully adjusted on the dynamometer so as to have equilibrium when the belt is off, and the weights are at the zeros.

When the dynamo is started, take a dynamometer reading with both field and armature circuits open. This, corrected for speed, gives the power consumed in friction. If the field is then separately excited to its normal value, hysteresis and foucault currents will be produced, causing a higher dynamometer reading. Subtract the first reading from this to get the power consumed in hysteresis and foucault currents. Take similar readings at the end of the run; also the approximate temperature of the armature by the use of a thermometer.

During the main run, take readings for current, difference of potential, and speed at convenient intervals. For the first one-third of the run, while the machine is warming up, readings need not be taken so frequently as for the remainder. The mean values of the readings for the last two-thirds of the run are used in filling out the following outline, in which are indicated the principal data and the computed results to be obtained.

Data and Results of Efficiency Test.

Test of____

Date of test,______________________. Duration of run,_________________

Kind of dynamo,____

Field,____

Data on winding,_______________________________________

Armature,____

Data on winding,_______________________________________

Dimensions,____

Commutator segments,____________________________________

Data from Tests.

Resistance of armature: cold, ________________; hot, ________________

" " series field, " ________________; " ________________

" " shunt " " ________________; " ________________

Extra resistance in shunt field circuit, ________________________________

External resistance, ____

Temperature of armature before run, ____________; of room, ______________

" " " at end of run, __________; " " ______________

Current: in armature, ________________; in shunt field, ________________

" in external circuit, ______________; in series field, ______________

Potential at terminals, _____________. Revolutions per min., _____________

Power consumed, measured by dynamometer, __________________________

" " in separately excited field, ________________________

" " total, watts (W), ________________________________

" " in friction (w), ________________________________

" " in hysteresis and foucault currents, ___________________

Computed Results.

Electrical power, external $(W_{E.E.})$, ______________________________

" " in self-excited series field, __________________________

" " " " shunt " __________________________

" " " armature, ________________________________

Total electrical power $(W_{T.E.})$, ________________________________

Efficiency of conversion, $\frac{W_{T.E.}}{W - w}$, ________________________________

Electrical efficiency, $\frac{W_{E.E.}}{W_{T.E.}}$, ________________________________

Commercial efficiency, $\frac{W_{E.E.}}{W}$, ________________________________

Temperature at end of run, computed from hot and cold resistance,

Armature, ___________. Series coils, ___________. Shunt coils, ___________.

Sometimes it is desirable to obtain a curve showing the variation of efficiency with change of output, especially for some machines with automatic regulation. Such a curve may be obtained as a continuation of this experiment or taken as a separate experiment.

Vary the output, or power in the external circuit, from no load to full load, and take data from which each different output and the corresponding total power supplied may be obtained and used to find the commercial efficiency, which is plotted as ordinates, with output as abscissas. Show what is indicated by the curve, and what use may be made of it. If a like curve has been obtained for the same machine by the method used in Exp. 17, the two may be compared and discussed.

EXPERIMENT 20. **Reversing motor.**

When a dynamo and a motor are connected under certain conditions, the armature of the motor refuses to revolve continuously, but turns a few revolutions in one direction and then in the other, and so on, back and forth.

The object of this experiment is to ascertain these conditions, to note carefully all of the phenomena taking place, and to explain fully the peculiar action observed.

The apparatus is to be arranged as follows: Connect the terminals of a small series-wound dynamo, which is to be used as the generator, with the brushes of another small dynamo, which is to be the motor, and if necessary put some resistance in the circuit, so that no part will be liable to be burned out. Separately excite the field of the motor; the amount of excitation need not, however, be very great. The brushes of the motor had better be of carbon; but if they are of metal, their form should be such that there will be no liability of their catching when the commutator turns towards them.

A quick-reading ammeter, which will read right and left, thus showing the direction as well as the amount of the current, should be placed in the generator circuit, and a quick-reading voltmeter should be connected across the brushes of the motor: this should also read right and left. Another ammeter, a direct-reading one, should be placed in the field circuit of the motor, and a compass needle should be held near one of the poles of the generator. The diagram, Fig. 22, will show how the con-

nections are to be made. *A* is the generator, *B* the motor, and *C* the exciting dynamo. The voltmeter is shown at *d*, and the ammeters at *e* and *f*.

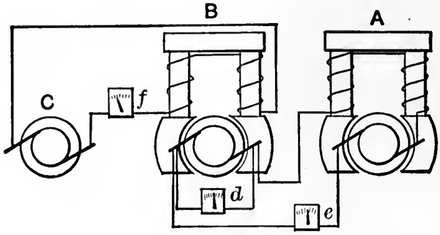


Fig. 22.

Carefully observe the movements of these instruments with reference to the movements of the armature of the motor. Observe the effect of putting upon the motor, while it is going in either direction, a small load by means of friction; also the effect of breaking the field circuit of the motor.

EXPERIMENT 21. **Study of arc-lamps.***

The lamp to be studied is first to be examined, and a diagram is to be made of the circuits and of the operating mechanism. Designate in this the winding of the magnets, whether series, shunt, or compound wound. Then a current is to be sent through in the proper direction, and adjusted to the specified amount. The movements of the parts are to be observed while starting and stopping the current, also while the feeding device is in operation. Explain the use of these different parts.

Examine the glowing carbon points through a smoked glass; also produce their images on a distant screen by means of a lens. Prove, by finding the direction of the current, that the positive carbon gives out the most light.

---

* See Arc-Lamps and their Mechanism, by S. P. Thompson, London Electrician, Vol. 22, p. 534. Also Electrical Engineering, Chap. XV., Slingo and Brooker.

From references cited answer questions as to

(*a*) Shape and temperature of points.
(*b*) Counter electromotive force of arc.
(*c*) Length and resistance of arc.
(*d*) Rate of consumption of + and − carbons.
(*e*) Effect of foreign substances in the pencils.
(*f*) Effect of copper coating.
(*g*) Candle power, its quality, quantity, and probable distribution.

Fill out the following schedule:

(1) Name of lamp.
(2) Long or short arc.
(3) Nature of supply.
(4) Current, potential difference, and watts.
(5) Resistance of parts.
(6) Driving power mechanism.
(7) Striking mechanism.
(8) Feeding mechanism.
(9) Moderating mechanism.
(10) Replacement mechanism.
(11) Focusing mechanism.
(12) Change over mechanism.
(13) Cut out mechanism.

Definitions of terms used in the above schedule.

By the supply is meant whether the kind of lamp examined is intended to be operated by a continuous or alternating current, and whether constant current or constant potential. The resistance of series coils range from 0.05 to 0.2 ohm, and of shunt coils from 200 to 400 or 500 ohms. The driving power is that which propels the carbons forward as they burn away. The striking mechanism touches the carbons together if they are apart, then separates them, and so starts the arc. The feeding mechanism relates to those parts by

which the driving power propels the carbons forward. The moderating attachment prevents too sudden movement of parts. The replacement device allows the carbon-holders to be pushed wide apart to admit new carbons. Both carbons need to be fed forward, but at different rates, if the luminous point is to be kept at a fixed position in space, as at the focus of a lens. The change over mechanism brings into action a second pair of carbons after the first are burned away. The cut-out completes the circuit around a lamp when it refuses to operate.

EXPERIMENT 22. **Determination of the constants of a tangent galvanometer from its dimensions.**

The great tangent galvanometer * has three pairs of coils arranged after the Helmholtz form, as shown in Fig. 23. Two pairs have a single turn each of large copper rod about 1.9 cm. in diameter, and are designated by the numbers 1, 4, and 2, 3. The largest pair are about 2 m. in diameter.† Each of the third pair, marked *w*, *w*, has two coils, of eighteen turns each, interwound so that they have the same mean radius, making thirty-six turns in each groove, wound six wires wide, and six deep, as shown by the enlarged section in Fig. 23, in which the cross-sections of the wires composing

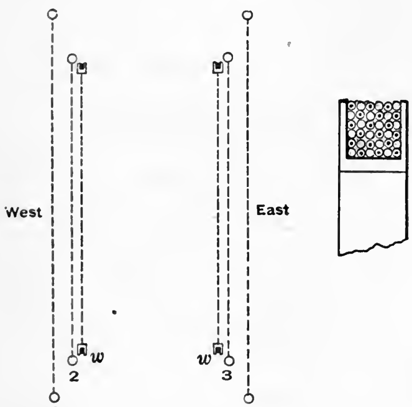


Fig. 23.

* An illustrated description of this galvanometer is given in The Electrical Engineer (N. Y.), Vol. 4, October, 1885, by Professor W. A. Anthony.

† Coil 1 originally belonged to the galvanometer used in making tests of dynamos at the electrical exhibit in Philadelphia in 1884. Material for the other three could not be obtained of exactly the same size as coil 1; hence the slight difference in the diameter of rod.

one coil are marked with a dot, and the remaining ones belong to the other.

Measure three or more diameters of each ring, from outside to outside, by means of the brass tube with adjustable collars provided for the purpose, and a meter scale.

Also measure, in several places, the horizontal distance between each pair of coils. Measure with micrometer calipers the diameter of each of the rods of which the four large rings are made. From these measurements, the mean radius of each coil, and the mean distance between each pair of coils, may be obtained. In taking measurements, avoid disturbing any adjustments of the galvanometer.

Derive the general formula *

$$I = \frac{H \tan \delta}{\dfrac{2\pi r_1^2 n_1}{(r_1^2 + x_1^2)^{\frac{3}{2}}} \pm \dfrac{2\pi r_2^2 n_2}{(r_2^2 + x_2^2)^{\frac{3}{2}}} \pm \text{etc.}} = K \tan \delta,$$

and compute from it the following constants, the coils or combinations for which they are obtained being denoted by the subscripts: K_1, K_2, K_3, K_4, K_{1+4}, K_{2+3}, $K_{1+2+3+4}$, $K_{(2+3)-(1+4)}$, K_{2-1}, K_{1w}, and K_{4w}. The plus sign indicates that the coils act together, and the minus sign that they oppose, tending to produce magnetic fields in opposite directions. K_{1w} and K_{4w} represent the constants for one wound coil and four wound coils in series respectively. Assume H to be 0.1710.

It is well to put the computations in the report. Tabulate the data, and the computed results for both the C. G. S. and the practical units. Give also the logarithms of the constants, since they are needed to add to the logarithms of the deflections in order to obtain the logarithms of the currents.

---

* For a derivation of this formula, see The Galvanometer by Edward L. Nichols, Electric Power, January, 1894; also in notes on electrical measurements in "The Crank" for November, 1891 (later called The Sibley Journal of Engineering).

EXPERIMENT 23. **Use of great tangent galvanometer and a verification of its constants.**

The galvanometer measures currents from a very small fraction of an ampere to more than two hundred amperes. The light from the scale falls on an inclined mirror which throws it upwards to a totally reflecting prism. This reflects it through the observing telescope, as shown in Fig. 24. Note that this arrangement gives a scale reading which measures the real

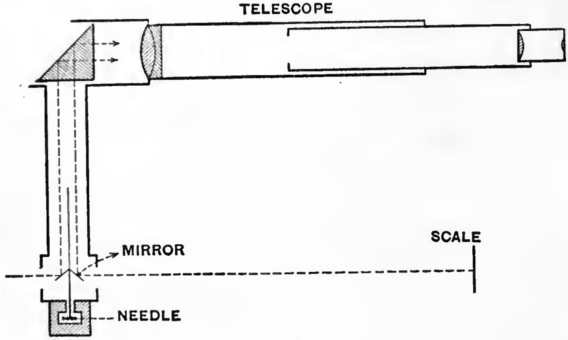


Fig. 24.

deflection instead of double the deflection of the needle, as is the case with ordinary mirror galvanometers. The logarithms of the tangents of the deflection angles corresponding to each scale division are tabulated for use with the instrument. Each large division of the circular scale is one-fourth inch, which is divided into ten parts. A deflection of 45° is equivalent to 78.614 scale divisions.

The four coils 1, 4, and 2, 3, are connected with a switchboard of massive bronze shown in diagram in Fig. 25, by means of which any coil can be used by itself, or any two, or three, or four, in series, directly or differentially. To connect in coil No. 1, plug the block marked 1 to the outside ring, and block 1′ to the center (or the reverse). Current in the coil is reversed by throwing the lever * of switch No. 1. Coils 2, 3,

* Each of the six switches has a lever like that shown for the cut-out switch where the current enters from the line.

and 4 used alone will deflect the needle in the same direction as coil 1 if the blocks with primed numbers are connected to the center in each case, and the switch-levers are thrown to the same side.

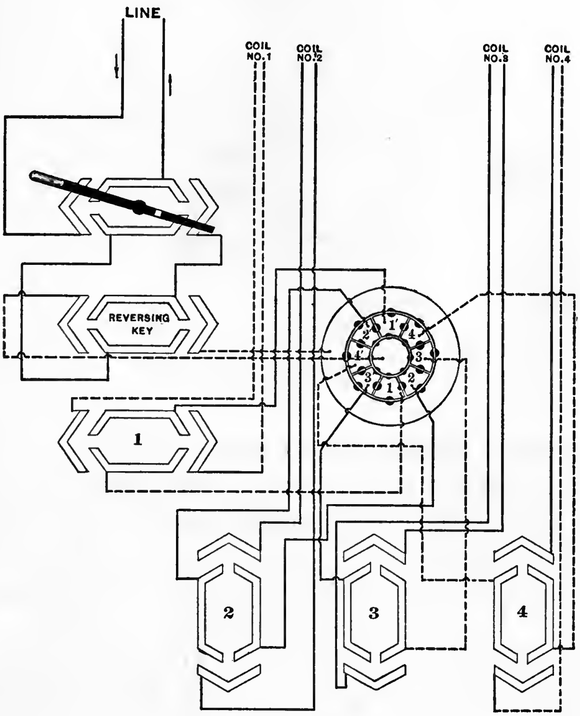


Fig. 25.

To check the values of the constants computed in the previous experiment from dimensions, they may be used in measuring, with the different coils and combinations, the same constant current obtained from some suitable source. If the measurements do not agree, the cause should be sought for, and eliminated if due to error in data or computations, or to lack of proper conditions of measurement.

EXPERIMENT 24. **Conditions for maximum sensitiveness of great tangent galvanometer.***

This experiment is to determine the current limits beyond which greater sensitiveness of the great tangent galvanometer may be secured by changing from one coil or combination to another.

Let α and α' represent the angles of deflection for which the sensitiveness of one of two combinations, having the constants k and k', is the same as that of the other for the same current. Then

$$i = k \tan \alpha = k' \tan \alpha', \text{ and } \frac{d\alpha}{di} = \frac{d\alpha'}{di}.$$

From these equations derive

$$\frac{k'}{k} = \tan^2 \alpha = \operatorname{cotan}^2 \alpha',$$

or an equivalent expression from which α and α' may be found. The scale division or current corresponding to the angle α gives the limit beyond which that combination is either more or less sensitive than the other.

The maximum single deflection that can be read on the scale is 58°. Each large scale division is 0.01 of a radian. Each degree is 1.747 large scale divisions. The logarithmic tangents of the angles corresponding to each scale division are tabulated in a book for use with the galvanometer.

Determine for each constant the corresponding limits of current within which no other combination would be more sensitive; also find the deflections for these limits in scale divisions as well as degrees. The results may be tabulated as indicated in the following table; in which the mean of two constants having very nearly the same values (as K_2 and K_3) may be used.

---

* See Experiment 22.

BEST LIMITS FOR CURRENT AND DEFLECTION.

Constants.		From			To		
			Deflections.			Deflections.	
Symbols.	Values.	Amperes.	Scale Div.	Degrees.	Amperes.	Scale Div.	Degrees.
K_w	—	—	—	—	—	—	—
$K_{1+2+3+4}$ . .	8.4568	—	—	—	11.35	93.12	53° 18′ 10″
K_{2+3} . . .	15.225	11.35	64.11	36° 41′ 50″	17.02	84.18	48° 11′ 4″
K_{1+4} . . .	19.024	17.02	73.05	41° 48′ 56″	—	—	—
K_2 or K_3 . .	—	—	—	—	—	—	—
K_1 or K_4 . .	—	—	—	—	—	—	—
$K_{(1+4)-(2+3)}$.	—	—	—	—	—	—	—
K_{1-2} . . .	—	—	—	—	—	—	—

Show for what deflection the sensitiveness of a tangent galvanometer is a maximum, and note its relation to the limiting deflections above.

EXPERIMENT 25. **Determination of H by the tangent galvanometer method.**

The galvanometer to be used is one having a single large copper coil at the center of which is supported the short needle carrying two fibers of glass at right angles, forming four pointers which move over the graduated dial of the compass-like box enclosing the needle.

After obtaining a clear understanding of the equation for a tangent galvanometer, make the measurements necessary for determining the constant; repeat and take the average; then pass a current, not exceeding 16 amperes, through the galvanometer, and also through a Thomson balance, or any standard ammeter by which the current may be accurately measured. The connections are shown in Fig. 26.

Take readings for current and galvanometer deflections, then reverse the current through the galvanometer, and read again. If the readings do not agree, seek for the cause. Take

observations for several values of current distributed over the range of the galvanometer.

Having obtained the above data and substituted in the formula, the value of H is given by the equation. Compute H for each different current and corresponding deflection, and take the average.

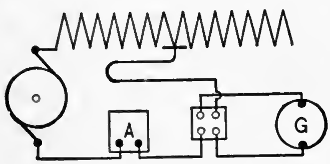


Fig. 26.

Using this value of H and the galvanometer constant above, the deflections corresponding to different currents may be computed from the formula, and a curve drawn showing them. As a check, the observed values may be plotted on the same sheet to the same scale.

If the curve for the computed values does not coincide with the one for the observed, the observations not giving points on or near a smooth curve may be assumed inaccurate, and the average of those remaining used to obtain H.

Explain what should be the position of the ring, and how to determine whether it is in that position or not; also what would be the effect on the value found for H if not properly adjusted, and what are probably the sources of greatest error in the determination.

EXPERIMENT 26. **Computation of the resistance necessary to render a potential galvanometer direct reading [from dimensions].**

The instrument is a tangent galvanometer of the Helmholtz type, having 64 turns of wire, No. 23 B. & S. gauge, in each of the two coils, which for this experiment are connected in series. The mean radius of the east coil is 82.38 mm., and of the west coil 82.57 mm. The scale, 120 cm. long, with centimeter and millimeter divisions, is placed a little less than $2\frac{3}{4}$ m. from the mirror. To measure this distance, place one end of a long pole against the scale, and let the other end pass by the side of the vertical tube around the needle. Mark on the pole the point

at which it is tangent to the tube. Be careful not to jar or move the galvanometer. Remove the pole, and measure the length marked. Avoid using steel scales or magnetized instruments of any kind near the needle.

It is required to find what resistance must be placed in series with the galvanometer in order that any given number of volts will give the same number of centimeters, double deflection. The instrument is not to be connected or used at all in this experiment. Assume some convenient value of potential difference, and 0.1710 as the value of H at that place. By use of the formula for the tangent galvanometer, find what current will give the deflection corresponding to the assumed difference of potential; then find the resistance through which the difference of potential will give that current. Taking the resistance of a mil-foot of copper wire to be 10.8 ohms, obtain approximately from dimensions the resistance of the galvanometer coils. It may thus be shown that their resistance is negligible in comparison with that to be added.

Compute the resistance for two or more differences of potential. Explain why the resistance for accurate direct reading must be slightly increased as the difference of potential increases; and why the increase per volt is greater for high values of potential difference than for low. Show whether this is small enough to be negligible or not.

Experiment 27. **Determination of magnetic dip.**

To find the magnetic dip, a piece of apparatus called Barrow's Circle may be used. This has a perfectly balanced needle supported upon a horizontal axis which rolls upon agate planes. The needle, being magnetized, is to be placed in the plane of the magnetic meridian, and then its inclination is to be found by setting the cross-hairs of a microscope upon one of the points and reading the circle by means of the verniers at the two ends of the arm carrying the microscopes, then setting the other microscope and reading again. A large num-

ber of such readings are required to eliminate the principal errors and make a complete set from which to obtain the final result.

The axis of the needle is to be supported in a pair of Y's at all times during the experiment except while a group of readings are being made.

To stop the vibrations of the needle, it should be raised and lowered several times; this will also put it in the right place on the agate planes. The body of the instrument turns upon an axis which can be made accurately vertical by means of leveling screws and a spirit level, the agate planes being horizontal when the axis of the instrument is vertical.

To put the needle in the magnetic meridian, proceed as follows: Turn the plane of the circle to such a position that the lower point of the needle is seen in the center of the field of the lower microscope, which is set at 90°; read the horizontal circle, then set the upper microscope at 90° by its vernier, and swing the instrument around till the upper end comes into the center of the field, then read the horizontal circle again. Now reverse the needle in its bearings and repeat, then turn the face of the instrument around 90° to the east from the mean of the four readings.

After putting the circle in the magnetic meridian, a group of eight readings may be made as follows (see blank form): After adjusting the lower microscope upon the needle, read the upper and lower verniers (these readings are to be recorded in *a* and *b*); then adjusting the upper microscope will in a similar manner give readings *c* and *d*. Now lift the needle from the agate planes and put it down again; then repeat the readings, recording them in *e* to *h*. After these are taken, turn the face of the instrument 180° to bring it to the west, and take the next group, as indicated in the blank form. After the four groups are taken, the polarity of the needle is to be changed by remagnetizing it. To do this, place it in the wooden block provided for that purpose, putting the

Magnetic Dip.

Station, ________________ . Date, ________________ , 18___

Circle No. ________________ . Needle No. ________________

Setting of azimuth circle, ________________

Remarks, ________________

Time, __________ to __________ | __________ to __________

Face of Needle	Face of Inst.	Poles direct, with — pole dipping. Lower End.	Upper End.	Mean.	Poles reversed, with — pole dipping. Lower End.	Upper End.	Mean.
		Readings of Needle.			Readings of Needle.		
		° ′	° ′	° ′	° ′	° ′	° ′
Face of Needle to Face of Instrument.	East.	*a*	*c*				
		b	*d*				
		e	*g*				
		f	*h*				
		Mean $= \alpha'$.			Mean $= \beta'$.		
	West.						
		Mean $= \alpha''$.			Mean $= \beta''$.		
Face of Needle Reversed.	West.						
		Mean $= \alpha'''$.			Mean $= \beta'''$.		
	East.						
		Mean $= \alpha''''$			Mean $= \beta''''$		
		α'''			β'''		
		α''			β''		
		α'			β'		
		Mean of Means $= \alpha$			Mean of Means $= \beta$		
					Mean of Means $= \alpha$		
					$\frac{\alpha+\beta}{2}$ Dip $=$		

Observer,

brass cap over the axis to protect it from injury; then take two bar magnets, one in each hand, and place their unlike poles on the needle near the cap, while their outer ends are slightly elevated. Draw them outward and repeat several times. Turn the needle over and treat it again in the same manner, then replace it in the instrument and take the four groups.

After computing the final result, discuss the experiment, showing how some of the errors are eliminated.

INTRODUCTORY TO THE CALIBRATION OF INSTRUMENTS.

Instruments for measuring currents, differences of potential, and other electrical quantities, frequently from use or abuse change their readings, so that considerable errors are introduced into the quantities observed. Especially is this true of that class of instruments in which permanent magnets are employed. Also new instruments are often thrown slightly out of adjustment by rough handling while on the way. From these and other causes it is frequently necessary to restandardize them. This is usually done by direct comparison with other instruments which are accepted as being correct, and the method ordinarily employed is to measure the same electrical quantity by both instruments, then change the amount a dozen or more times, so that a number of different points along the scale may be secured. These observations are to be used in plotting a curve, from which the correct value for any point on the scale of the restandardized instrument may be found. Then if desired, a table of readings may be made from the curve.

EXPERIMENT 28. **Calibrating an ammeter.**

The ammeter is to be connected in series with the instrument which is to be taken as the standard; then a steady current is to be sent through them, and both are to be read at as nearly the same time as possible, so that there is not likely to be any change of current between the readings. Several different

amounts of current are to be used, so that enough points will be obtained to accurately locate the curve which is to be plotted.

To avoid the wasting of energy through high resistances while calibrating, it is often advisable to run the dynamo at as low a potential as possible, provided that potential is not too low to cause the required current to be maintained.

EXPERIMENT 29. **Calibrating an electrodynamometer.**

An electrodynamometer is based upon the principle of the attractions and repulsions of currents flowing in adjacent wires, and hence is suitable for the measurement of alternate as well as direct currents, because whatever change takes place in one part of the circuit through the instrument the other parts suffer a like change, and so the force produced always tends to move the parts in the same direction. This force is usually opposed by means of a spring, as in the Siemens dynamometer (see Slingo and Brooker, p. 110), or by means of weights, as in the Thomson balance.

If a direct current is used in calibrating, the process is similar to the calibration of a direct current ammeter, and for a standard a direct current ammeter may be taken; but if an alternate current is to be used, the standard will have to be some kind of alternate current ammeter, such as a Thomson balance.

After the curve has been plotted, see if the relation of forces applied and currents flowing conform to theory.

EXPERIMENT 30. **Calibrating a voltmeter.**

The voltmeter to be calibrated is to be connected in multiple with the voltmeter or potential instrument, which is to be taken as the standard, and the difference of potential of their terminals is to be changed by approximately equal steps throughout the range of the instrument being calibrated. With each change both instruments are to be read as accurately as possible, estimating to tenths of divisions. If either instrument

has a contact key, it must be observed whether the closing of this has any effect upon the other, and if such an effect is noticed, the key should be closed when the readings of both are being made, otherwise an error will be introduced. If the instrument being calibrated has a right and left deflection, depending upon the direction of the current, a reversing key should be placed in circuit with it.

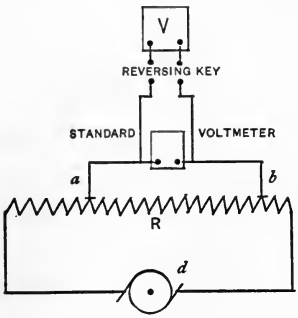


Fig. 27.

To easily obtain the steps for the readings, connections may be made to a dynamo or battery, as shown in Fig. 27; *d* is the source, or supply, which sends a current through the resistance *R*. The terminals *a* and *b* of the instruments are to be connected to two points upon this resistance which have the desired difference of potential. Increasing the distance apart of the terminals increases the deflection. After the readings are obtained, a curve should be plotted.

Experiment 31. **Constants of a graded ammeter.***

The Thomson graded ammeter is an instrument of the tangent galvanometer type, but it differs in this respect that it has a controlling magnet. The coil, consisting of one or more turns of a thick copper strip, is fastened to one end of a small wooden platform, which has a V groove that is perpendicular to the plane of the coil. A metal box, containing the needle and scale, and supporting the magnet, is arranged to slide along this groove, so that the distance of the needle from the plane of the coil may be varied at will. By varying this distance the effect upon the needle is varied, thus varying the deflection for the same current. The effect will vary as $\frac{r^3}{b^3}$, where r is the

* See Electrical Engineering, by Slingo and Brooker.

radius of the coil, and b the mean distance between the coil and the needle.

The needle, like in a tangent galvanometer, may be deflected to the right or to the left, according to the direction of the current. There are certain lines on the wooden platform, crossing the groove at right angles. When the curved part of the needle box is placed tangent to the line marked 1, a certain current should deflect the needle one division to the right or left. The value of that current in amperes is called the constant of the instrument. For any of the marked positions, the constant $= \frac{I \times P}{D}$, where I equals the current, P the position, and D the deflection.

The current is to be measured by means of a standard instrument or a voltmeter.

A table of constants for right and left deflections at each position should be made, and the mean constant of all the positions should be obtained.

EXPERIMENT 32. **Constants of a graded voltmeter.***

The graded voltmeter is similar to the graded ammeter, except that instead of a coarse coil of few turns it has a fine coil of many turns and high resistance.

By connecting it to a circuit, and putting it in multiple with a standard voltmeter, readings for the various positions may be taken. A reversing switch will be necessary to make the instrument deflect to the right and to the left.

The formula is the same as for the ammeter, except that instead of I for amperes, write v for volts. A table of constants for right and left deflections and the mean constant should be obtained as in the case of the ammeter.

EXPERIMENT 33. **Reliability test of a voltmeter.**

When a voltmeter is connected between two points which have a difference of potential, it will, if it is within the range

---

* See Experiment, Constants of a Graded Ammeter.

of the instrument, give a certain reading. If the voltmeter is not very reliable, this may be greater or less than the true amount.

The following experiment may be performed to ascertain if the needle will always return to the same reading for the same voltage, or whether it is liable to vary. Connect the voltmeter and two resistances in series, and through a reversing switch to a storage battery which should deflect the needle to some point well along its scale. Let S in Fig. 28 be the storage battery, and R and R' the resistances. There should be three mercury cups, a, b, and c, connecting the resistances to each other and to the line as shown. Then by putting in a wire staple across from a to b, the resistance R will be shunted out. On account of this the needle will be deflected to some higher reading, which, for convenience, we will call x; this should lie in that part of the scale which it is desired to test. By shunting out R' for a moment, the needle will be deflected farther along the scale; then it will, when R' is restored, return to x or some point near it.

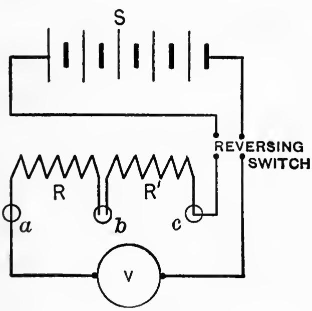


Fig. 28.

By using the two staples and the reversing switch, the needle may be made to shift from the position x by a number of different combinations, but it should come back to exactly the same reading each time the circuit is restored to the original condition which gave x in the first place. If the needle should fail to return to exactly the same reading, there will then be several values for the same potential, and these may be treated by the method of least squares to find the probable error of a single observation.

EXPERIMENT 34. **Reliability test of an ammeter.**

The reliability of an ammeter is determined in much the same way as is used in the case of a voltmeter. But instead of

the mercury cups, other forms of cut-out switches should be used; for when large sparks are made in a mercury switch, the vapors produced are very injurious.

The resistances must be small enough to permit the required currents to flow, and the supply of current must be obtained from a constant potential generator, or from a storage battery.

The results are to be worked up in the same manner as in the preceding experiment.

EXPERIMENT 35. **Exploration of the field of a dynamo.**

When the armature of a dynamo is revolving, the difference of potential between the two ends of any one of its coils is continually changing from a maximum value in one direction through zero to a maximum in the other.

These changes are going on at different rates in different parts of the revolution, and are not the same for full load that they are for no load. They can be represented by means of curves constructed from data obtained while the armature is in motion, and by the aid of these curves we are enabled to determine the relative value of different parts of the field, and see whether there is a proper distribution of the lines of force. They also show the best position for the brushes, and the amount of shifting necessary for a given change of load.

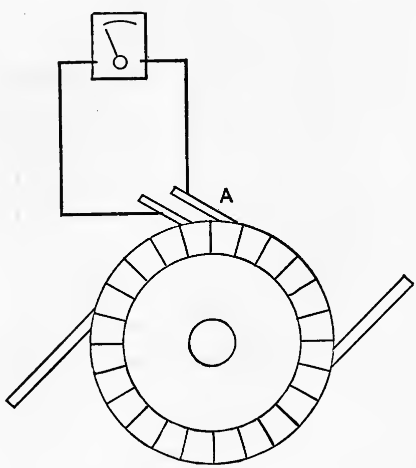


Fig. 29.

A method due to Dr. S. P. Thompson, and which applies to closed coil continuous current dynamos, is as follows (see Dynamo Electric Machinery, p. 68, S. P. Thompson): Two small

brushes are to be fixed to an arm or carrier (see *A*, Fig. 29), with their distance apart equal to the distance between two consecutive commutator bars, and the arm is to be attached to a graduated arc or circle, so that it can be moved 5° or 10° at a time. The brushes of the dynamo, if necessary, are to be moved over to one end of the commutator, so that the small brushes can pass them. For 100 to 150 volt dynamos the readings will probably all come within the range of a voltmeter, with a 15-volt scale, which is to be connected to the small brushes.

A single-brush method, suggested by Mr. W. M. Mordey, can be performed with part of the apparatus used for the first method, but the voltmeter will have to be changed to one having a range equal to the voltage of the dynamo. One of the voltmeter terminals is to be connected to one of the small brushes, and the other to one of the main brushes of the dynamo; then as the small brush is moved around to different positions, the readings will vary according to the distance from the stationary brush.

Two sets of readings are to be taken by each method, one with the dynamo at full load, and the other with no load, and the field excited to give the same potential at the brushes as before. If it is a shunt-wound dynamo, so that only a small current is required for the field, it may be self-excited when running at no load, but otherwise it should be separately excited.

Fig. 30.

Each set of observations is to be plotted with degrees as abscissas, and volt readings as ordinates. Along the axis of abscissas are to be indicated the position and width of the poles, and also the position of the brushes.

Discuss the curves, point out where the field is most active, where least. Explain what they show of the magnetic reactions between the field magnets and armature.

In Fig. 30 are typical curves taken by the two-brush

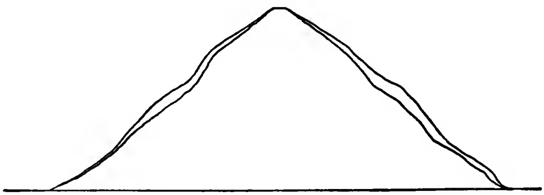


Fig. 31.

method from a shunt-wound dynamo, and in Fig. 31 are curves from the same dynamo by the single brush method.

Experiment 36. **Exploration curves by means of a dynamo indicator.***

This instrument is a portable voltmeter, having a needle which will vibrate very rapidly, and a lightly smoked cylinder against which the pointer of the needle can be made to press at the

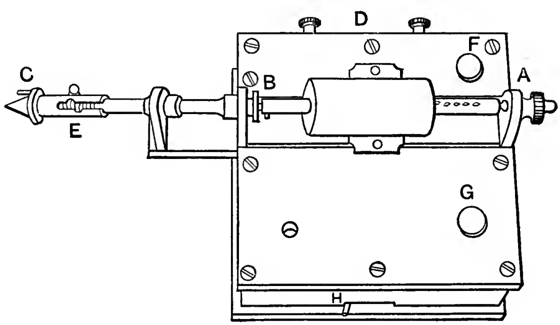


Fig. 32.

required moment. The cylinder is mounted upon a short shaft, and is arranged so that it can be slid to several different positions: a spring dropping into notches holds it at each place. It can also be removed for smoking, or for transferring the curves.

* Moler, "A Dynamo Indicator." Transactions of the American Institute of Electrical Engineers, Vol. 9.

This shaft is connected with the driving shaft of the instrument by projecting pins at *B*, Fig. 32, so that it will always maintain the same relative position with the pin at *C*. The pin *C* is attached to a sliding sleeve, which is pushed toward the point by a spring. A hole or a pin in the end of the armature shaft is to engage with the pin *C*, so that the cylinder will always have the same relative position with the armature. To operate the instrument, two insulated wire loops or bands are wound around some convenient part of the shaft or commutator; one is connected to one commutator bar, and the other to an adjacent one. Small brushes press upon these bands, and are connected to the binding screws at *D*. The cylinder is lightly smoked by removing it and revolving it over a candle or gas flame; it is then placed in position. The sleeve carrying the pin *C* is pulled back and latched in the notch *E*. Then the point is pressed into the center hole in the end of the armature shaft; this will put the drum in motion; then by letting the projecting knob strike the finger the sleeve will be unlatched, and will spring forward and engage with the pin in the end of the armature shaft. Then pressing the button *F* closes the circuit, and pressing *G* brings the pointer against the drum. *H* is a reversing switch. The base line of the curve is drawn by pressing *G* only. Another form of curve will be obtained if one of the terminals of the indicator is attached to one of the permanent brushes of the dynamo, and the other to the small brush pressing upon one of the bands connected to the commutator bars. A suitable resistance must be in series with the instrument in this last case. The curves are transferred from the smoked cylinder by damping a sheet of paper and then rolling the cylinder over it.

In Fig. 33 are shown some of the curves produced by exploring the field of a 20-ampere shunt-wound dynamo running at 1075 revolutions and 115 volts. The curves *a* and *b* were made while 20 amperes were flowing, and *c* and *d* while the outside circuit was broken. The curves *a* and *d* show the

changes in potential of the two ends of a single coil, and *b* and *c* between one end of a coil and one of the dynamo brushes. The line *g* shows the position of the center of the

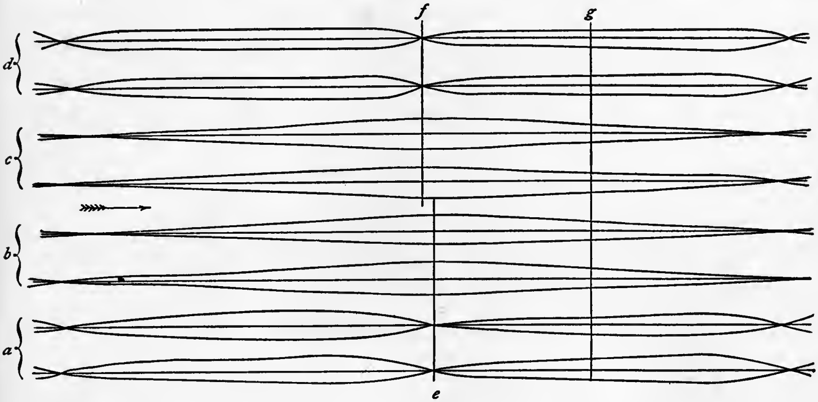


Fig. 33.

pole-piece, and the arrow the forward direction of the curves. The line *e* shows the position of the brush when 20 amperes are flowing, and *f* when the line is open.

EXPERIMENT 37. **Determination of the coefficient of magnetic leakage in a dynamo.**

Owing to the imperfect nature of the magnetic circuit of dynamos and motors, the flux is by no means uniform throughout. Only a portion of the total flux generated in the magnet limbs is available for the establishment of electromotive forces in the armature conductors; the remainder leaks around by various paths, determined to some extent by the design of the machine. The ratio of maximum to useful magnetic flux is a coefficient of much importance in the calculation of magnetic circuits; it is usually denoted by v.*

In any actual machine this coefficient is best determined by a temporary secondary circuit containing a ballistic galvanom-

* Reference should be made to the papers of Drs. J. and E. Hopkinson, Phil. Trans., May 6, 1886, and Feb. 15, 1892.

eter (Fig. 34). Thus if a single turn of fine wire be placed around the magnet core, and this wire be made part of a circuit containing a ballistic galvanometer and suitable resistance, then on suddenly making the field circuit of the dynamo a throw of the ballistic needle will be produced, and the magnitude of this throw is a measure of the magnetic induction within the field core. On breaking the field circuit a throw

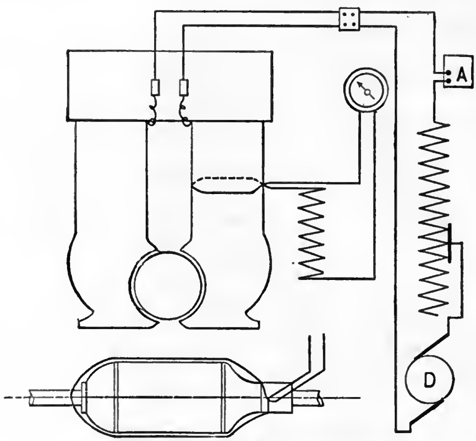


Fig. 34.

will be produced in the opposite sense. Residual magnetism is in this case neglected. The exploring coil should then be wound longitudinally around the armature in such a plane as to include the maximum flux, and deflections of the ballistic needle obtained on making and breaking the field circuit as before. From a large number of observations the mean throw for each position should be obtained; the ratio of these is the value of v sought. The position of maximum induction may not coincide with the mid-length cross-section of the field limb; it should be found by trial. Values of v should be obtained for different field currents.

In lieu of a ballistic galvanometer a low-reading Weston voltmeter may be used with good results.* In this case it will

* See Ives, Electrical World, Vol. 19, p. 12, Jan. 2, 1892.

be necessary to wind more than one turn around the field limb and armature. When a galvanometer is used, the resistance must be the same in the secondary circuit in both of its positions; with a voltmeter small changes in the resistance of this circuit will not influence the result.

The following results will serve to illustrate the two methods as applied to a 10 K. W. generator:

WITH BALLISTIC GALVANOMETER.

Field.			Armature.		
Make.	Break.	Diff.	Make.	Break.	Diff.
0.60	7.1	6.50	2.4	6.8	4.4
0.55	7.2	6.65	2.3	6.5	4.2
0.60	7.1	6.50	2.2	6.4	4.2
0.65	7.1	6.45	2.1	6.3	4.2
0.70	6.9	6.20			
0.60	7.0	6.40			

$$v = \text{ratio of means} = \frac{6.45}{4.25} = 1.517.$$

WITH VOLTMETER.

Field.			Armature.		
Make.	Break.	Diff.	Make.	Break.	Diff.
0.150	0.60	0.375	0.10	0.45	0.275
0.150	0.60	0.375	0.10	0.45	0.275
0.150	0.50	0.325	0.10	0.40	0.250
0.175	0.65	0.412	0.10	0.40	0.250
0.175	0.65	0.412	0.09	0.39	0.240
0.150	0.60	0.375	0.10	0.38	0.240
			0.10	0.39	0.245
			0.10	0.39	0.245
			0.10	0.38	0.240

$$v = \frac{0.379}{0.251} = 1.51.$$

EXPERIMENT 38. **Distribution of waste magnetic flux in a dynamo.**

The total amount of waste flux having been determined in Exp. 37, a relative measure of its distribution may be obtained

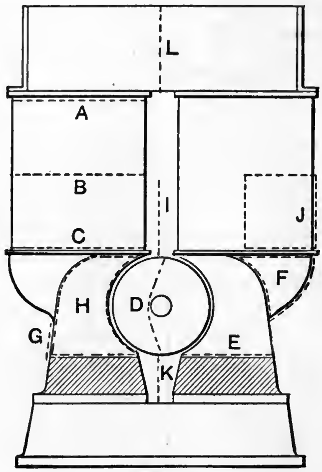


Fig. 35.

by a simple extension of the method there used. Thus if an exploring coil of a known number of turns be placed in the rear of the pole-piece (Fig. 35), the throw obtained in a ballistic galvanometer in series with the coil when the field circuit is broken will be a measure of the leakage from the surface. This may be conveniently expressed as a percentage of the maximum induction obtained near the middle of the field limb. Further, coils may be placed at such other positions on the machine that together they will include, in the judgment of the experimenter, the whole of the waste flux, and throws of the galvanometer may be obtained for each position. In the following illustrative results, the waste flux was considered to take place from the two sides, the base and the rear of the pole-piece, and between the lower halves of the magnet limbs.

That is, waste flux $= E + 2F + G + 2H + I + 2J$,

and also waste flux $= B - D$.

MAGNETIC LEAKAGE IN A 5 K. W. SHUNT GENERATOR.

Coil.	Turns.	Def.	Def. per 100 turns.	Per cent of max.
A	9	6.48	72.10	99.00
B	9	6.55	72.80	100.00
C	9	6.00	66.60	9.15
D	10	4.07	40.70	55.90
E	13	1.16	8.92	12.26
F	98	2.56	2.62	3.60
G	98	1.20	1.23	1.69
H	98	4.10	4.19	5.75
I	98	4.15	4.24	5.83
J	98	1.81	1.85	2.54
K	98	2.13	2.17	2.98
L	10	6.86	68.60	94.30

E	12.26
$2F$	7.20
G	1.69
$2H$	11.50
I	5.83
$2J$	5.08
	43.56

Loss by subtraction = 44.1 per cent.
Loss unaccounted for = 0.54 per cent.

These results indicate a nearly uniform induction in the upper halves of the magnet limbs and in the yoke, and also that the maximum leakage occurs through the zinc to the bed-plate. It is important to note, however, that much of the induction through H, and some of that through E, passes into the armature, and hence is not waste, but useful flux. The discrepancy between the loss by subtraction and that found by direct measurement should therefore be greater than the foregoing figures indicate.

EXPERIMENT 39. **Measurement of resistance by the ammeter and voltmeter method.**

If while a current is made to flow through a circuit, that current be measured, and the difference of potential between

the terminals of a dead resistance in that circuit be also measured, the amount of that resistance can be computed by Ohm's law. This is also known as the fall of potential method.

In some cases this method has its advantages; for instance, if the given resistance is in a line which is used to convey a certain amount of current, there will be a certain amount of heat generated in the resistance, which will maintain it at some temperature higher than the surrounding medium, and this will, unless it has a temperature coefficient of zero, give it a resistance different from that which it has at ordinary temperatures.

The voltmeter and ammeter used should have ranges such that the values of current and difference of potential may be determined within a small fraction of one per cent.

When measuring the resistance of a Siemens drum armature, or any armature having a similar commutator, the wires from the terminals of the voltmeter should be put between the brushes and the commutator bars, and should be applied to diametrically opposite bars. The armature should be turned to several positions and the average of the results taken.

Usually in measuring a resistance, currents of several different amounts had better be employed to determine whether the result obtained is constant, or whether it varies according to some law.

Experiment 40. **Study of a resistance.**

In the dynamo laboratory, resistances must be employed for regulating field currents, loading dynamos, etc.; for such purposes incandescent lamps, tin resistances, and resistances of other forms are in common use. This experiment consists in a study of such a resistance, including a determination of its resistance for different currents, the maximum current allowable, and the consumption of power for different potentials and currents. In Fig. 36 is shown the change in the resistance of a set of tin strips for currents ranging from zero to 30 amperes, at which current the strips burned. In making the

study, note the condition of the tin approximately as to temperature. Do this by the sense of feeling, using the back of the hand when small currents are flowing.

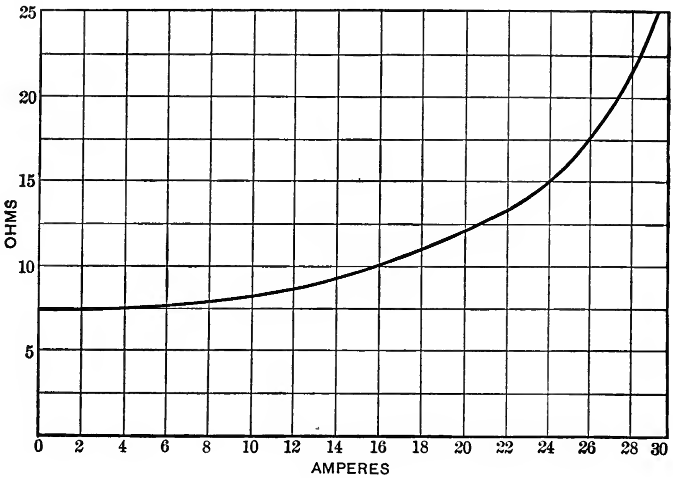


Fig. 36.

Tin resistances may be made as follows: Large sheets of roofing tin are to be cut first from one end and then from the other, making strips of about five-sixteenths of an inch wide, and by cutting to within about one inch of each end, the strips are left joined together in a continuous zigzag manner as shown in Fig. 37. After cutting the strips they are nailed to a suitable wooden frame, and terminal connections are attached. The broad ends are not heated by large currents, so as to endanger the wood. By

Fig. 37.

nailing the tin to the two sides of a frame and connecting the sides together at one end, so that the current will go along through one side and back through the other, a metal piece or bent wire can easily be used to slide along the top to adjust to the amount of resistance needed.

EXPERIMENT 41. **Construction and test of fuse wire.**

The reliability of easily fusible wires as circuit protectors is dependent upon a number of conditions, some of which at the outset are not likely to be apparent. A due appreciation of these is best gained through the actual construction of a small amount of fuse wire. The alloy to be used may consist, for example, of a mixture of lead and tin in any desired proportion. The metals should be melted in a small ladle, and the alloy cast in the form of cylindrical slugs about 4 mm. in diameter and 10 cm. long. The mould for casting these may be very simply made by pinning together two smooth plates of iron of suitable size, and drilling them in such a way that the axis of the drill hole lies in the contact plane of the plates. After this operation the pins may be removed, pointed slightly, and reinserted: the mould is then complete. The slugs, when cast, should be drawn into wire by the use of draw plates. The drawing of wire from so soft material must be done with care in order that the product may be of uniform diameter and smooth surface. The slightest fouling of the die may reduce the cross-section, or by abrasion of the surface alter the latter's radiating power. The samples drawn should be at least 15 cm. in length, and of diameters varying from 0.254 to 2.5 mm. (10 to 100 mils).

The experimental work should include at least the following tests:

I. A determination of the influence of the length of the specimen upon its fusing current.

II. A study of the *time element* in fusing.

III. A verification of the law connecting diameter with fusing current.

In test I., samples of a fixed diameter, but of varying lengths, should be successively tested between rather massive terminals, the distance between which is capable of being varied. The results obtained should be exhibited in a curve. The increased fusing current, which will be here observed for short lengths,

is due to the conduction of heat into the terminals. The magnitude of this effect is dependent upon other quantities than the length of the specimen, such as the mass and specific heat of the terminals. The curve obtained in this individual case may be taken, however, as representing usual conditions; and from it may be chosen a length for use in practical work, and in the subsequent tests of this experiment such that the cooling effect of the terminals will not seriously affect the true fusing current. In these tests by *true fusing currents* is meant that constant current which will fuse the wire after it has flowed for a sufficient time to bring about permanent temperature conditions, the length of the specimen being such that the conduction loss is relatively nil. Accordingly, the current should be brought to a definite value by means of a shunt circuit around the fuse of approximately the same resistance as the fuse circuit. By means of a suitable switch it should then be thrown on the fuse circuit, and the duration of flow in tests I. and III. should be at least 60 seconds. Attempts to obtain values of the fusing current of wires by gradually increasing the current, however slowly, will result in too high current values.

In test II. the object should be to gain an idea of the rapidity of action of the fuse. A series of wires should be fused by currents whose duration is limited to 20 seconds. The process should be repeated for a time limit of 10 seconds. Finally, a set of wires should be subjected to currents of momentary duration, and values of fusing current observed as before. All these results should be plotted, using diameters as abscissas, and currents as ordinates.

As a final test (III.), the law of fusion should be observed. When current flows through a conductor, heat is developed in accordance with Joule's law, and losses of heat take place by radiation, by convection, and to a greater or less extent by conduction into the terminals. Neglecting the last, the other losses may be assumed to be proportional to the surface of the

conductor, and to the excess of its temperature above that of the surrounding medium. The wire will come to a constant temperature when it loses heat at a rate equal that at which heat is developed by the current. The experimenter should strive to obtain the value of the current when this permanent temperature is just that of fusion of the alloy employed. By equating the expressions for the rates of gain and loss of heat in accordance with these assumptions, it is easy to show* that

$$C=ad^{\frac{3}{2}},$$

where C is the fusing current, and a is a constant for any given alloy with constant surface conditions. It is numerically equal to the current which will fuse a wire of unit diameter of the given material. From each set of values of fusing current and diameter a value of a should be computed; from the mean of these a theoretical curve should be constructed.

Diameter.	Current.	Calculated Current.	Constant.
0.0125	2.12	2.10	1552
0.0143	2.54	2.50	1488
0.0196	3.79	3.79	1384
0.0205	4.30	4.30	1465
0.0287	6.38	6.60	1416
0.0312	7.75	7.75	1406
0.0333	8.26	8.28	1360
0.0363	9.57	9.57	1377
0.0380	10.74	10.61	1448
0.0430	12.52	12.70	1385
0.0509	16.16	16.40	1407
0.0607	21.65	21.62	1447
0.0735	29.17	28.80	1464
0.0875	37.80	37.50	1460
0.0950	42.66	42.50	1457
0.1000	45.99	45.80	1454

Mean constant 1441.

* This law was first enunciated by Professor Forbes, before the British Association. See Reports of the British Association, 1882.

The accompanying data represent the behavior of an alloy of tin two parts, lead one part, when tested under conditions similar to those described above. The curve of Fig. 38 is plotted from calculated fusing currents, using the mean value of *a* (1441) for this alloy. The reader will find results obtained from a number of the lead-tin alloys in a paper by Mr. C. P. Matthews,* while values of *a* for all the common metals have been obtained by W. H. Preece, F.R.S.,† who has made an extended investigation of fuse metals.

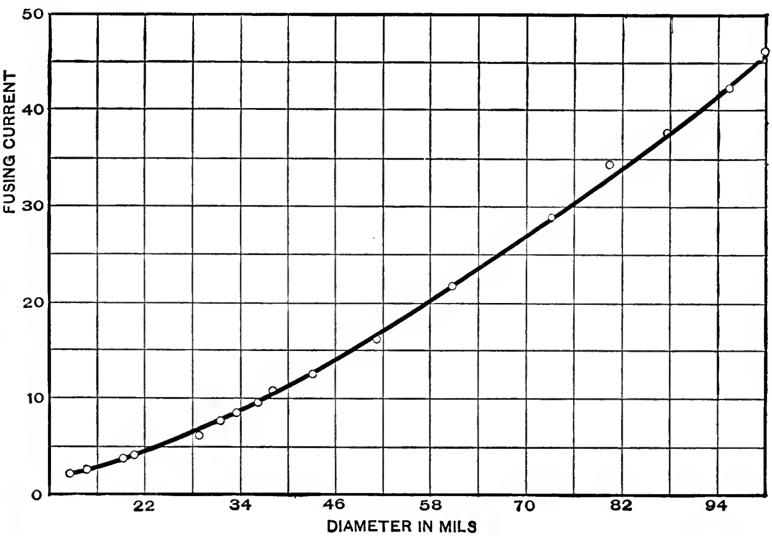


Fig. 38.

Experiment 42. **Adjustment of a secondary cell.**

This experiment is intended to give the student some familiarity with the construction of an ordinary secondary cell, and especially of the Brush-Faure or "pasted" type. Before undertaking the work, the student should pursue a short course of

---

* Transactions of American Institute of Electrical Engineers. Vol. 10, p. 251 (1893).

† Proceedings of the Royal Society, April 3, 1884, and December 22, 1887.

reading,* and embody in his report a brief discussion of such topics as:

(1) The distinction between a primary and a secondary battery.

(2) The characteristics of the earlier types of cells.

(3) The Planté cell.

(4) The Brush-Faure cell.

(5) The usual values of electromotive force, resistance, and capacity, with a careful distinction between the *quantity* efficiency and the *energy* efficiency of a cell.

(6) The chemical reactions which take place.

(7) The defects and diseases peculiar to secondary cells.

The lead grids and necessary acid and oxide of lead having been obtained, the student may prepare the pastes, and set up the cell as follows:

For the positive grids use a paste of minium (red lead, Pb_3O_4) mixed with dilute sulphuric acid of density 1.1, care being taken to have the acid cold when the paste is made. Apply this evenly to both sides of the grid, using a trowel, or wooden paddle. For the negative grids use, in the same way, litharge (PbO) mixed with acid, of density 1.2. Set the plates up edgewise and allow the pastes to harden. Then connect two positives and two negatives in such a way that the two positives shall constitute one element and the two negatives the other. The positive and negative grids should alternate in position. The four grids are held firmly by rods and cross-pieces of hard rubber. Connected in this way, the cell is ready for the process of "formation," the ultimate result of which is the production of peroxide of lead (PbO_2) on the positive grid, and the reduction of the litharge to spongy lead. Figure 39 shows grid before and after pasting.

It frequently happens that the active material of the positive grid will crumble and fall from the grid during the process

* See article and bibliography by C. P. Matthews, in Proceedings Cornell Electrical Society, 1893–94.

of formation. This is a defect dependent to some extent upon the form of the grid. It will be reduced by making the paste for the positive grid of four parts red lead to one of litharge.

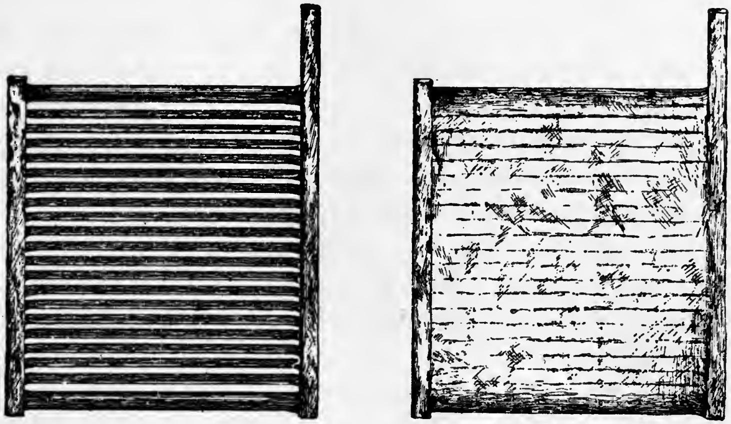


Fig. 39.

Owing to the length of time required for formation and also for efficiency tests, these matters can be taken up only as special investigations outside of the prescribed senior course. Specific directions must in such cases be given by the instructor.

EXPERIMENT 43. **Effects of self-induction and armature reaction on the difference of potential at the brushes of a dynamo.***

The difference between the electromotive force of a dynamo having a closed coil armature and the potential difference at the brushes would equal the fall of potential due to current through the armature resistance, if there were no other influences affecting this difference. It is, however, somewhat increased by self-induction caused by the reversal of direction of current through the coils as they pass the brushes; and the potential difference at the brushes is also affected by the weakening and distortion of field due to armature reaction.

* Kapp's Electric Transmission of Energy, pp. 136–138.

Separately excite the field of the dynamo to about its normal value. Maintain the excitation and speed as constant as possible. Close the external circuit through a resistance, which may be varied to give different currents up to full load. Observe the difference of potential at the brushes for each different current. If it is found difficult to keep the speed and excitation constant, the open circuit electromotive force may be adjusted between observations to the constant value chosen; then note how much the potential difference drops when current is permitted to flow through the armature.

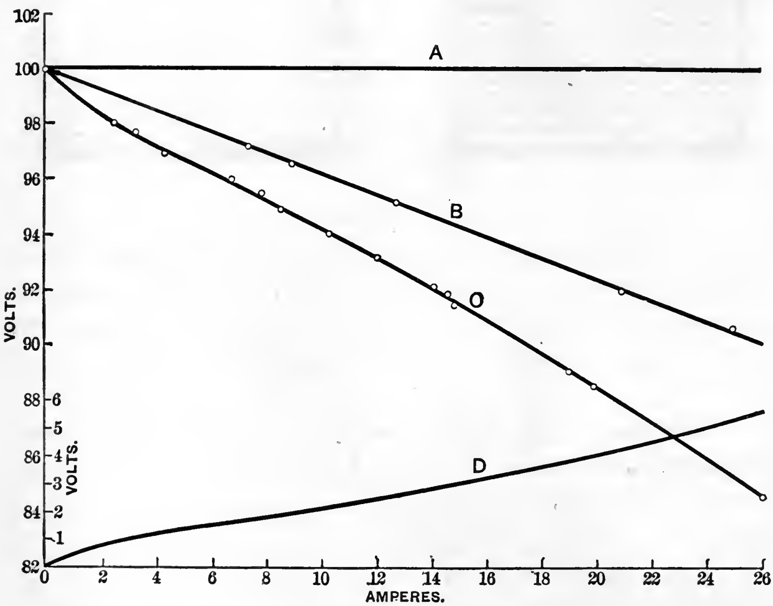


Fig. 40.

In Fig. 40 are shown the results obtained from a shunt-wound dynamo, with the brushes at the neutral points on open circuit. The line *A* represents the electromotive force that the constant excitation would produce if there were no reactions. The line *B* represents the difference of potential that would be found at the brushes if there were no reaction,

and is obtained by plotting downward from line *A* the fall of potential equal to the product of current and armature resistance, measured with the armature at rest, care being taken to have good contact with only two opposite commutator segments. Curve *C* gives the difference of potential observed at the brushes. The portions of the ordinates between the lines *B* and *C* are plotted from the axis of abscissas, giving the curve *D*, which represents the portion of the difference between open and closed circuit potential differences that is not due to armature resistance, but is due principally, if not entirely, to self-induction and weakening and distortion of field.

The effects of trail and lead of the brushes may also be considered.

EXPERIMENT 44. **Power lost in an armature due to hysteresis and foucault currents.**

One of the machines already mounted on a cradle dynamometer may be used for this experiment. Balance the dynamometer before starting; then the reading when running with no field excitation, and the armature circuit open will give the watts lost in friction. The reading for the friction run may not be due to friction alone if residual magnetism is large. Results more nearly accurate may be obtained by first demagnetizing the field cores.

When no current flows in the armature circuit, and the machine is separately excited, the number of watts required to drive it, less the number lost in friction, gives the number lost in hysteresis and eddy currents. Vary the field excitation, and note the corresponding dynamometer readings and speed. For constant speed, the curve expressing the results may be plotted with total watts indicated by the dynamometer as ordinates, and field currents as abscissas. Then, assuming friction constant, draw a line parallel to the axis of abscissas, with ordinates equal to friction. The portion of the ordinates above this line will represent the hysteresis and foucault current loss for that speed.

Explain why the curve has the form obtained, and find approximately what percentage of the normal output equals the armature loss for normal field current and speed.

If different speeds can be readily obtained, the field current may be maintained constant near its normal value, and the sum of hysteresis and eddy current losses found for different speeds.

EXPERIMENT 45. **Separation of losses in a dynamo.**

The principal causes of loss in a dynamo are: (1) the I^2R loss in the conductors; (2) foucault or eddy currents in iron and copper; (3) hysteresis in the armature core; (4) friction of bearings, brushes, and air.

In Exp. 17, the I^2R loss is obtained separate from the rest, and in Exp. 44, the loss by friction alone, and the sum of the hysteresis and eddy current losses are found. It remains to separate the loss due to hysteresis from that due to eddy currents when their sum is known. The different methods that may be used depend upon the fact that the eddy current loss varies approximately as the square of the speed, while hysteresis varies as the first power of the speed.

Mr. Housman and Mr. Kapp published a method of separating dynamo losses, which is explained and illustrated in S. P. Thompson's *Dynamo-Electric Machinery*, pp. 791–793. Mr. Mordey used another method, the details and illustrations of which may be found in Slingo and Brooker's *Electrical Engineering*, pp. 376–379. Similar methods are given in *Electromagnetism and Construction of Dynamos*, by Dugal C. Jackson, pp. 249–255; and in *Dynamos and Motors*, by F. P. Cox, pp. 194–196.

Any of these methods may be used in performing the experiment. If the first is used, note that the losses are expressed by the product of the co-ordinates of points on the curve instead of the ordinates. Additional curves may be drawn in which ordinates represent losses obtained for different speeds.

EXPERIMENT 46. **Effects of speed variation with a series dynamo.**

This experiment is to determine the effect of change of speed on the electromotive force of the dynamo for two or more cases. One when the field is separately excited with a constant current, and the external circuit is open; and another, when the dynamo is self-excited, and the circuit is closed through enough resistance to give, at normal speed, a current equal to the field excitation in the first case. Other cases may be included or substituted if desired.

The dynamo should be belted to the cone pulleys or driven by a motor so arranged that the speed may be regulated to any value within a wide range.

For the first case, electromotive force at the brushes and speed are observed. Make note of the constant field current used. If difference of potential at the brushes is observed in the second case, a curve for that and speed may be plotted, and then another derived for electromotive force by adding the fall of potential through the armature. Compare and interpret the curves; also draw conclusions regarding the correction of readings of potential difference for fluctuations of speed in taking characteristics of series dynamos.

EXPERIMENT 47. **Effects of speed variation with a shunt dynamo.**

An experiment, similar in some respects to the previous one on a series dynamo, may be performed with a shunt dynamo.

The variation of electromotive force with change of speed may be determined for separate constant field excitation, and also for self-excitation with constant field circuit resistance, the external circuit being open in each case.

When the external circuit is closed through a constant resistance, a change of speed changes both electromotive force and current, or the number of watts delivered. A curve may be

obtained showing the variation of output as speed is changed, the resistance being such as to give normal load at normal speed.

State what is shown by the results, and what conclusions may be drawn from them.

EXPERIMENT 48. **Mechanical characteristic of a motor.**

As the load on a motor is varied, there is a corresponding change of torque, or speed, or both; according to the winding of the motor, and the conditions of supply from the generator. A curve, with torque and speed of a motor as co-ordinates, has been called a mechanical characteristic * of the motor. No electrical measurements need be taken. Use a dynamometer to measure the torque, and observe the corresponding speed.

Show what the results indicate, and what uses may be made of them. If difference of potential and current are also observed, other useful curves may be obtained.

EXPERIMENT 49. **To "compound" a shunt dynamo from its armature characteristic.**

The number of added series turns necessary to maintain a constant potential difference, either at the terminals of the machine, or at any point in the external circuit, is readily obtained from the *armature characteristic.* This curve, and the method of obtaining it, have been discussed under the head of characteristics. Briefly, it is a curve showing the field current necessary to maintain a constant potential difference at a given point for different currents in the external circuit. It is obtained by separately exciting the machine, and varying this excitation so as to keep the potential difference constant for all external currents within the safe carrying capacity of the machine. Let A (Fig. 41) represent the curve thus obtained; then for a current I in the external circuit, the field current has

* Mechanical characteristics of different kinds of motors are discussed in S. P. Thompson's Dynamo-Electric Machinery, pp. 577–585.

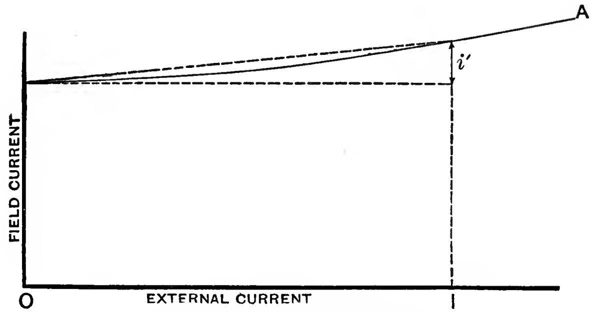


Fig. 41.—Armature Characteristic.

been increased by an amount i' in excess of its value on open circuit. If S be the required number of series turns, and S' the existing number of shunt turns, we have

$$S = \frac{S'i'}{I}.$$

I should be taken near the maximum current capacity of the machine. It is evident that it is impossible to satisfactorily compound a machine having an armature characteristic that departs greatly from a straight line.

EXPERIMENT 50. **To "compound" by added turns.**

The result obtained in Exp. 49 may be reached by the direct process of winding a certain number of turns of large wire (the smaller the wire, the greater the number of turns necessary) around the magnet limbs of the machine. Current from another dynamo may then be sent through these turns so as to maintain a constant potential difference at the terminals as the external current is increased. The results may be exhibited in a curve which shows for each value of the external current the required number of added ampere-turns. Let S'' be the number of added turns, and i'' the current in them for maximum current I in the external circuit; then the required number of series turns is

$$\frac{S''i''}{I}.$$

It is important that the wire should be wound uniformly on the limbs.

EXPERIMENT 51. **Variation in economic coefficient. Series dynamo.**

In any dynamo-electric machine, the ratio of the useful power at the terminals to the total power developed in the armature is called sometimes the *economic coefficient*, sometimes the *electrical efficiency*. It is a ratio depending wholly upon the electrical design of the machine, and hence differing from the gross and net efficiencies, being in fact the ratio of the net to the gross efficiency.

In a series machine the economic coefficient is

$$\eta = \frac{ei}{Ei} = \frac{e}{E} = \frac{R}{R + r_a + r_m} = \frac{R}{R + r},$$

where $r = r_a + r_m$. Since η may be expressed as a function of the resistances in circuit simply, the student may plot and discuss a curve which shows, for the given machine, the relation between R and η.

It is, however, more desirable to know in what way this ratio varies as a function of the current. It is evident, from the second equality above, that this relation may be very simply obtained, provided the external characteristic of the machine is available. Values of η may be computed and plotted on the same sheet. It will be found instructive also to plot the symmetrical curve showing the per cent of waste power as given by the equation

$$1 - \eta = \frac{i}{E} r.$$

The student should give a full discussion of the two curves, basing his reasoning on the analytical expressions, and answering such questions as the following: If the series curve starts from the origin, what will be the maximum value of η? Through what range will it be constant? About what line are the curves for η and $1 - \eta$ symmetrical? If the machine possesses residual magnetism, how are these questions answered? What is the effect of change of speed on the economic coefficient? Show how arc-light machines with drooping characteristics sacrifice

efficiency to regulation. Given several series machines of different capacities, whose economic coefficients are to be compared: for what values of speed and current should the comparison be made?

The curves in Fig. 42 illustrate the variation of η and $1-\eta$ in a 2 K.W. series machine.

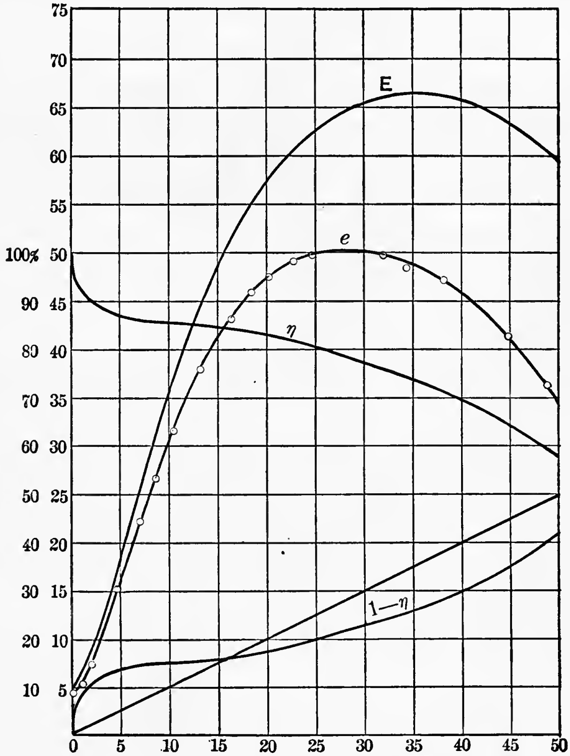


Fig. 42.—Economic coefficient for a series dynamo.

EXPERIMENT 52. **Variation in economic coefficient. Shunt dynamo.**

The expression for the economic coefficient of a shunt dynamo is complicated slightly by the existence of a divided circuit. Here

$$\eta = \frac{ei}{ei + i_s^2 r_s + i_a^2 r_a}.$$

This may also be expressed as a function of the resistances in circuit. By a few transformations based on the fact that the external and shunt circuits are in parallel, and that hence $i = i + i_s$, we have

$$\eta = \frac{1}{1 + \frac{Rr}{r_s^2} + \frac{r_a}{R} + 2\frac{r_a}{r_s}},$$

wherein, for brevity, $r = r_a + r_s$.* Since R is the only variable here, the student may show that η is a maximum for

$$R = r_s \sqrt{\frac{r_a}{r}}$$
$$= \sqrt{r_s r_a},$$

quite approximately. The fact that the most economical working resistance in the external circuit of a shunt dynamo is a

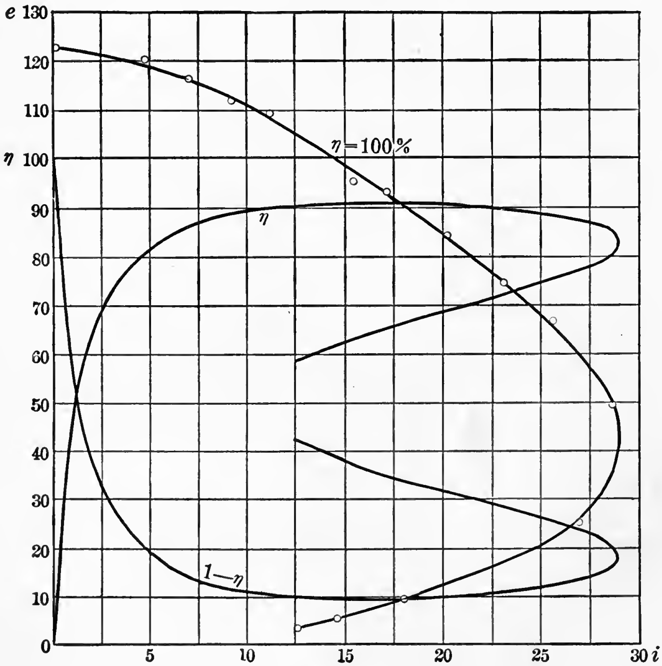


Fig. 43.—Economic Coefficient for a Shunt Dynamo.

---

* See S. P. Thompson's Dynamo-Electric Machinery, p. 240.

mean proportional between the armature and field resistances was first pointed out by Sir W. Thomson.*

The curves showing the variation of η and $1-\eta$ with the external current may be obtained by taking the external characteristic and measuring the field and armature resistances after the run has been made. Or, if a low-reading ammeter is available, it may be inserted in the field circuit, and i measured for each value of the potential difference at the brushes. The power lost in the armature must be computed from its resistance in either case.

Figure 43 illustrates the variation in the per cent useful and waste power in a 2 K.W. shunt-wound generator. A number of interesting facts, which the student should bring out in his discussion, are revealed by the curves.

EXPERIMENT 53. **Experimental determinations of the air-gap in a dynamo.**

If the magnetization curves of the separate parts of the magnetic circuit of a dynamo be plotted on the same sheet, the total magnetizing force may be obtained by summing the abscissas of all the curves for each chosen ordinate. The curve showing this relation for the air-gap is a straight line, the permeability of air being unity. An inspection of such curves† shows that the principal magnetic resistance in the circuit of the dynamo for small magnetizing forces is that of the air-gap. The *pitch* of the straight portion of the magnetization curve of a dynamo is determined almost wholly by the magnetic resistance of the air-space. This fact furnishes a means of determining the depth of the air-space in a machine, which dimension, owing to the presence of wires and insulation, does not often allow of direct measurement.

---

* British Association Reports, 1881.

† See S. P. Thompson's Dynamo-Electric Machinery, p. 150; Ewing's Magnetic Induction in Iron and Other Metals, p. 218.

Take the magnetization curve of the machine, starting with a positive field current of normal value. Reduce this current to zero and then increase it in the opposite sense. This will carry the straight part of the curve across the X-axis to the left of the origin. For any point on this straight part, we have the relation

$$d=\frac{1.26\,Si_s}{N_0}A,$$

where S is the number of turns in the field, i_s the current measured from the intersection referred to above, A the area of the pole face, N_0 the total number of lines through the armature for this field current, and d the required depth of the double air-space. N_0 may be found from the fundamental equation of the dynamo,

$$E_0=\frac{N_0 Cn}{10^8}.$$

Example.

40 ampere shunt-wound dynamo.

From curve taken as above indicated: $E=90$, $i_s=0.4$.

Further, $A=425$ sq. cm., $C=160$, $S=5364$.

Revolutions per second, $n=44.3$.

From the formula, $d=0.903$ cm.

By actual measurement, $d=0.904$ cm.

Volume II. — Part II.

ALTERNATING CURRENT EXPERIMENTS.

By Frederick Bedell.

INTRODUCTORY TO ALTERNATING CURRENT EXPERIMENTS.

INTRODUCTORY TO ALTERNATING CURRENT EXPERIMENTS.

In alternating current measurements, we have to deal with quantities rapidly varying from instant to instant, and we are concerned not only with the values of the quantities themselves, but also with rate at which these values are changing, and the dependence of one quantity upon the rate of change of another. The value of a varying quantity may be expressed in several ways. It may be expressed as the value of the quantity at a particular instant. In experimental investigations, these instantaneous values are commonly ascertained, and are of particular importance in certain lines of research. In commercial work they play no part. The value of a quantity varying periodically is sometimes expressed in terms of the maximum value which the quantity attains in each period. These maximum values

become the more significant when the quantity varies harmonically, or nearly so, as is ordinarily the case with alternating currents. For a current differing widely from a sine-function, the maximum value indicates little as to the magnitude of the current, in the usual sense of the term. When a current is periodically varying, it is most commonly the case that we wish to know, not the value of the current from instant to instant, nor its value when a maximum, but the value of an equivalent unvarying current, by which we mean a current equivalent in heating and dynamic effects. This value of an alternating current is called its *virtual* value. It is the one commercially used, and usually given as the value of a periodically varying quantity, unless otherwise stated. It is this value which is given by most measuring instruments. The virtual value of a quantity is equal to the *square root of the mean square* of the instantaneous values. The square root of the mean square of the instantaneous values of an *harmonically* varying quantity is equal to the maximum value divided by $\sqrt{2}$; or virtual value is equal to 0.707 times the maximum. The *mean* or *average* value of an harmonic function is equal to the maximum value multiplied by 0.6369. This mean value is, however, of little importance.

Besides knowing the magnitudes of the periodically varying quantities under consideration in alternating current work, we should know their *phase* relations, which depend upon the times at which the different quantities attain their maximum values. These relations may be seen from curves drawn to represent the instantaneous values of the quantities, as in Fig. 117. If one quantity reaches a maximum after another, it is said to *lag* behind it in phase. Lag and advance are relative terms.

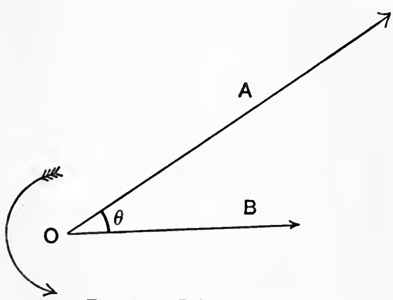


Fig. 44.—Polar Diagram.

When the quantities considered vary *harmonically*, they may be represented by vectors in a *polar diagram*. Thus, in Fig. 44, A and B represent the maximum values of two harmonic or sine functions, and are supposed to revolve in a counter-clockwise direction indicated by the arrow. This is arbitrarily taken as the positive direction of rotation, and will be so considered in the following pages. B lags behind A by an angle θ. The instantaneous values of these quantities are represented at any time by the projection of A and B on any fixed line of reference.

In a large part of the following work, it will be assumed that the currents and electromotive forces are harmonic, or rather that they are *equivalent* to certain harmonic currents and electromotive forces, and may accordingly be represented by vectors in a polar diagram. These equivalent harmonic currents have the same square root of mean square value, and are in such relative directions that they represent the same amount of power.

Strictly speaking, lines in a polar diagram should represent the maximum values of the several quantities. Inasmuch as virtual values are equal to maximum values multiplied by a constant factor, relative magnitudes and phase relations are preserved if the scale of the diagram be such that virtual and not maximum values are represented. Polar diagrams may represent then either maximum or virtual values. For theoretical work, the former is to be preferred as being more logical. In experimental work, the latter is the more convenient inasmuch as virtual values are obtained from most measuring instruments. That the relation between the magnitudes of vector quantities is the same whether maximum or virtual values are employed is exemplified in the value of impedance, which is equal to electromotive force divided by current, when these two quantities are expressed either in maximum or in virtual values. Where power and energy are involved, maximum and virtual values cannot be interchanged.

Current, electromotive force and quantity will be commonly represented by the letters I, E, and Q. Where it is essential that either maximum values or virtual values be exclusively employed, these will be denoted by $I_{max.}$, $E_{max.}$, $Q_{max.}$ and $\bar{I}$, $\bar{E}$, $\bar{Q}$, respectively. For instantaneous values the corresponding small letters i, e, and q are used.

For a discussion of alternating current apparatus, experimental work and matters of practice, the reader is referred to *The Alternate Current Transformer*, by Fleming; for theory, to *Alternating Currents*, by Bedell and Crehore, which will be designated by its title alone, when referred to. The conventions of this latter work are here adopted.

The instruments needed in performing the following experiments are for the most part such as are well known, and need no description here. The student should familiarize himself with the principles of the instruments he uses by reference to standard works. Small alternators of low potential, such as those made for ten lights by the Westinghouse Company, are suitable for most of the experiments. For general student work upon transformers, it is well to use transformers with small coefficients of transformation. High potentials are thus avoided, rendering the experiments simpler and more safe; furthermore, the measurements are more easily made. Every laboratory has special apparatus of its own, but such apparatus is usually similar in many respects to that which is standard and well known, and needs, therefore, no particular description. Each laboratory adapts itself to various lines of experiments according to the apparatus at hand.

The development of the method of instantaneous contact is receiving more and more attention; but the method has usually been accompanied with errors due to change in the contact during the course of an investigation. These errors are, however, obviated in a contact-maker in which contact is made by a liquid jet rather than by a mechanical device. Such a contact-maker is here described, inasmuch as it is a recent device, and has been found to work satisfactorily.

A general view of the revolving contact-maker* is given in Fig. 45, and a detailed view in Fig. 46. The whole instrument is supported by a stationary frame *F*. The shaft *S* is connected to the armature shaft of a dynamo by a coupling (not shown)

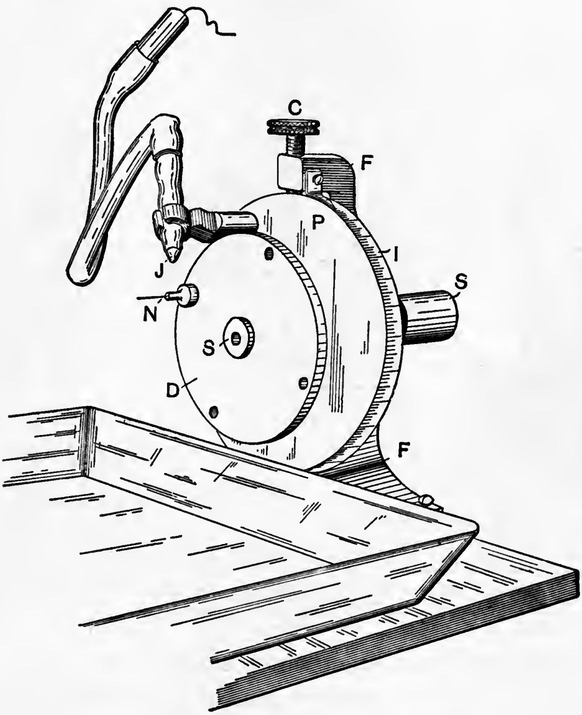


Fig. 45.—The Bedell-Ryan Contact-Maker.

on the end of the rod *R*, and carries the disk *D*, which revolves with it. The needle *N* projects from this disk and forms one of the electrodes of the contact. The other electrode is a fine water-jet (not shown) issuing from the nozzle *J*, well insulated by hard rubber from the rest of the instrument. This fine jet

* Described by Bedell, Miller, and Wagner in the Transactions of the American Institute of Electrical Engineers, Vol. 10, p. 500, from which this description and figures are taken.

is maintained from a jar of water, several feet above, to which it is connected by a flexible rubber tube. Electrical connection is maintained with the water-jet by a wire *W*, which passes through this tube and is soldered to the nozzle *J*. Electrical connection with the needle-point electrode is obtained through the shaft and frame of the instrument. The needle cuts the water-jet once in every revolution of the armature of the

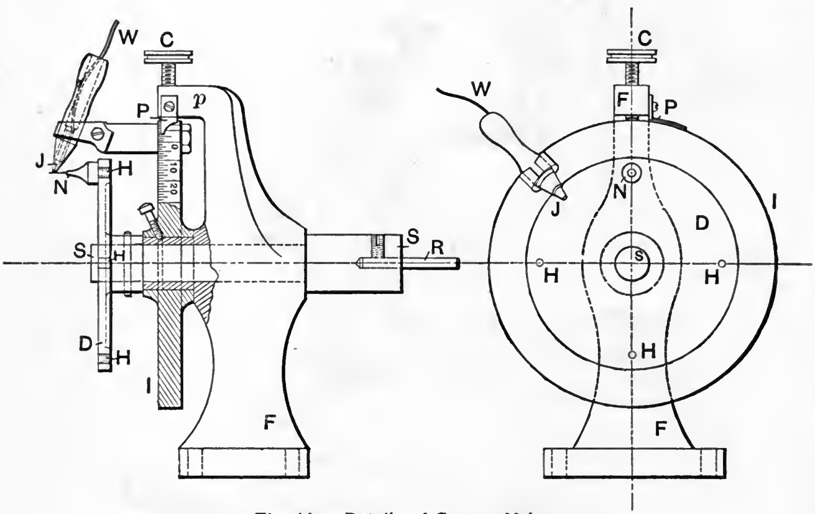


Fig. 46.—Details of Contact-Maker.

dynamo and makes a contact which is well defined and unvarying. The nozzle of the water-jet is carried upon an index-disk *I*, which can be turned into any position by revolving it upon a projecting collar of the frame, which forms its bearing. The water-jet is held in any position by securing the index-disk by means of the set-screw *C*, in the top of the frame. Its position is indicated by the pointer *P*, upon the scale on the circumference of the index-disk, which is graduated in degrees. The needle cuts the water-jet very near the nozzle, at a point where the jet is quite stiff, due to the head of water. The nozzle is radial, and the jet keeps the direction of the nozzle for some little distance before being materially deviated by gravity. A

little salt put in the water increases the conductivity of the jet; acidulated water corrodes the nozzle, thus changing the jet.

The contact made in this way is perfectly constant and reliable, and free from the changes always found in a mechanical device, due to the wearing away of the contacts. This constancy is particularly necessary in an instrument which is to be used in an extended investigation, during which any change would be fatal. By maintaining a fine, strong jet, for which a head of 5 or 6 feet is ample, and using a fine needle close to the nozzle, the instrument may be used with great precision, and needs but little attention. Of course, the accuracy of the instrument is increased as the diameters of the disks are made greater. For use in investigations, as Exp. 51, in which it is desired to obtain measurements for several consecutive cycles without interruption, the instrument is made so that the needle can be secured in the disk *D*, by screwing it into any of the four holes *H*, thus making it possible to have the contact made with the armature in any desired position, without moving the nozzle more than 45° from the vertical.

The following experiments are intended, first, to familiarize the student with the phenomena due to self-induction, and the significance of the coefficient of self-induction, impedance, and lag, the understanding of which is essential. Where possible, results should be represented graphically both by polar and rectangular diagrams. The frequency should be given in all cases, and the arrangements of the various circuits, together with the values of resistances, self-inductions, and other quantities, when known, even if not required for the experiment in hand. With complete data given, it is possible to refer to former work and derive results other than those originally sought, which could not be otherwise obtained without the repetition of the entire experiment.

Experiments are given under separate heads, which may well be taken up and performed together. Thus, Exps. 1 and

3 may well be grouped together, and the external characteristic of an alternator taken in connection with its curve of magnetization. Similarly, Exps. 48 and 51 may be combined as one, and the method of instantaneous contact with the multicellular voltmeter used in determining the irregularities in alternating current curves. Such are not included in one experiment in this manual in order to make it possible to group the experiments together in such ways as may seem best.

Under the head of alternating current experiments are given some experiments not strictly of this class, — as the determination of the curves of magnetization for iron, study of hot-wire instruments, and non-inductive resistances, — these being included in this group on account of their close association with the subject of alternating currents.

The experiments here given can in no sense be supposed to cover the field. They familiarize the student with the general phenomena, and bring out some of the essential principles of alternating currents. Many of the experiments are capable of precision; others are merely practise methods, capable of refinement, however, in the hands of advanced students. The assumption of a resistance or instrument being practically free from self-induction, is not allowable in the most accurate work, but may at times be made in practise experiments. In some cases the instruments ordinarily employed do not allow accurate results to be obtained without complicated corrections. Thus in some experiments the current taken by the voltmeter may introduce an error which is ordinarily negligible, but which may be eliminated by proper correction, or may be avoided by the use of electrostatic instruments. Under some circumstances, a small error of this kind may be allowed in practise work.

Many experiments will suggest themselves in the way of modification of these here given and as additions. For further advance work, the student who is ready for such needs no detailed directions. For such work, reference should be made

to the current technical literature. Inasmuch as this book is prepared especially for students at Cornell University, many of the references cited are descriptive of investigations conducted there. In all the work, introductory as well as advanced, the student should be encouraged to devise experiments of his own, and to extend those here given beyond their present limits, and to obtain relations and quantities besides those indicated in this manual.

EXPERIMENT I. **Curve of magnetization of an alternating current generator.***

The object of this experiment is to obtain the relation between the electromotive force and the exciting current of an alternating current dynamo. The electromotive force generated is directly proportional to the magnetization, or the strength of the magnetic field in which the armature revolves, while the exciting current is a measure of the magnetizing force to which this field is due. Hence there is the same relation between the electromotive force and the field current as there is between the magnetization and magnetizing force in the magnetic circuit of the alternator.

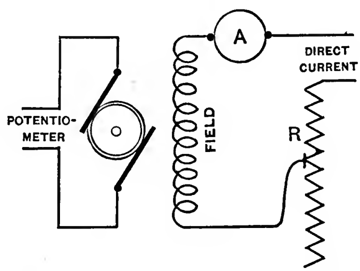


Fig. 47.—Connections for Curve of Magnetization.

Start the dynamo on open circuit with no field excitation; then excite the field with a continuous current, and gradually increase this exciting current by small increments. Take simultaneous measurements of the exciting current and the electromotive force at the brushes. The exciting current can usually be increased up to the safe carrying capacity of the field winding. In some machines, which are not perfectly

* Compare experiments on dynamo characteristics.

true and symmetrical, the armature shaft is liable to be sprung by too strong a field. The connections are shown in Fig. 47. Small low-potential alternators, such as those made by the Westinghouse Company for ten lights, are suitable for this experiment, and many which follow. For measuring the electromotive force at the brushes, the potentiometer described in a following experiment should be used; for current, a Siemens dynamometer or Thomson balance.

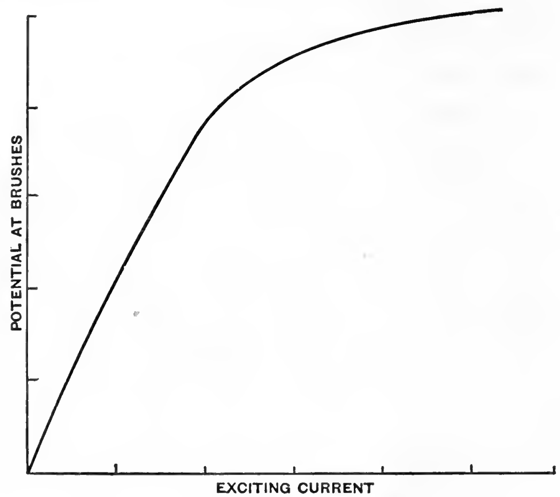


Fig. 48.—Curve of Magnetization.

The speed should be taken for each observation, and the observed values of electromotive force corrected accordingly. Corrected values of electromotive force should be plotted as ordinates, and exciting current as abscissas. This curve is the curve of magnetization* for an iron magnetic circuit with an air-gap. Such a curve is shown in Fig. 48. It is not always possible to reach the knee of the curve, as in the curve here given. If the speed can be varied, the experiment should be repeated for different speeds and curves plotted. Results should be interpreted.

* Compare Exp. 62 on magnetization of iron.

EXPERIMENT 2. **Study of an alternating current generator.***

Give a description of the machine with essential dimensions and data, including output, electromotive force, and current; field current; hot and cold field resistance; hot and cold armature resistance; self-induction of armature†; rise of temperature‡ of armature when run on open circuit; frequency n corresponding to speed at which the dynamo is run; the value of $\omega = 2\pi n$; watts consumed in the field for normal excitation.

EXPERIMENT 3. **External characteristic of an alternating current generator.**

In this experiment the field excitation is to be kept constant. The load is varied by varying the resistance in the external circuit. Simultaneous readings are taken of the current in the external circuit and the electromotive force at the brushes. (See Fig. 49.) The external current may be increased by small increments up to the normal full load of the machine. A few measurements may then be made by running the machine for a *few moments* far beyond full load. Speed and exciting current are to be noted. In plotting the results, abscissas should be used to represent current and ordinates to represent electromotive force. Curves should be taken for different exciting currents. Such curves are given in Fig. 50.

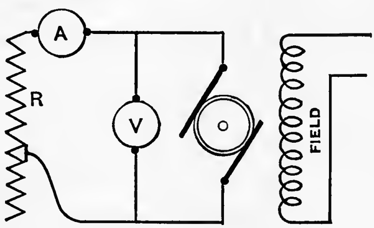


Fig. 49.—Connections for External Characteristic.

---

* Compare: Study of a continuous current dynamo, Exp. 1, Part I.

† This may be deferred until some of the subsequent experiments have been performed. See Exp. 7.

‡ Obtained at the same time as hot resistances.

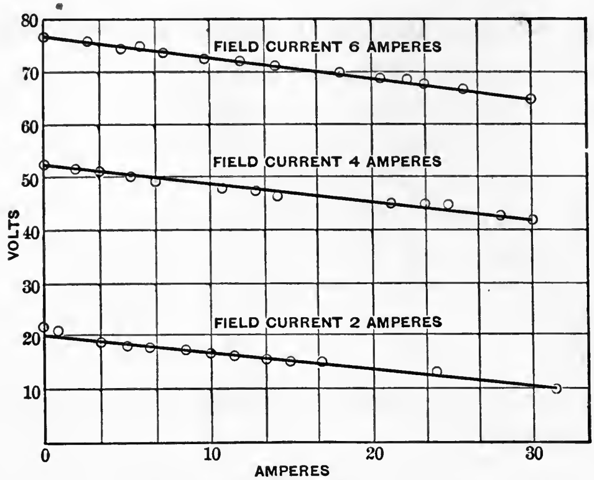


Fig. 50.—External Characteristic of an Alternator.

EXPERIMENT 4. **Alternate current potentiometer: adjustment and test for sensitiveness.***

For measuring differences of potential in alternating current circuits, a lamp potentiometer† is in many ways convenient. This consists of an incandescent lamp, so arranged that it can be switched back and forth from the terminals of the alternating current circuit whose potential difference is to be measured, to the terminals of a continuous current circuit, the difference of potential between which can be readily adjusted until the lamp glows with the same intensity when fed from either circuit. While the lamp is connected with the continuous current circuit, its difference of potential is measured by a continuous current voltmeter, and the voltage thus read is equivalent to that of the alternating circuit which produces the same effect in the lamp.

The arrangement of the switch and a diagram of connections is shown in Fig. 51. A switch-arm *S* carries two lamp terminals,

---

* This experiment may best be taken up in connection with one of those just preceding.

† Moler, "An Alternate Current Potentiometer," Transactions American Institute of Electrical Engineers, 1891.

l, *l*, which may be thrown into connection with the spring contacts *a*, *b*, or *c*, *d*. The contacts *a*, *b*, are connected with the points *A*, *B*, of the alternating current circuit, whose potential is to be measured. The contacts *c*, *d*, are connected with a continuous current circuit, and the difference of potential between these points is regulated by an adjustable resistance *R*, and sliding contact *C*. When this resistance is so adjusted that no change in the brilliancy of the lamp is observable as it is switched

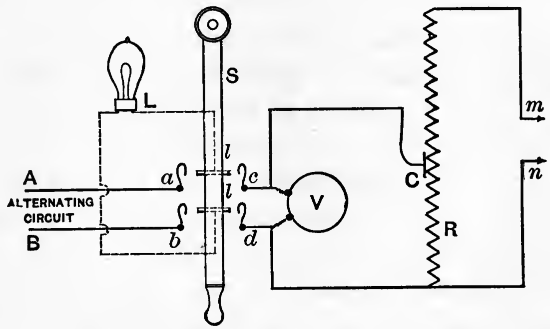


Fig. 51.—Moler Alternate Current Potentiometer.

back and forth between the two circuits, it is known that the lamp is subjected to the same potential difference in each case. When the lamp is connected to *c*, *d*, of the continuous current circuit, its potential difference is measured by means of a standard continuous current voltmeter. The alternating current difference of potential (square root of mean square value) is thus ascertained.

The resistance *R* is connected with a direct current circuit *mn*, of higher potential than that to be measured. The resistance should be so large that the current flowing through it is small, and the consumption of energy not objectionable. The switch and lamp may be connected in series with the resistance, instead of forming a shunt circuit as just described and shown in Fig. 51. The energy consumed is then less, but a higher resistance is necessary in order to afford the same range in reading.

The switch is controlled by strong springs so as to snap back and forth quickly. A slight change in the intensity of the light is thus readily detected. If the lamp has over it a blackened hood, with an opening in one side through which it is viewed while the sliding resistance is being adjusted, a close setting may be obtained, and a small difference of potential detected. Instead of observing the light directly, the reflection from the hood may be observed, or the intensity of the shadow cast by the lamp. When the light is bright, it may be observed by holding a white paper screen between the lamp and the eye.

By having a number of lamps of different voltages from which to select, it is possible to secure a greater range for the instrument than can be obtained with only one lamp. These may be successively tried, higher voltage lamps first, until the one is found best suited for the measurement to be made.

For higher voltages than the highest voltage lamp can stand, a number of lamps may be used in series, and the fall of potential for all, or for each individual lamp, can be measured by the voltmeter. By this later arrangement, the potential of the alternating current circuit measured may be greater than that of the storage battery or continuous current dynamo.

This experiment consists in the adjustment of the apparatus and test for sensitiveness by the use of a direct current supplied to each pair of terminals. The potentiometer is to be tested for sensitiveness in the followng way: Connect the terminals a, b, to the same resistance R, by two other contact points, which are to be changed several times so as to include different amounts of that resistance. Each time the sliding contact C is to be moved until the lamp is equally illuminated for each position of the switch. With a standard potential instrument connected to the lamp terminals l, l, take readings of the potential of the lamp with the switch in each position. The differences between these readings indicate the errors of observation in the use of the instrument.

Determine the sensitiveness for a range of lamps, testing

each at various voltages, from that just sufficient to bring the lamp to a perceptible glow up to that of full candle-power. Investigate the relative advantage of the shunt, and series arrangement of the resistance. Compare the several methods of observation.

A modification of the potentiometer as above described may be used, in which two lamps are employed instead of one. One of these lamps is supplied from the alternating current circuit, and the other from the continuous; a switch device rapidly interchanges them. The lamps are brought to the same brilliancy, and the potential obtained as before. What are the relative merits of the one-lamp and two-lamp arrangement?

EXPERIMENT 5. **Curve of magnetization of an alternating current generator. Ballistic method.**

The object of this experiment is the same as in Exp. 1. Instead of running the machine and measuring the magnetization by the electromotive force generated, as in the experiment referred to, the armature is blocked, and the change

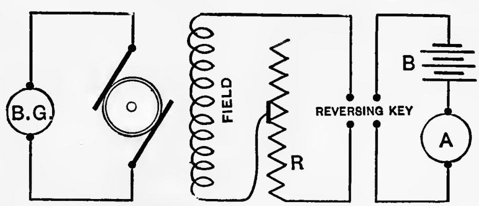


Fig. 52.—Connections for Curve of Magnetization by Ballistic Method.

in magnetization, caused by a reversal of the exciting current, is measured by the throw of a ballistic galvanometer connected with the armature. This is repeated for various values of exciting current, and a curve plotted as before. The connections are shown in Fig. 52. Instead of the *method of reversals* above described, the *step-by-step method* * may be employed, in

* Compare Magnetization of Iron, Exp. 62.

which the exciting current is changed by sudden increments, and the corresponding change in the magnetization measured by the throw of the ballistic galvanometer.

Instead of the ordinary ballistic galvanometer, a D'Arsonval galvanometer may be employed, or even a sensitive ammeter or voltmeter; for instance, a Weston voltmeter.

The armature of the dynamo should be blocked in the position which gives the maximum throw of the needle of the measuring instrument. In some cases, the curve of magnetization can be carried higher by this method than by the method of Exp. 1.

Compare the curves for increasing and decreasing exciting current, obtained by reversal and the step-by-step process, with each other, and with the curve obtained in Exp. 1.

EXPERIMENT 6. **Exploring field of an alternator.**

In this experiment the machine is not run; the armature is connected with a ballistic galvanometer, and used as a test coil, as in Exp. 5. The field is excited with a constant current, which may be reversed at will. A circular scale is so arranged that the position of the armature is definitely known. The armature is placed in a certain position, the field current reversed, and the throw of the galvanometer noted. It is a measure of the lines of magnetization passing through the armature. The armature is then turned a few degrees to another position. The field current (of the same value as before) is now reversed, and the throw of galvanometer observed. This is repeated until data are obtained, showing the throw of the galvanometer corresponding to each position of the armature. These throws are plotted, as the solid curve in Fig. 53, to an arbitrary scale. Abscissæ here represent armature position. The curve for electromotive force is obtained from the curve of magnetization in accordance with the relation that the electromotive force set up in the armature is proportional to the rate of change of the magnetic induction through it; that is,

$e = -\frac{dN}{dt}$. This electromotive force curve is shown by the dotted line. A graphical means of constructing it is indicated. The perpendicular OA is erected at O. From a convenient point O', the lines a', b', etc., are drawn parallel to a, b, etc., tangent to the magnetization curves at p_1, p_2, etc. The points

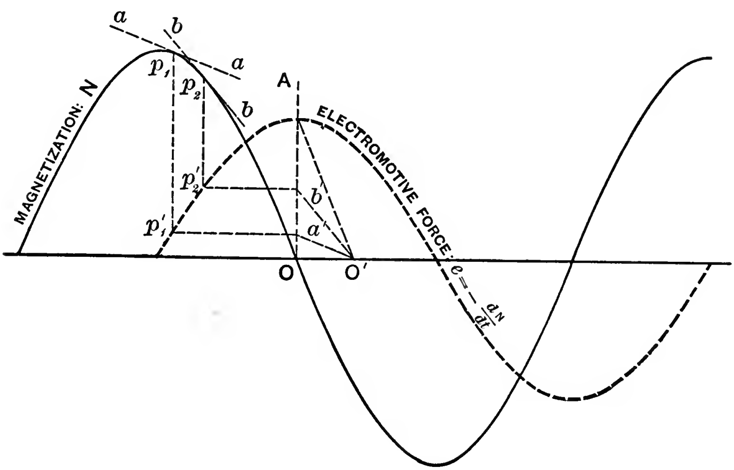


Fig. 53.

p_1', p_2', on the electromotive force curve are found by the construction indicated.

Where it is possible to excite the field above saturation, curves of magnetization and electromotive force should be found for field excitations both above and below saturation. These curves should be compared, and any departure from a sinusoidal form should be interpreted.

Experiment 7. **Measurement of the coefficient of self-induction. Impedance method.**

The value of the coefficient of self-induction of a coil may be computed from the value of the impedance, found by passing

an alternating current through the coil and measuring the fall of potential. The value of the current * is

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}}.$$

This gives the current in amperes, when E is the electromotive in volts at the terminals of the coil; R the resistance in ohms; and L the coefficient of self-induction in henrys. ω is $2\pi \times$ frequency. The ratio of the electromotive force to the current, $\sqrt{R^2+L^2\omega^2}$, is the impedance, and may be expressed in ohms.

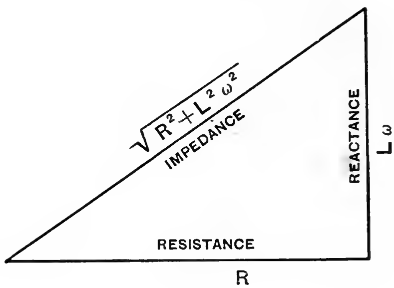


Fig. 54.

The name *reactance*† has been given to the term $L\omega$, which may also be expressed in ohms.

To obtain the value of the coefficient of self-induction, the electromotive force supplied to the coil is measured, and also the current which flows through it. The resistance is obtained by previous measurement, and the value of ω is computed from the speed of the dynamo.

The value of L is found by substituting the values for E, I, R, and ω, thus found, in the above formula. Draw a triangle,

---

* Alternating Currents, Bedell and Crehore, p. 53 et seq. Hereafter when reference is made to this work, it will be referred to as "ALTERNATING CURRENTS."

† This name has become authorized while this book is in press. For further explanation of the significance of the term, see paper on "Reactance," by Steinmetz and Bedell, Transactions American Institute of Electrical Engineers, 1894.

as in Fig. 54, with the three sides equal to the impedance, resistance, and reactance respectively, and measure the angle of lag. Compute the time constant of the circuit.

If the instruments are so connected that the voltmeter does not include the ammeter, the voltmeter reading is correct, but the ammeter reading includes the current through the voltmeter circuit, as well as the current through the coil. This error is generally negligible, and does not occur at all with an electro-

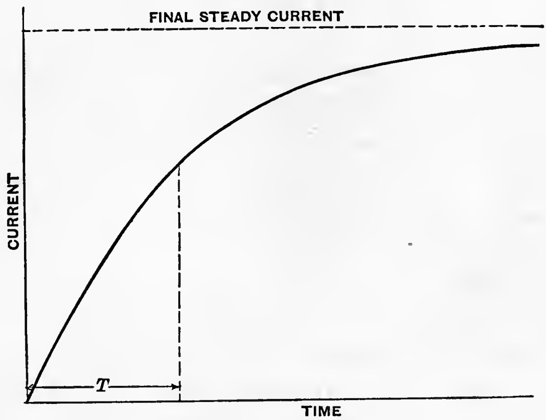


Fig. 55.—Establishment of a Current.

static voltmeter. If the ammeter is placed so as to measure the current in the coil only, the value of the coefficient of self-induction obtained includes the self-induction of the ammeter, which may, however, be deducted from the observed result.

The measurement of resistance would ordinarily be by the fall of potential method. The determination of the coefficient of self-induction then depends upon an ammeter and voltmeter reading. The value of the self-induction may be made to depend upon ammeter readings and measurements of resistance by two sets of observations, taken with different resistances in the circuit, as follows. When the impressed electromotive force is E and the resistance is R_a, the current is found to be I_a. Additional non-inductive resistance is then placed in the circuit,

so that the whole resistance is R_b. With the same electromotive force as before, the current is now I. From the relations

$$I_a=\frac{E}{\sqrt{R_a^2+L^2\omega^2}}, \text{ and } I_b=\frac{E}{\sqrt{R_b^2+L^2\omega^2}},$$

we may eliminate E and obtain

$$\frac{I_a^2}{I_b^2}=\frac{R_b^2+L^2\omega^2}{R_a^2+L^2\omega^2}.$$

The value of the currents, resistances, and frequency being known, L is determined. By this method it is not necessary to have an accurate potential instrument.

In a similar way, the self-induction can be made to depend upon resistance and potential measurements by eliminating the current by taking two sets of observations with the same current and different resistances.

When an inductive circuit is supplied with current from a constant source of electromotive force, the current does not immediately assume its final steady value, but approaches it gradually. If the value of the coefficient of self-induction is constant, the curve of approach is an exponential curve, as in Fig. 55. Similarly, if the source of electromotive force is removed and the circuit closed on itself, the current dies away and gradually approaches zero,* as in Fig. 56. The value of the current at any time t, after its establishment, is given by the formula $i=\frac{E}{R}(1-\epsilon^{-\frac{Rt}{L}})$; the formula $i=\frac{E}{R}\epsilon^{-\frac{Rt}{L}}$ gives its value while dying away at any time t, after the removal of the electromotive force. It is to be borne in mind that the above holds true under the assumption that the value of the coefficient of self-induction is constant for different values of the current. When this is not true, the effects are modified. (See Exp. 9.)

* Alternating Currents, pp. 46, 50.

From the values found for the resistance and self-induction of the coil, plot exponential curves for the establishment and dying away of the current in the coil when the constant electromotive force employed has some assumed value. When the

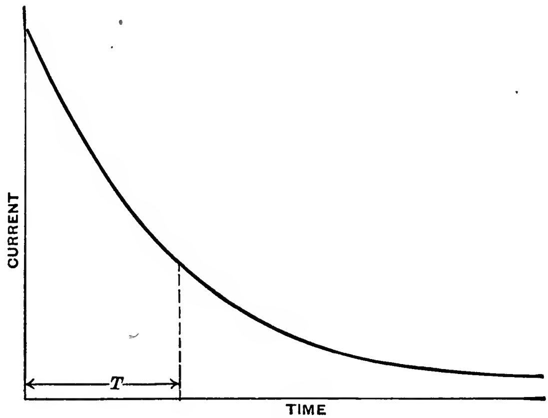


Fig. 56.—Dying away of Current.

circuit is made, in what time will the current rise to one-tenth of its final value? When the electromotive force is removed, in what time will it die away to one-tenth of its steady value? Give full explanation of the term "time constant" and its effect.

Experiment 8. **Measurement of the coefficient of self-induction. Three-voltmeter method.**

A non-inductive resistance is placed in series with the coil whose self-induction is to be obtained. In Fig. 57, R is a

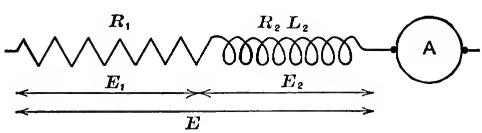


Fig. 57.

non-inductive resistance in series with an inductive resistance R_2L_2; L_2 is to be determined. An alternating current is

passed through the circuit and measured. Three voltmeter readings are taken: the total electromotive force E, the electromotive force E_1 around the non-inductive resistance, and the electromotive force E_2 around the coil. A triangle OBA is drawn, Fig. 58, with the three sides equal to the three voltmeter readings. If $\overline{OB}$ is produced to C, so that OCA is a right triangle, $\overline{AC}$ is equal to $L_2\omega I$. The value of $L_2\omega I$ may be taken directly from the figure, and the value of L_2 found from the known values of frequency and current. An expression may be found* from which L_2 may be calculated from the observed readings, but the graphical method is the more convenient and quite as accurate.

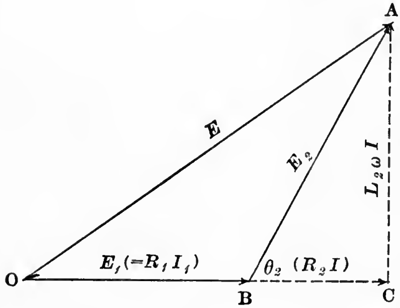


Fig. 58.

In this experiment, and in many others, the three-ammeter method, Exp. 15, may be substituted for the three-voltmeter method.

EXPERIMENT 9. **Variation in the coefficient of self-induction with the current.**

In case of a coil surrounding an iron core, as the primary or secondary coil of a transformer, the coefficient of self-induction is different for different currents, decreasing as the iron core becomes saturated. The value for the coefficient of self-induction for different currents can be obtained by the foregoing methods. It should be borne in mind that, in an alternating current circuit containing iron, the coefficient of self-induction goes through a cyclic variation each period. These instantaneous values are summed up by the methods of

* Alternating Currents, p. 231.

Exps. 7 and 8 so as to give the effective result. It is this value of the coefficient of self-induction which is of value in alternating current work. The curve in Fig. 59 is plotted from

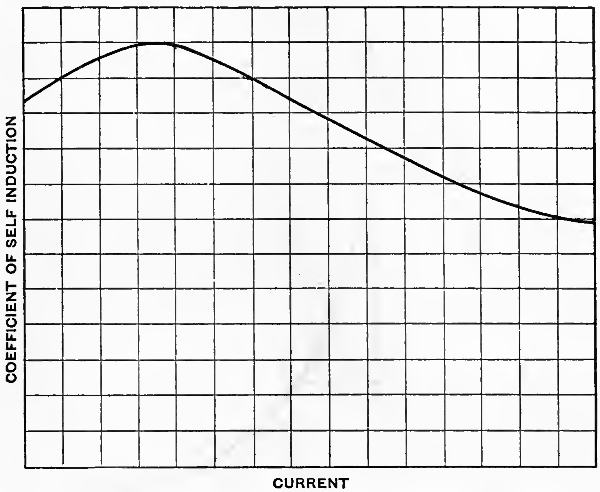


Fig. 59.—Variation of the Coefficient of Self-Induction with the Current.

experimental data obtained in this way. The shape of the curve in any particular case depends upon the character of the iron core. The results should be interpreted with reference to a curve of magnetization, — either a typical curve or a curve for the iron core used in the experiment. In this connection, see Fleming, Alternate Current Transformer, Vol. 1, pp. 54–63.

The coefficient of self-induction should be measured for different currents in a coil without iron, and the results compared with the above.

In Exp. 7 curves were obtained for the establishment and dying away of the current in a circuit with constant coefficient of self-induction. If the coefficient of self-induction is variable, these curves will differ from the exponential ones there obtained, and may be constructed from the curve showing

the value of the self-induction for different currents, by the following graphical method due to Dr. Sumpner.*

In the case of the dying away of the current, when the electromotive force is removed, the electromotive force of

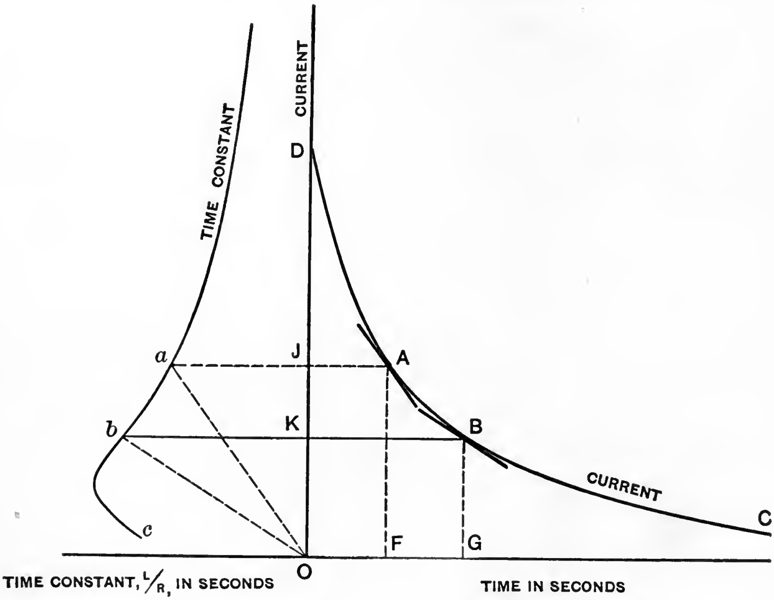


Fig. 60.—Dying away of Current in a Circuit with Variable Self-Induction.

self-induction, $L\frac{di}{dt}$, is still active in the circuit, and is expended in overcoming the ohmic resistance of the circuit. Hence

$$L\frac{di}{dt}+Ri=0; \quad \text{or,} \quad \frac{di}{dt}=\frac{i}{-\frac{L}{R}}.$$

This means that the slope of the curve representing the current for each instant of time is equal to the ratio of the current at that time to the corresponding value of the time constant.

* Philosophical Magazine, June, 1888.

In Fig. 60 a curve is drawn showing the dying away of the current after the removal of the electromotive force and the graphical method of constructing the curve from the time-constant curve is indicated. To the left of the origin the curve *a*, *b*, *c*, is drawn, showing the value of the time-constant in seconds for different values of the current. The slope of the current curve as it dies away is shown by lines drawn from O to the time constant curve. Thus, when the current has a value $\overline{OJ}$, the value of the time constant is $\overline{aJ}$; hence

$$\frac{\overline{OJ}}{\overline{aJ}}=\frac{i}{-\frac{L}{R}}=\frac{di}{dt}.$$

The tangent to the current curve is thus determined point by point. Thus, the tangent at A is parallel to $\overline{Oa}$; at B, parallel to $\overline{Ob}$, etc. The curve may be constructed by beginning at any point, and proceeding in each direction by taking points at short intervals. $\overline{OD}$ is the initial current, the value of which is assumed.

The curve for the establishment of the current, when the circuit is subjected to a constant electromotive force E, is obtained by a similar construction shown in Fig. 61. Here we have for any point B,

$$\frac{\overline{DK}}{\overline{Kb}}=\frac{\frac{E}{R}-i}{\frac{L}{R}}=\frac{di}{dt},$$

in accordance with the equation $E=Ri+L\frac{di}{dt}.$ The final current, $\overline{OD}$, depends upon the electromotive force assumed.

These curves differ from exponential curves according to the amount of variation in the self-induction and time-constant. Curves constructed by this method should be compared by the student with exponential curves. It should be shown that when the coefficient of self-induction is constant, this construction

gives an exponential curve. It is to be noted that the inverted curve for the dying away of the current is the same as that for the establishment when the self-induction is constant, but not otherwise.

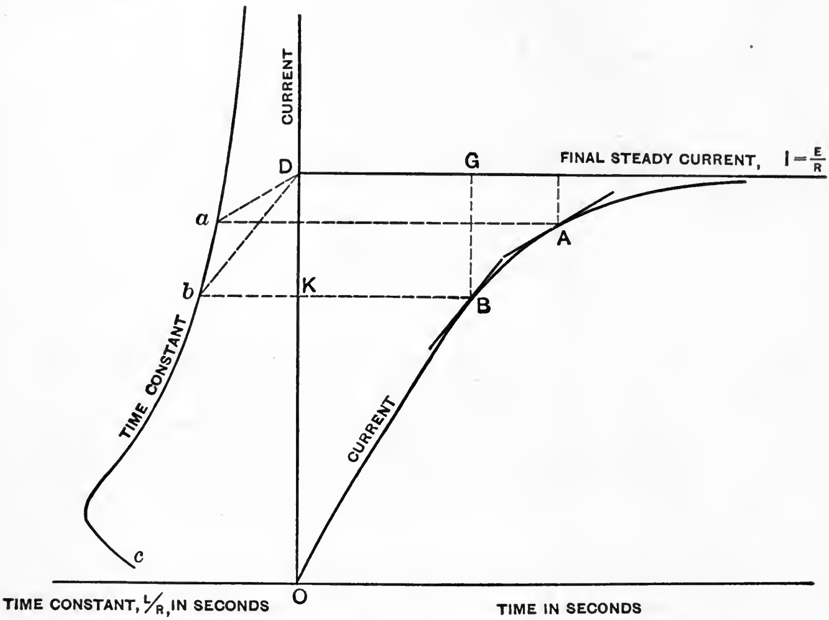


Fig. 61.—Establishment of Current in a Circuit with Variable Self-Induction.

EXPERIMENT 10. **Variation in the coefficient of self-induction with the saturation of the iron core.***

To show that the variation in the self-induction obtained in Exp. 9 is due to the saturation of the iron core, the coefficient of self-induction may be determined by one of the previous methods for a coil around an iron core whose magnetic saturation may be varied by passing a continuous current through an auxiliary coil wound around the core.

This experiment may be most easily performed by using for the coil to be tested one of the coils of a transformer. The

* See Fleming, I. p. 63; Sumpner, Phil. Mag., June, 1888, p. 468.

coefficient of self-induction of one coil of the transformer is to be determined when a constant continuous current of 1, 2, 3, etc., amperes is passing through the other coil of the transformer. A curve obtained in this way is given in Fig. 62 in which ordinates represent the coefficient of self-induction of the coil which is tested, and abscissæ denote the current in the auxiliary magnetizing coil. It is to be noted that the results are affected by previous changes in the magnetization.

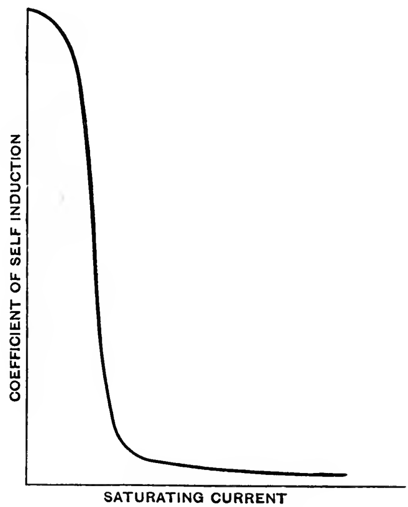


Fig. 62.—Variation in the Coefficient of Self-Induction with Saturation.

When the coefficient of self-induction is determined by one of the alternating current methods given above, the alternating current should not be large enough to affect the saturation of the coil.

Many of the remarks made in connection with the previous experiment apply equally well here. The results should be interpreted.

Experiment 11. **Effects of the variation of the resistance in a series circuit.**

If the electromotive force supplied to a series circuit containing resistance and self-induction is maintained constant, and the ohmic resistance is varied, the current will be changed not only in magnitude, but also in direction; that is, its position with reference to the electromotive force will be changed. As the resistance is diminished, the current will be increased, but will lag further behind the electromotive force in phase.

Let the series circuit be as shown in Fig. 57, in which R_1 is a non-inductive resistance, R_2L_2 is a coil with self-induction L_2 and resistance R_2. Measure the current with an ammeter A, and the various falls of potential with a voltmeter as follows: E, the impressed electromotive force; E_1, the fall of poten-

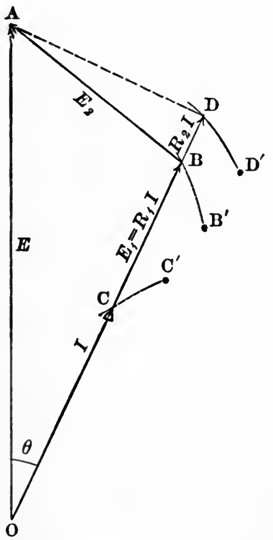


Fig. 63.

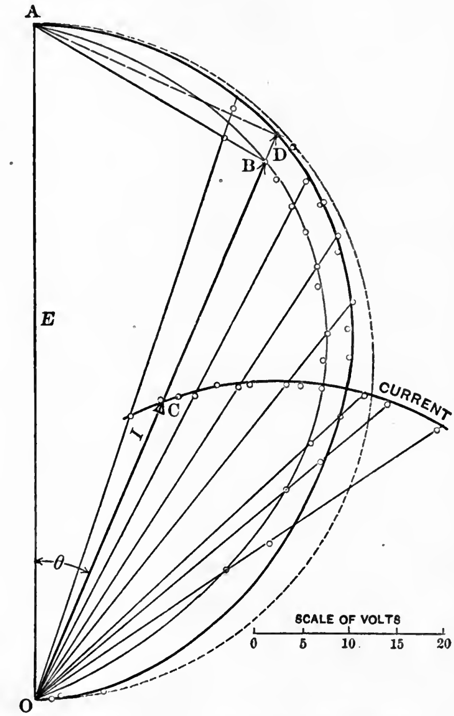


Fig. 64.—Effect of Variation of Resistance in a Series Circuit

tial around the non-inductive resistance R_1; E_2, that around the coil R_2L_2.

Draw a triangle, OAB, Fig. 63, with the observed values of E, E_1, and E_2, as the three sides. Lay off $\overline{OC}$ in the direction of $\overline{OB}$ equal to the current I, in any convenient scale. Produce $\overline{OB}$ to D by an amount $\overline{BD}=R_2I$, the electromotive force to overcome the ohmic resistance in the coil. $\overline{OD}$ is the electromotive force to overcome ohmic resistance in the entire circuit.

The current and various electromotive forces are now represented in magnitude and direction for one value of the resistance.

Change the resistance R_1, maintain the impressed electromotive force E the same as before, and repeat all measurements. Locate the points B', C', D', in the figure according to the new measurements in the same way as the points B, C, D, were located. Again change the resistance and take measurements,

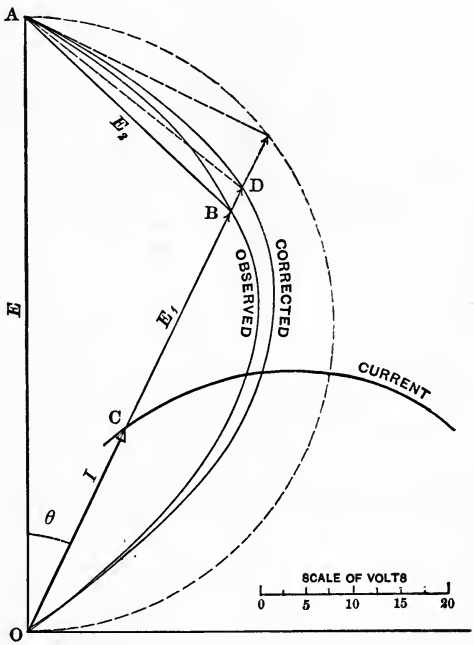


Fig. 65.

and repeat until points enough are obtained to define the curves forming the loci of the points B, C, and D.

It is not necessary to maintain the impressed electromotive force constant; for if it varies, all readings may be reduced by direct proportion to correspond to some constant value. The speed of the alternator should, however, be kept constant so that the frequency is constant throughout the experiment.

In Fig. 64 the loci for the points B, C, and D are plotted according to the method given above from measurements upon a coil with no iron. The dotted curve is a semicircle, showing the theoretical * locus of D, under the assumption that the current was a perfect sinusoid, and that there was no loss due to hysteresis or foucault currents. The departure of the observed locus from this semicircle is in this case largely due to the foucault currents set up in the coil itself. In Fig. 65 are shown curves from measurements upon a coil wound upon a hollow brass spool. The foucault current losses in this case were very great, and caused a marked deviation of the locus of D from a semicircle. An iron core would likewise cause a departure from a semicircle on account of hysteresis as well as foucault currents.

The results of this experiment may also be shown by plotting curves with rectangular instead of polar co-ordinates. The total resistance of the circuit in this case should be plotted as abscissæ, and current and angle of lag as ordinates.

These curves should be compared with curves plotted according to the relations,

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}}, \text{ and } \tan\theta = \frac{L\omega}{R}.$$

The value of the self-induction may be obtained from the line $\overline{AD}$, in Fig. 63, which is equal to $L\omega I$. Similarly, the values of R_1 may be found from $\overline{OB}$.

Experiment 12. **Effects of the variation of the self-induction in a series circuit.**

This experiment is in many respects similar to the previous one in which the resistance of a series circuit was altered. We have to find the variation in the direction and magnitude of the current in a series circuit in which the ohmic resistance

---

* Alternating Currents, p. 223.

is constant and the self-induction varied. The arrangement of the circuit is the same as in the previous experiment. The various falls in potential, E, E_1, and E_2, are measured as before, and the current found by direct measurement or calculated from E_1 and R_1; for $E_1 = R_1 I$. In order to be able to compare the results with those derived theoretically,* the self-induction must be varied without change of resistance. Hysteresis and foucault currents must be absent. This variation of the self-induction may be obtained by two concentric solenoids, one of which slides within the other. It is difficult, however, to obtain sufficient self-induction without iron. When coils with iron cores are used, the effect of hysteresis and eddy-currents must be borne in mind.

EXPERIMENT 13. **Electromotive forces in a series circuit.**

The object of this experiment is to ascertain the magnitudes and directions of the electromotive forces in the several portions of a series circuit consisting of a number of inductive and non-inductive resistances.

Pass an alternating current through a series circuit, such as that shown in Fig. 66. Measure the current and the total

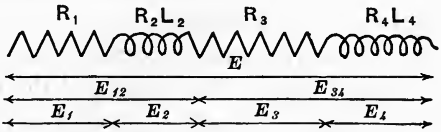


Fig. 66.

impressed electromotive force, and the falls in potential around the different portions of the circuit, separately and combined.

The observed electromotive forces, E, E_{12}, E_1, etc., are drawn as the lines $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, etc., in Fig. 67. The current I is in the direction of the electromotive force E_1, in one of the non-inductive portions of the circuit. *If hysteresis is*

* Alternating Currents, p. 223.

negligible, the projections $\overline{OC}$, $\overline{CJ}$, etc., of the observed electromotive forces, upon a line drawn in the direction of the current, represent the electromotive forces used in overcoming the resistance in each portion of the circuit. The projections upon a line $\overline{GA}$ at right angles to the current represent the electromotive forces to overcome self-induction. It is to be noted that, treating the circuit as a whole, the total resistance and total self-induction are obtained by adding algebraically the separate resistances and self-inductions.

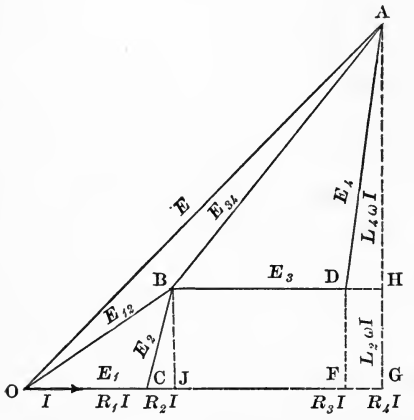


Fig. 67.—Electromotive Forces in a Series Circuit.

EXPERIMENT 14. **Measurement of power. Three-voltmeter method.**

In a non-inductive circuit the power may always be directly found from the product of the electromotive force supplied to the circuit, and the current flowing in it. In an inductive circuit, in which the current and electromotive force are not in phase, the power is equal to the product of the electromotive force and current (square root of mean square values), and a power factor equal to $\cos\theta$, where θ is the angle by which the current lags behind the electromotive force; that is, $W = \overline{EI}\cos\theta$.

When the current and electromotive force are harmonic, the angle θ is the true angle of lag of the current behind the electromotive force. If the current and electromotive force are not harmonic, strictly speaking there is no such thing as angle of lag. However, we may represent the current and electromotive force by equivalent sinusoids (with the same square

root of mean square values and the same power) and take $\cos\theta$ as the power factor.

To measure the power in an inductive circuit, by the three-voltmeter method, a non-inductive resistance is placed in series with it as in Fig. 57, in which R_2L_2 is the circuit whose power is to be measured, and R_1 is the non-inductive resistance in series. The measurements consist in three voltmeter readings and the measurement of the current. Voltmeter readings are taken of the total electromotive force $\overline{E}$, supplied to both the inductive and the non-inductive portions of the circuit, and of $\overline{E}_2$ and $\overline{E}_1$, between the terminals of the inductive and non-inductive portions separately. One voltmeter may be used for the three readings. If an electrostatic voltmeter is used, no error is introduced. A voltmeter taking current introduces an error which is only negligible when the current taken by the instrument is small compared with the main current.

The power * supplied to the inductive circuit R_2L_2 is

$$W_2=\frac{I}{2\,\overline{E}_1}(\overline{E}^2-\overline{E}_1^2-\overline{E}_2^2).$$

If a known non-inductive resistance R_1 is used, it is not necessary to measure the current, for it is equal to $\frac{\overline{E}_1}{R_1}$. In this case the formula for the power is

$$W_2=\frac{1}{2\,R_1}(\overline{E}^2-\overline{E}_1^2-\overline{E}_2^2).$$

The angle of lag between the current and electromotive force E_2 of the inductive circuits is

$$\cos\theta_2=\frac{\overline{E}^2-\overline{E}_1^2-\overline{E}_2^2}{2\,\overline{E}_1\overline{E}_2}.$$

The results should be graphically shown, as in Fig. 58, and the above formulæ proved geometrically for the case where the

* Fleming, Alternate Current Transformer, Vol. 2, p. 555; also Alternating Currents, p. 232.

current is harmonic. The three-voltmeter method is, however, true for any circuit whether the current is harmonic * or not. It possesses the disadvantage that small errors in observation may introduce large errors in the results, especially where the angle of lag is quite large. For maximum accuracy, $\overline{E}_1=\overline{E}_2$. The method is useful, however, even in cases where it cannot be directly applied, in checking up other methods.

Ordinarily as much power is consumed in the non-inductive resistance as in the inductive circuit upon which the measurements are made. Furthermore, the electromotive force which must be supplied is nearly twice that required by the inductive circuit alone.

EXPERIMENT 15. **Measurement of power. Three-ammeter method.**

The power in an inductive circuit may be measured by means of three ammeter readings by a method quite analogous to the three-voltmeter method given in the preceding experiment. A non-inductive resistance R_1 (Fig. 68) is placed in parallel with the inductive circuit R_2L_2, the power in which is to be measured. Measurements are made of $\overline{I}_1$ and $\overline{I}_2$, the currents in the non-inductive and inductive circuits, and of $\overline{I}$, the main current. These may be graphically represented as in Fig. 69. The current $\overline{I}_2$, in the inductive circuit, lags behind the impressed electromotive force by an angle θ_2. The value of this angle of lag may be obtained by the student from the geometry of Fig. 69; thus, †

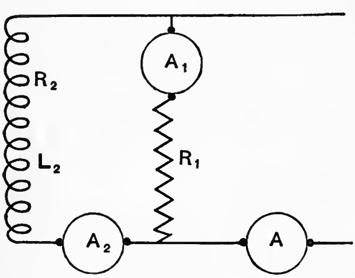


Fig. 68. Three-Ammeter Method.

$$\cos\theta_2=\frac{\overline{I}^2-\overline{I}_1^2-\overline{I}_2^2}{2\,\overline{I}_1\overline{I}_2}.$$

* Ayrton and Sumpner, Proc. Roy. Soc., Vol. 49, 1891, p. 424; The Measurement of Power given by Any Electric Current to Any Circuit.

† Note that this is the power factor even for currents that are not sinusoidal. See previous experiment.

By substituting the value for $\cos \theta_2$ in the expression $\bar{E}\bar{I}_2 \cos \theta_2$, for the power expended in the inductive circuit, and writing $R_1\bar{I}_1$ for $\bar{E}$, we obtain an expression for the power in terms of the three current readings; thus,

$$W_2 = \tfrac{1}{2} R_1(\bar{I}^2 - \bar{I}_1^2 - \bar{I}_2^2).$$

Where the value of the non-inductive resistance R_1 is not known, the power may be computed from the three ammeter

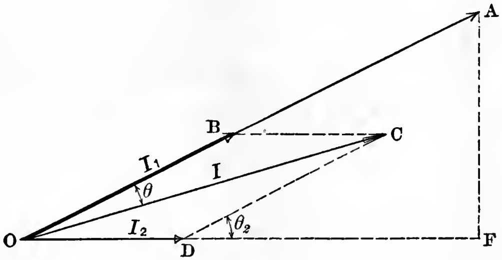


Fig. 69.

readings and the impressed electromotive force which must then be measured. We then have

$$W_2 = \frac{\bar{E}}{2\,\bar{I}_1}(\bar{I}^2 - \bar{I}_1^2 - \bar{I}_2^2).$$

This method is open to some of the same objections as the three-voltmeter method. A small error in observation produces a large error in the results. The non-inductive resistance consumes as much power as the inductive circuit being measured. In fact, it consumes more than the inductive circuit when the branch currents are made approximately equal for greatest accuracy. The one advantage of the three-ammeter method over the preceding is that it does not require a greater impressed electromotive force than is needed to supply the inductive circuit. Furthermore, in the three-voltmeter method, the three readings could be made with but one voltmeter without introducing much error. Considerable error, however, is introduced

in the three-ammeter method if one instrument is used and shifted from one position to another. Error is introduced by self-induction in the ammeter used to measure $\bar{I}_1$.

Combination Method.—A modification of the three-ammeter method — sometimes called the combined voltmeter and ammeter method—is shown in Fig. 70. Evidently, if the resistance R_1 is known, the power is obtained from the readings of the two ammeters and the voltmeter; thus,

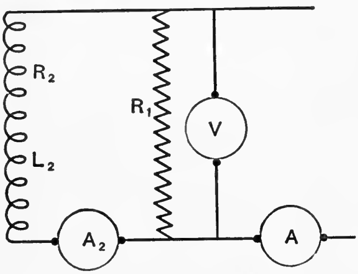


Fig. 70. Combined Ammeter and Voltmeter Method.

$$W_2 = \tfrac{1}{2} R_1 \left\{ \bar{I}^2 - \left(\frac{\bar{E}}{R_1}\right)^2 - \bar{I}_2^2 \right\}.$$

In most cases this is the most accurate of these methods. The ammeter A_2 may be compared with A by breaking the circuit through the resistance R and the voltmeter V. By breaking the inductive circuit, the ammeter A measures the current through the non-inductive resistance and voltmeter. The voltmeter reading divided by the ammeter reading gives the value of R_1 to be used in the above formula. (The value of the resistance thus obtained is really that of the non-inductive resistance and voltmeter in parallel, which gives greater accuracy than the use of R_1 alone.)

When the power measured is small, a deduction should be made for the power consumed in the ammeter A_2. When only one ammeter is available, and that is shifted back and forth between the two circuits, an error is introduced which becomes small when the power measured is large.

Experiment 16. **Equivalent resistance and self-induction of parallel circuits.**

When an alternating current is passed through two or more coils connected in parallel, the effect produced by the several

coils is the same as would be produced by a single coil with proper resistance and self-induction substituted in place of the several coils in parallel. The resistance R' and self-induction L' of the equivalent single coil are called the equivalent resistance and equivalent self-induction, respectively, of the parallel coils. For the experimental study of equivalent resistance and self-induction, take two coils between which there is no mutual induction. First determine the impedances, J_1 and J_2, the resist-

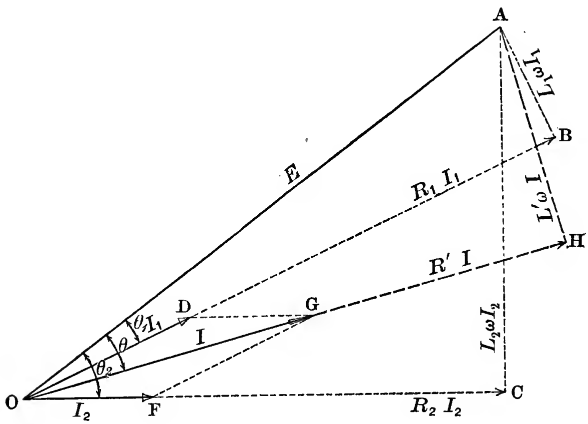


Fig. 71.

ances, R_1 and R_2, and the coefficients of self-induction, L_1 and L_2, for each coil separately.

Connect the two coils in parallel, and subject to an alternating impressed electromotive force E. Measure the total current I which flows through the two coils. Compute the current in each branch by dividing the impressed electromotive force by the impedance; thus,

$$I_1 = \frac{E}{J_1}; \quad I_2 = \frac{E}{J_2}.$$

The results may be shown graphically as in Fig. 71. The electromotive force impressed is represented (to scale) by $\overline{OA}$. The current I_1 in the first branch is drawn in the direction of

$\overline{OB}$, lagging behind the impressed electromotive force by an angle θ_1, the tangent of which is $\frac{L_1\omega}{R_1}$. Note that $\overline{OB}=R_1I_1$; $\overline{BA}=L_1\omega I_1$. The current I_2, in the second branch, is similarly drawn in the direction of $\overline{OC}$, which is equal to R_2I_2; $\overline{CA}=L_2\omega I_2$. The line $\overline{OC}$ is the resultant or geometrical sum of I_1 and I_2, and represents the total current. It should equal the value of the current obtained by measurement. The construction is completed by producing the line $\overline{OG}$ to H so that OHA is a right angle. The line $\overline{OH}$ is equal to $R'I$, the product of the equivalent resistance of the parallel circuits and the total current through them. The current being known, the value of the equivalent resistance R' may be determined. Similarly, $\overline{HA}=L'\omega I$, from which the value of L', the equivalent self-induction of the coils, is found. This value should be compared with those found analytically, according to the formula given on page 241 of ALTERNATING CURRENTS.

The impedance is calculated from these results; thus, $J'=\sqrt{R'^2+L'^2\omega^2}$. The impedance may be found graphically as follows: Lay off the reciprocal of the impedance of each branch in the direction of the corresponding current. The geometrical sum of these reciprocals is the reciprocal of the impedance of the combination. The reciprocal of impedance has been called *admittance*, and is measured in *mhos*, corresponding to conductance, which is the reciprocal of resistance. The admittance of a system of parallel circuits is the vector sum of the admittances of the separate branches.

The impedance may be found directly by dividing the impressed electromotive force by the total current.

What is said here in regard to two parallel circuits is true of any number of circuits, provided there is no mutual induction between them.

The advanced student may take up the case of two parallel circuits mutually related. This case would be obtained by connecting the primary and secondary of a transformer in parallel. The current in each branch, as well as the total current, should be measured. The total current may be less than that in one of the branches. If the coefficient of mutual induction is

determined by separate experiment, the results may be laid off graphically, the impressed electromotive force in each circuit being equal to the vector sum of three components due to resistance, self-induction, and mutual induction. Reversing the connections of one of the coils reverses the direction of the effect of the mutual induction, but leaves other actions unchanged.

EXPERIMENT 17. **Effect of frequency upon impedance of a circuit containing resistance and self-induction.**

For this experiment it is necessary to be able to vary the frequency of alternation of the alternating current used. The variable frequency may be obtained by varying the speed at which the alternator is run. The speed of the alternator may be readily controlled if it is driven by a cone pulley, or by a motor, the speed of which may be regulated.

The values of the resistance and self-induction of each circuit or coil used in performing this experiment should first be determined.

Pass an alternating current through the coil; measure the current and the fall in potential around the coil, taking these measurements for as wide a range of frequencies as possible.

Compute the value of the impedance from each observation by taking the ratio of the fall in potential around the coil to the current; thus, **Impedance** $= E \div I$. Compute the impedance for each frequency also from the expression, **Impedance** $= \sqrt{R^2 + L^2\omega^2}$. Plot the results as two curves, with frequency or ω as abscissae; one curve from observation and one from calculation.

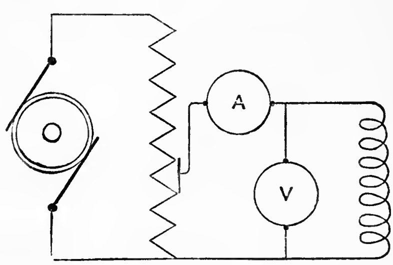


Fig. 72.

Plot a curve showing the value of the current for different frequencies with a constant impressed electromotive force. The data for this curve may be obtained in two ways: first, by maintaining the electromotive force constant; and second, by reducing results, by proportion, to correspond to some

constant electromotive force, when the electromotive force is not kept constant. The electromotive force of the alternator depends directly upon its speed; it may be kept constant for different speeds by regulating the exciting current. When the electromotive force of the alternator is not maintained constant, the electromotive force impressed upon the coil may be kept constant by an adjustable resistance in series, or shunt

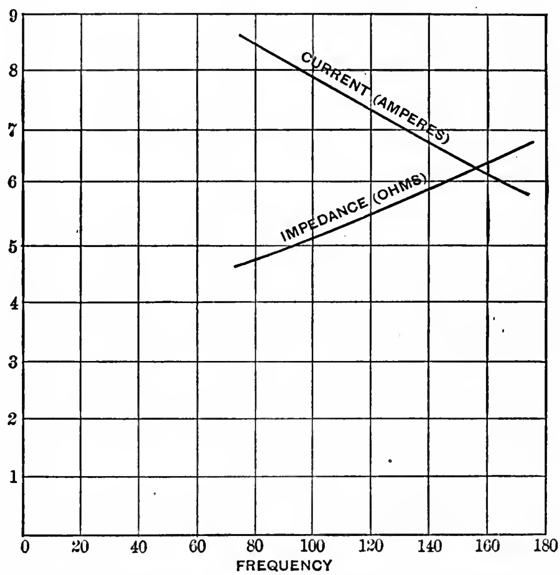


Fig. 73.—Dependence of Current and Impedance upon Frequency in a Circuit with Resistance of 4 Ohms and Self-Induction of .005 Henry.

as in Fig. 72. The value of the current can also be computed from the expression $I=\frac{E}{\sqrt{R^2+L^2\omega^2}}$. Values so obtained should be plotted and compared with the curve found above.

In Fig. 73 is shown the value of the impedance for different frequencies in a circuit with a resistance of 4 ohms and self-induction of .005 henry. In the same figure a curve is drawn showing the current which would flow with an impressed electromotive force of 40 volts. The forms of these curves

depend upon the values of the resistance and self-induction, and will differ considerably with different circuits.

The experiment should be performed with a coil, in which the self-induction is large compared with the resistance (that is, one with a large value for the time-constant $\frac{L}{R}$), and with a coil with a small time-constant. It should likewise be performed with a non-inductive resistance.

EXPERIMENT 18. **Effect of frequency upon angle of lag in a circuit containing resistance and self-induction.**

Connect a coil with known resistance and self-induction in series with a non-inductive resistance, as in Fig. 57. Measure

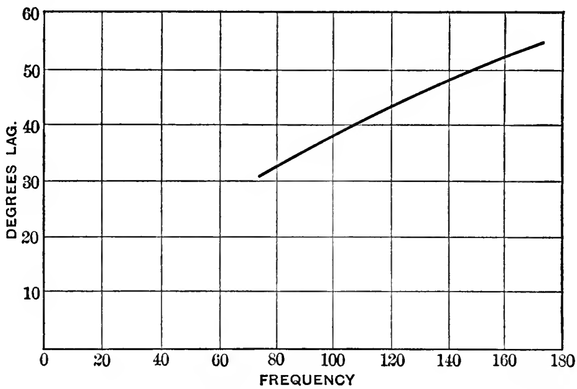


Fig. 74.—Dependence of Angle of Lag upon Frequency in a Circuit with Resistance of 4 Ohms and Self-Induction of .005 Henry

the fall of potential around the coil, around the non-inductive resistance, and around the two. Repeat these measurements for different frequencies. If the three voltmeter readings for any one observation are plotted, as E, E_1, and E_2, Fig. 58, the angle of lag of the current in the inductive part of the circuit, R_2L_2, is θ_2.* Such a diagram should be plotted from

* Alternating Currents, p. 230.

each set of observations, and the angle of lag corresponding to each frequency thus obtained. The results can be shown by a curve such as Fig. 74.

The value of the angle of lag may be computed for each frequency by constructing a right triangle, with sides equal to R_2 and $L_2\omega$ respectively. The triangle thus constructed will correspond to the triangle ABC in Fig. 58. Tan $\theta_2 = \frac{L_2\omega}{R_2}$. The value of the angle of lag thus calculated should be compared with the result found by the three-voltmeter measurements.

Experiment 19. **Measurement of mutual induction. Ballistic method.**

The coefficient of mutual induction between two coils may be determined by ascertaining the quantity of electricity which flows in one of the circuits when a constant current in the other circuit is made or broken. Connect one coil, which we will call the primary, with a constant source of current, and include in the circuit an ammeter and reversing switch. Connect a ballistic galvanometer with the secondary. Reverse the primary current and note the throw of the galvanometer. The quantity of electricity which flows in the secondary when the primary current I_1 is reversed is

$$Q = \frac{2\,MI_1}{R_2},$$

where R_2 is the total resistance of the secondary circuit including the galvanometer. This quantity is determined from the throw of the galvanometer according to the relation $Q = k\theta$, where k is the galvanometer constant* and θ the throw.

The value of the coefficient of self-induction, M, should be determined from a series of observations taken with one

* See Exp. 60.

coil as the primary, and then from a second series taken with the other coil as primary.

If the coils have an iron core, the value of the mutual induction will depend upon the saturation of the iron core. Measurements should be made with different primary currents and a curve plotted showing the coefficient of mutual induction for different magnetizing currents, or magnetizing ampere turns where the number of primary turns is known. The results should be interpreted with reference to the magnetization of the iron. (See Exps. 9, 10, and 62.) If the primary current is broken instead of reversed, a measure of the residual magnetism is obtained.

Experiment 20. **Measurement of mutual induction. Alternating current method.**

When an alternating current flows in one of two coils, an electromotive force is induced in the other coil. This electromotive force is proportional to the coefficient of mutual induction of the two coils, and may therefore be taken as a measure of it. When the current is harmonic, the secondary induced electromotive force is $E_2 = M\omega I_1$, where $\omega = 2\pi \times$ frequency. The coefficient of mutual induction may be thus determined by measuring the primary current, secondary electromotive force, and the frequency. It is to be noted that the secondary is to be kept on open circuit, otherwise the measured secondary electromotive force is different from the induced electromotive force, on account of the fall of potential due to secondary resistance and self-induction.

The mutual induction should be found by using each coil in turn as primary.

The value of the coefficient of mutual induction changes, if iron be present, with the saturation of the iron. It should be measured for different currents and the variation shown by a curve in which the coefficient of mutual induction is plotted as ordinates, and ampere turns as abscissas.

Similar curves should be obtained for the coefficients of self-induction of each of the coils, found by the methods described in previous experiments, these curves being also plotted with ampere turns as abscissas.

From a comparison of the foregoing results, the amount of magnetic leakage may be obtained. In case of no magnetic leakage, the value of the coefficient of mutual induction is a mean proportional to the values of the two coefficients of self-induction; thus $M=\sqrt{L_1L_2}$. This value of the coefficient of mutual induction is to be compared with the measured value for different degrees of saturation expressed in ampere turns. The difference between the two values is the amount of magnetic leakage, and may be expressed in per cent. In the case of a transformer, this is the magnetic leakage for no load, *i.e.* with the secondary open. The magnetic leakage when a current is flowing through the secondary is somewhat greater.

The coefficients of induction found by experiment are to be compared with the values

$$L_1=\frac{4\pi S_1^2A\mu}{l},$$

$$L_2=\frac{4\pi S_2^2A\mu}{l},$$

$$M=\frac{4\pi S_1S_2A\mu}{l}.$$

These are the values in case of no magnetic leakage for two coils wound upon a uniform closed magnetic circuit of length l, cross-section A, and permeability μ. From them it follows that

$$L_1 : L_2 : M :: S_1^2 : S_2^2 : S_1S_2.$$

From the ratio of the values of L_1 and L_2 the ratio of the number of turns in the two coils may be computed.

EXPERIMENT 21. **Study of a transformer.**

The conditions under which the transformer is designed to be operated should be given (constant potential or constant

current, step-up or step-down), the ratio of transformation, output, and a general description of the arrangement of parts. The following data should be given:

Magnetic Circuit. — Form and dimension (in centimeters), including cross-section; mean length and volume; nature of lamination; weight of iron.

Primary and Secondary Circuits. — Number of turns; size wire; thickness of insulation; length of each turn; total length; weight of copper; normal (full load) current; circular mils per ampere; resistance hot; resistance cold; coefficient of self-induction; power expended in heat (RI^2); fall in potential due to ohmic resistance (RI).

The coefficient of mutual induction and magnetic leakage at no load should be obtained as in the previous experiment.

When a measure of the insulation is desired, the insulation resistance between the two coils and between each coil and the iron may be found by the method of Exp. 32.

EXPERIMENT 22. **Transformer test. Three-voltmeter method at no load.**

Before operating the transformer, a complete study should be made of it as in Exp. 21. The following experiments should be made with a constant potential at the terminals of the primary of the transformer. The transformer should first be run at no load, that is with the secondary open, and the no-load losses determined by the three-voltmeter method described in Exp. 14. This necessitates a non-inductive resistance in series with the primary, and a source of electromotive force considerably greater than that which is to be maintained at the primary terminals. For maximum accuracy, the fall in potential through this resistance should equal the fall in potential through the primary. A series of readings should be taken in order to eliminate errors of observation, inasmuch as a small error in observation may make a large error in the result.

The three voltmeter readings should be drawn as a triangle so as to show the angle by which the current lags behind the electromotive force at no load. The value of this angle and the power factor should be determined.

The losses due to the heating of the primary conductor during this no-load run are $W_{Cu_1}=R_1 I_1^2$, where R_1 is the hot primary resistance, and I_1 the primary current at no load. By subtracting these copper losses from the total no-load losses, we find the iron losses, W_{Fe}, due to hysteresis and foucault or eddy currents. Inasmuch as the magnetization is constant in a constant potential transformer, it follows that the iron losses are constant for all loads.

A second run should be made to ascertain the efficiency and regulation of the transformer at different loads. The primary of the transformer is supplied with the same constant potential as in the first run and the secondary connected to a variable non-inductive load. The secondary load is changed from no load to full load, or up to the carrying capacity of the conductors. Measurements are taken, at each load, of the primary potential (in order to maintain it constant), primary current, and of the secondary potential and current.

The power, W_2, expended in the secondary load is equal to the product of the secondary current and the difference of potential at the secondary terminals, inasmuch as the load is non-inductive and the current is in phase with the electromotive force. The primary and secondary copper losses are directly computed; thus,

$$W_{Cu_1}=R_1 I_1^2; \quad W_{Cu_2}=R_2 I_2^2.$$

The iron losses are already known from the first run. The power put into the transformer is equal to the power taken out and made use of in the secondary load, plus the several losses;

thus, $$W_1=W_2+W_{Cu_1}+W_{Cu_2}+W_{Fe}.$$

All these quantities being known, the efficiency is directly

obtained by dividing the output by the power supplied to the transformer; thus,

$$\text{Efficiency} = W_2 \div W_1.$$

The power factor should be computed for each load by dividing the primary power by the product of the primary current and electromotive force.

In the determination of efficiency by this method, the primary current enters only in the expression for the loss in

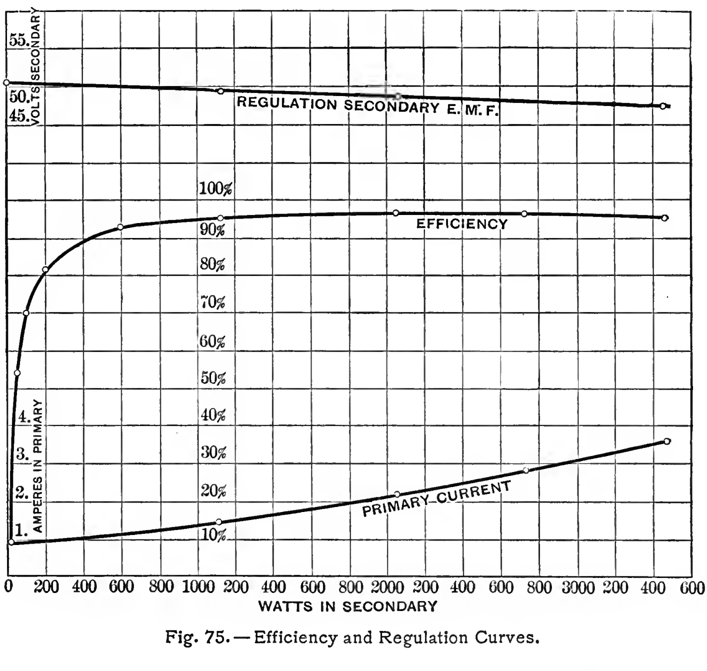


Fig. 75.—Efficiency and Regulation Curves.

the primary conductor, and may be obtained, with sufficient accuracy in most cases, by computation from the secondary current. The value of the primary current at any load is equal to the diagonal of a right angle triangle of which one side is equal to the no-load primary current I_0, and the other side is equal to the secondary current divided by the ratio of transformation; thus,

$$I_1 = \sqrt{I_0^2 + \frac{S_2^2}{S_1^2} I_2^2}.$$

Curves should be plotted with secondary current or with secondary output as abscissas, and all other quantities as ordinates; namely, E_2, I_1, W_2, W_{Cu_2}, W_{Cu_1}, W_{Fe}, W_1, and efficiency. In Fig. 75 are given curves for an open-magnetic-circuit transformer possessing good regulation and particularly high efficiency.

EXPERIMENT 23. **Transformer test. Three-voltmeter method at all loads.**

This experiment is substantially the same as the preceding; the primary power is, however, measured at all loads by the three-voltmeter method, instead of being computed by adding the losses to the secondary power. All other measurements are made as before. Triangles should be drawn to represent the three-voltmeter readings at each load, and the angle of lag between the primary current and electromotive force determined, together with the power factor.

EXPERIMENT 24. **Transformer test. Three-ammeter method at no load.**

The no-load losses of the transformer are determined according to the three-ammeter or combination method described in Exp. 15. The transformer is then run at different loads to determine its efficiency and regulation, as in Exp. 22. The advantage of this method over the three-voltmeter method used in the two preceding experiments is that it is not necessary to have an electromotive force greater than that supplied to the transformer — a requirement which renders the voltmeter method undesirable in many cases.

EXPERIMENT 24 *a*. **Transformer test. Three-ammeter method at all loads.**

This experiment consists in an efficiency test of a transformer. The primary power at each load is measured by the three-ammeter method. All other measurements and computations are made as in the preceding experiments.

EXPERIMENT 25. **Variations in transformer diagrams.***

In this experiment the transformer is run at different loads, and data are obtained so that a polar diagram can be drawn for each load showing the magnitude and direction of the primary and secondary currents and electromotive forces, as in Fig. 76. The measurements are the same throughout, as in Exp. 22. From the primary measurements the values of the primary current and electromotive force are obtained, and the angle between them, together with the power supplied the transformer. In the secondary, the electromotive force and current are obtained by direct measurement. The product of the two gives the secondary power, inasmuch as the load is non-inductive. The ratio of the two gives the value of the resistance of the secondary load which, added to the resistance of the secondary of the transformer, gives the total resistance of the secondary circuit. The secondary angle of lag (*i.e.* the angle by which the secondary current lags behind the *induced* electromotive force in the secondary) is computed thus:

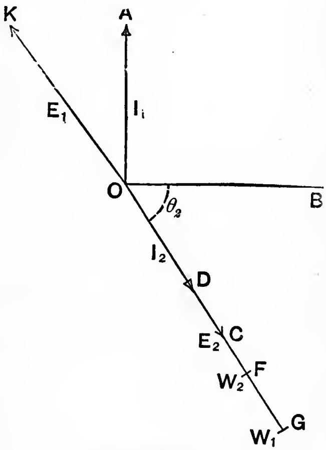


Fig. 76.—Polar Transformer Diagram.

$$\theta_2 = \frac{L_2\omega}{R_2}.$$

The polar diagram (Fig. 76) is constructed as follows: $\overline{OA}$ represents the primary current in magnitude and direction, the vertical direction being arbitrarily taken. $\overline{OK}$ represents the primary electromotive force which is in advance of the primary current by an angle which is calculated from the three-voltmeter

* See "Theory of the Transformer," Bedell and Crehore, Electrical World, 1893, Vols. 21 and 22; also, "Transformer Diagrams Experimentally Determined," Bedell, Proceedings of the International Electrical Congress held at Chicago, 1893.

measurements. (In all polar diagrams rotation is counter-clockwise.) The electromotive force *induced* in the secondary is 90° behind the primary current. This is not measured, but is represented in direction by the line $\overline{OB}$ drawn at right angles to $\overline{OA}$. The secondary angle of lag, θ_2, computed as above, is laid off from $\overline{OB}$. In this direction $\overline{OD}$ is drawn to represent the secondary current to scale, and $\overline{OC}$ to represent the secondary

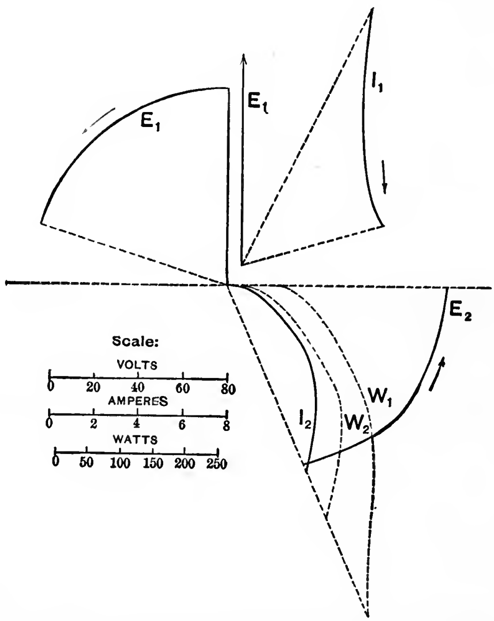


Fig. 77.—Polar Diagram for a Constant Potential Transformer at Different Loads.

electromotive force. This is the electromotive force measured at the terminals of the transformer, and is available to supply a secondary line. The lines $\overline{OF}$ and $\overline{OG}$ are laid off to represent the power in the secondary and in the primary.

A diagram like Fig. 76 may be constructed for each load. The diagrams taken for successive loads differ by small amounts both in the magnitude and direction of the various quantities represented. These variations should be shown by drawing

the diagrams for different loads in one figure, and marking the curves which form the loci of different quantities. Such curves are shown in Fig. 77 for a constant potential transformer. In this diagram there are two parts which will be considered separately. If the direction of the primary current be taken in all cases as vertical, the locus of the primary electromotive force, which is constant, will be the arc of a circle E_1, drawn to the left. The variation in primary electromotive force is in direc-

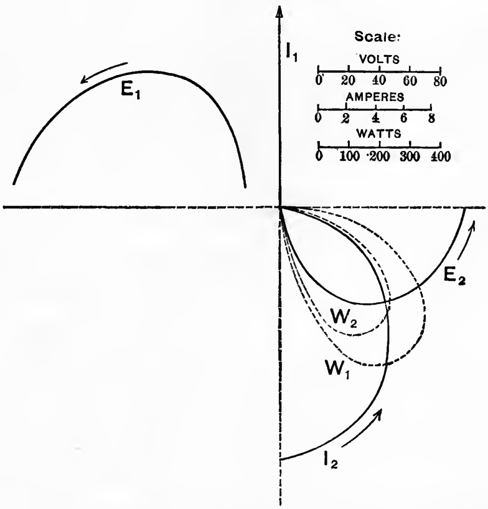


Fig. 78.—Polar Diagram for a Constant Current Transformer at Different Loads.

tion only. The curves for the secondary current and electromotive force, and for the secondary and primary power, are obtained point by point from the positions of D, C, F, and G. The variations in the primary current cannot be shown in the diagram thus constructed, with the primary current always in a constant vertical direction. The auxiliary diagram in the upper right-hand corner of the figure is drawn in order to show the changes in the primary current with different loads. In this auxiliary diagram, the primary electromotive force is represented by the vertical line E_1. It is constant both in magnitude

and direction. The locus of the primary is then found to be a curve I_1. The changes in the magnitude and direction of the primary current are thus shown.

The meaning of these curves is that the vectors representing the various quantities vary in magnitude and direction as the load of the transformer is changed; the curves form loci upon which the vectors lie. The arrows indicate the direction of change as a secondary resistance is increased. In the constant potential transformer, this represents a decrease of load.

For the sake of comparison, curves are given in Fig. 78 for a constant current transformer. In this case, the loci of the various currents and electromotive forces are found by experiment to be approximately semicircles, and may theoretically be shown to be such.

The variations in the different quantities should be shown by curves plotted with rectangular co-ordinates as well as by these polar diagrams.

The foregoing experiment includes an investigation of the effects of a change in the load of a transformer; that is, of a change in the secondary resistance. In this same way the effects may be studied which are produced by other changes of condition, such as change of frequency, change of the reluctance of the magnetic circuit, or of a self-induction in the secondary load.

For a fuller treatment of transformer diagrams, see the references given above.

Experiment 26. **Operation of a synchronous motor.**

If the field of an alternator be excited by a direct current, and the armature be supplied with an alternating current, the alternator will run as a motor if first brought up to the proper speed by some external means; but it will not start itself. The motor must be run at such a speed that, acting as a dynamo, it would generate an electromotive force of the same frequency as the current with which it is supplied as a motor. When

this condition is reached, the motor is said to be running in synchronism with the alternator; and if the two have the same number of poles, the speeds of the two will be the same. If the number of poles are not the same, the speed of the motor would not be the same as that of the alternator when in synchronism with it. In the laboratory for experimental work, in which the motor and alternator have the same number of poles, the former may be brought to speed by coupling direct to the latter, the two being separated when synchronism is reached; usually, however, this would not be possible. The motor may be brought to speed by a direct current motor, the current supplying which is broken when synchronism is reached. The direct current motor is now driven by the alternate current motor as a dynamo which may be loaded down at will. The field excitation of the alternate current motor should be such that the counter-electromotive force in the armature should not quite equal that of the generator supplying current to it. In the case of two similar machines with field circuits connected in parallel, this condition is brought about by adjusting a resistance in the motor field-circuit. When running, the brushes of the motor are connected directly to the line from the generator. While the motor is being brought into synchronism, it should not be connected directly to the line, but through an incandescent lamp which is to be cut out when synchronism is reached. This lamp serves two purposes: first, it prevents short-circuiting the generator through the armature of the motor when stationary, and likewise prevents excessive flow of current when the two machines are generating electromotive forces in the same direction; secondly, it acts as a phase indicator. When the two machines are synchronous, the lamp will be bright or dark, according to whether they be in the same or opposite phase. When the machines are not synchronous, beats will occur, and the intervals between these, as seen by the lighting and extinguishing of the lamp, will increase as synchronism is approached. The alternate current motor is

driven by the direct current motor until synchronism is reached. When the machines are practically synchronous, the lamp is to be cut out at a moment when the machines are in opposite phase (that is, when the lamp is dark); and, at the same moment, the currents supplying the direct current motor should be broken. When successfully done, the alternate current motor will continue to run on account of the current received from the alternator, and to drive the direct current motor as a dynamo. The cutting out of the lamp and the breaking of the circuit of the direct current motor should be done practically simultaneously, and this can be most easily accomplished by a suitably constructed switch. The experiment should be repeated under different conditions. The field current of the motor should be varied. The direction of the field current in the motor and generator should be reversed separately, and both at once, and the motor also run with armature connections reversed.

For investigating the armature lag of the motor the following phase indicator* may be used. The pilot lamp above described affords a simple and efficient means of starting a synchronous motor. It does not, however, indicate the moment when exact synchronism is reached, nor does it show whether the motor is running at a greater or less speed than that corresponding to the generator. It does not show the exact phase difference between the motor and generator, and does not indicate the phase relations after the motor has been connected to the alternator and is being driven by it. The phase indicator gives definite information in regard to the relative speeds and phase positions of the motor and generator. It shows

(1) When the machines are synchronous;

(2) Which machine is running the faster when they are not synchronous;

(3) The angle by which the motor lags behind the generator.

---

* "An Optical Phase Indicator and Synchronizer," by Moler and Bedell, Trans. Amer. Inst. Electrical Engineers, 1894.

In the simplest laboratory form of the phase indicator, the motor and generator are placed together with shafts in line and abutting, but not quite touching. The two machines must have the same number of poles, so that the revolution of the armature of each represents the same number of alternations. The abutting ends of the shafts carry two disks, one connected rigidly with the motor armature, the other similarly connected with the armature of the generator. In these disks are curved slits, one slit for each pair of poles of the machines. These slits are shown in Fig. 79 for an eight-

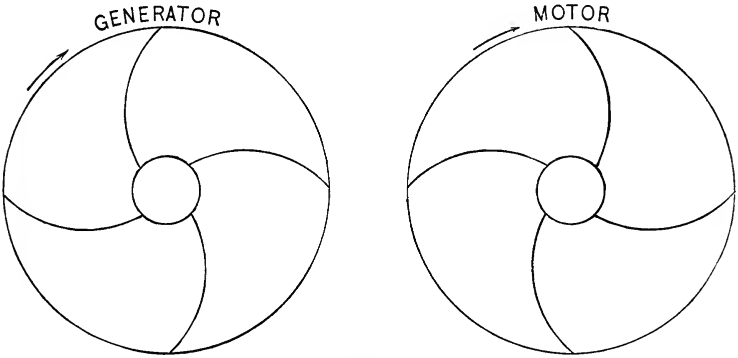


Fig. 79.

pole machine. The two disks are in every way similar, the one being the reverse of the other. The two disks are practically superimposed, and together form one disk with four holes, where the slits of one disk cross over the slits of the other. Evidently the distances of these four holes from the center depends upon the relative position of the two armatures; they move in and out as the armatures shift their relative positions. From the symmetrical arrangement of the slits, if one armature is stationary and the other is moved past two pole pieces, or through 90° of arc (corresponding to a complete period of alternation or 360° of phase), the intersection of the slits will be the same distance from the center as before. The curves in the slits are such that the distance to or from

the center that the intersections of the two sets of slits move is proportional to the change in relative position of the two armatures.

When the two armatures are running at the same speed in the same direction and there is a source of light on one side of the disks, the intersection of the slits, as seen from the other side, appears as a continuous ring of light. A slight difference of speed causes this ring of light to move outward or inward,

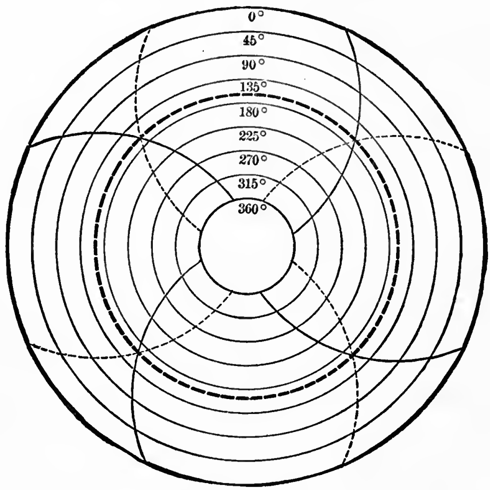


Fig. 80.—Bedell-Moler Phase Indicator.

according to which disk is revolving the faster. The more rapidly the ring moves in or out, the greater the difference in speed of the two disks. If the ring is moving out, a new ring starts at the center when one ring reaches the edges, and these rings keep following one another outward. If the difference in speed is the other way, the successive rings move inward.

In Fig. 80 the heavy dotted line represents the ring of light for a particular position of the two disks.

The position of the ring of light indicates the relative position of the two armatures. The disks may be secured to the

shafts so that when the armatures are in the same positions with reference to the pole pieces (*i.e.* the machines are in the same phase), the ring of light will be at the inner or outer ends of the slits. The concentric rings in Fig. 80 represent the phase differences corresponding to positions of the ring of light in this case.

A simple arrangement of the apparatus is as follows: On one side of the pair of disks is placed an incandescent lamp enclosed in a box. One side of the box is close to the disks, and has a slit in it about half an inch wide, extending from the shaft to the circumference of the disk. This slit is covered with a piece of oiled paper, so as to give a diffused light upon the disks. A complete ring of light is no longer seen from the other side, but only a small portion corresponding to the width of the half-inch slit. A stationary scale is fixed so as to extend from the shaft to the edge of the disks on the opposite side from the lamp, so that the distance of the changing line of light from the center may be read so as to give the phase difference of the two machines by direct reading. To enable one to see the scale and line of light most conveniently, a mirror is arranged at 45° with the disks, so that the line of sight is at right angles to the shaft.

The disks may be arranged, in the manner just described, upon the abutting ends of the motor and generator shafts only in case the two machines have the same number of poles. Where such is not the case, one or both of the disks can be driven by gears, which will give the proper relative speeds to the two disks.

The armature lag should be measured under different conditions of load and excitation, and both with and without self-induction in the line.

INTRODUCTORY TO EXPERIMENTS WITH CONDENSERS.

Although we usually treat the capacity of a condenser as a definite quantity denoting the ratio of the charge of electricity on either plate to the difference of potential between the plates, the actual value of the capacity* of a condenser is not as definite as this simple definition would imply. In the ideal condenser in which a perfect dielectric was used, the relation between the potential and charge would be a constant one under all conditions; in a real condenser, the presence of leakage, absorption, residual charge and hysteresis in the dielectric, causes the apparently simple relation between potential and charge to vary with different conditions, since it depends upon the absolute value of the potential and charge, the length of time the condenser has been charged, and also depends upon whether the charge has been obtained through a series of ascending or descending values.

The method to be employed in any particular measurement must be selected for each particular case with reference to the nature of the capacity to be measured and the instruments at hand. Various methods for the measurement of capacity, including many that are here given, are described by Kempe and Gray, and by Stewart and Gee. In the following experiments the student is to pay particular attention to the way in which the results are influenced by the various conditions of experiment as hereafter pointed out. Absorption of charge may vitiate one method and yet introduce no error in another in which, perhaps, perfect insulation is paramount. When a discharge is employed, as in the direct deflection and bridge methods, excessive residual

---

* The relation $Q = CV$ is true in C.G.S. units; it is also true in the practical system in which the coulomb, the farad, and the volt are the units for quantity, capacity, and potential, respectfully. The farad is too large for convenient use, and capacity is accordingly expressed in microfarads. One microfarad $= 10^{-6}$ farads $= 10^{-15}$ C.G.S. unit. Evidently the equation $Q = CV$ is not true where capacity is expressed in microfarads. For the relation between C.G.S. and practical units, see Alternating Currents, p. 312.

charge is fatal; so, too, is inductive retardation in long cables. The method of mixtures and Gott's method are less influenced by these causes, but give erroneous results unless the insulation resistance is high. Here, too, it may be noted that in the one the apparent capacity is smaller, and in the other larger than the real capacity. The time occupied in manipulation affects differently the errors in the two methods. The limitations imposed by apparatus may be such as to determine the

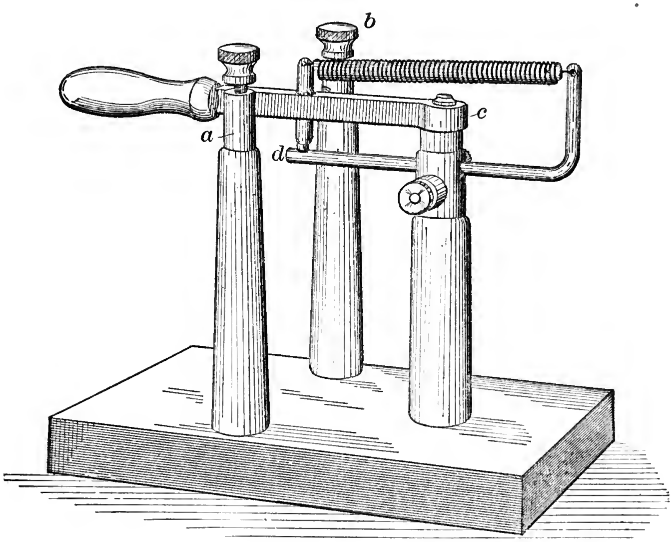


Fig. 81.—Discharge Key.

best method. It may be, for instance, that the galvanometer makes a zero method preferable. For most methods high battery insulation is necessary, but not so in the direct deflection and Gott's method. The loss of charge methods require a standard resistance of several megohms.

In general, high insulation should be maintained throughout; the lines should pass through air, and the battery and keys be well insulated from the ground. This is particularly necessary in testing grounded cables. In cable testing, measurements

should be made with the ground positive and also negative. This is readily done by a reversing key in the battery circuit. Discharge keys of various types are described by Kempe, and by Stewart and Gee. Keys and switches may be easily constructed for particular uses. A discharge key of convenient form is shown in Fig. 81, in which *a* and *b* are the charge and discharge terminals. The arm may be caught midway at *d* in the "insulate" position.

Attention has been called to variation in the value of capacity; the insulation resistance of cables is subject to change from the same causes. The two are mutually dependent. This lack of constancy of resistance, as well as constancy of capacity, must be borne in mind in measurements involving these values.

Experiment 27. **Study of a standard condenser.**

Standard condensers are usually arranged in boxes of convenient form, as shown in Fig. 82 and Fig. 83. By this study,

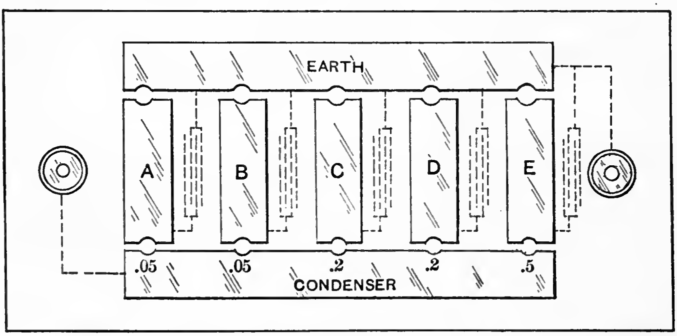


Fig. 82.—Standard Microfarad Condenser.

one should become familiar with the use of the instrument, and test its accuracy and insulation resistance. The connections of the form shown in Fig. 82 are indicated by dotted lines. Such portions of the condenser *A*, *B*, *C*, *D*, *E*, as are to be used are connected to the terminal marked **Condenser**; other

portions, to **Earth.** Thus, a capacity of 0.25 microfarads is given when *A* and *D*, Fig. 82, are connected by plugs to the **Condenser.** A capacity of 0.4 microfarads is represented in Fig. 83.

If certain portions of a condenser have faulty insulation, they cannot be used. A portion with faulty insulation will not hold its charge. Its charge at any time may be ascertained by discharging through a ballistic galvanometer and noting the throw, or by an electrostatic voltmeter. A good condenser should not lose an appreciable part of its charge when allowed to stand for an hour. The resistance, which should be in the thousands of megohms, may be determined, if desired, by the method of Exp. 32.

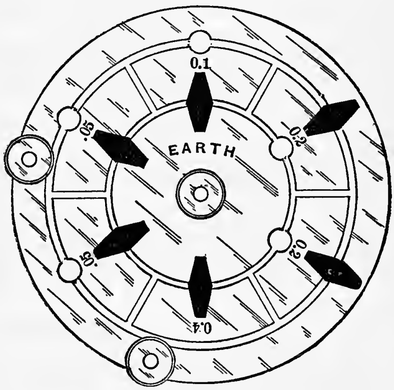


Fig. 83.—Standard Microfarad Condenser.

To test the accuracy of a standard condenser, the separate parts may be compared with each other. Thus, in Fig. 82, charge *A*, *B*, *C*, *D* simultaneously, and remove all plugs. Discharge *D* entirely by plugging to **Earth.** Then divide the charge of *C* between *C* and *D* by plugging *C* and *D* to **Condenser.** If *C* and *D* have equal capacity, each has now one-half its original charge. Again divide the charge of *C* between *C* and *D*, by first connecting *D* to **Earth,** and afterwards connecting *C* and *D* to **Condenser.** *A*, *B*, *C*, and *D* should now have equal charges, and should give equal throws of the galvanometer. If not, the capacities are not correct. The capacities of the various portions of the condenser can thus be compared with one taken as a standard. The capacities may be compared more accurately by the methods which follow.

EXPERIMENT 28. **Curves of condenser discharge. Ballistic method.**

The object of this experiment is to ascertain the value of the charge remaining in a condenser at any time as it is *discharged* through a resistance; and similar values of the charge at any time while the condenser is being *charged* through a resistance.

Discharge. — The condenser is first charged to a certain known potential and then discharged completely through a ballistic galvanometer. It is again charged to the same potential, and is allowed to discharge for a time t, through a resistance of several megohms, when it is completely discharged through the ballistic galvanometer. This operation is repeated, the time t during which the condenser is allowed to discharge through the resistance being increased with each observation.

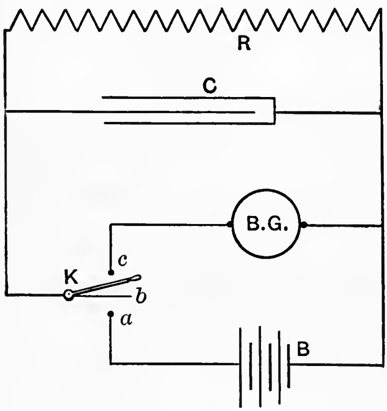


Fig. 84. — Condenser Discharge. Ballistic Method.

The arrangement of apparatus is shown in Fig. 84. The condenser is charged by placing the switch k in contact with the terminal a. During the time t, the condenser is allowed to discharge through the resistance R by placing the switch k in the position b. When the switch is thrown to c, the charge remaining in the condenser is discharged through the ballistic galvanometer and thus measured. The initial charge is found by throwing the switch directly from a to c.

Charge. — The values for the quantity of electricity in the condenser at any time while being charged through a high resistance are similarly obtained by an arrangement of apparatus

as shown in Fig. 85. The condenser is charged for a time t by placing the switch k in the position a. The quantity of charge in the condenser at any time is found by discharging through the galvanometer by placing k in the position b.

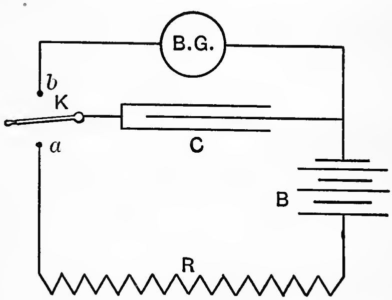


Fig. 85.—Condenser Charge. Ballistic Method.

The experiment should be repeated for different values of the capacity; also for different values of the resistance R through which the condenser is charged or discharged, and of the potential to which it is charged. All results should be plotted as curves, with quantity of charge for ordinates, and time for abscissas;

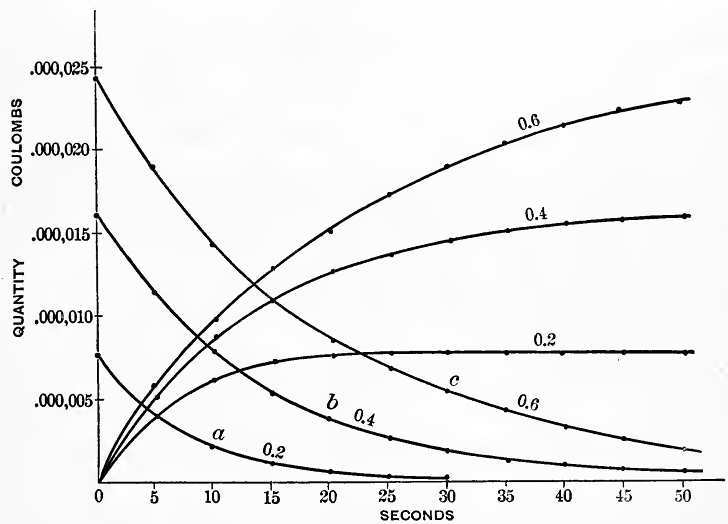


Fig. 86.—Curves of Discharge and Charge of Condensers with Capacities of 0.6, 0.4, and 0.2 Microfarad through a Resistance of 32.5 Megohms. Condensers charged to a Potential of 40 Volts.

the differences between curves taken under different conditions should be explained. Curves of charge and discharge, plotted

from observed data for different capacities, are given in Fig. 86. Note that the points a, b, and c, and corresponding points, are in a straight line from the origin.

For a perfect condenser the curves would be exponential in form, according to the following relations.* During discharge, the value of the charge q, at any time t, is

$$q = Q\epsilon^{-\frac{t}{RC}},$$

where Q represents the initial charge. While being charged, the corresponding expression for the quantity at any time is

$$q = Q(1 - \epsilon^{-\frac{t}{RC}}).$$

If theoretical curves according to these equations be plotted in dotted lines, together with the observed curves, any difference can be noted. These results should be explained. The corresponding curves of charge and discharge should be compared as to their symmetry with respect to each other. The time-constant for the different curves should be computed.

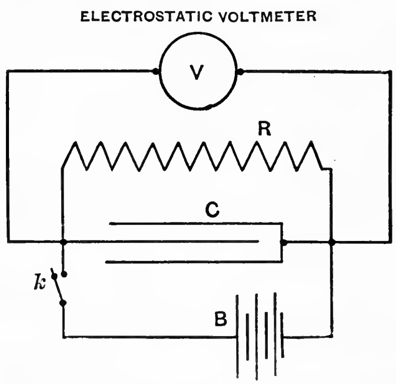


Fig. 87.—Connections for Curve of Condenser Discharge.

EXPERIMENT 29. **Curves of condenser discharge. Potential method.**

The object of this experiment is to determine the decrease in potential of a condenser while being discharged through a

* Alternating Currents, p. 72 *et seq.*

resistance, and the rise of potential while being charged through a resistance. Inasmuch as potential varies directly as charge, the results of this experiment are substantially the same as of the preceding experiment.

Discharge. — The connections are as given in Fig. 87. The condenser is charged by closing the switch *k*. The condenser is allowed to discharge through the resistance by opening the switch. The potential falls, as the condenser becomes discharged, and is measured by an electrostatic voltmeter.

Charge. — The arrangement of apparatus for obtaining the rise of potential during charging is shown in Fig. 88.

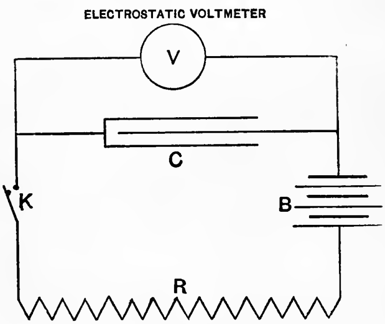


Fig. 88.—Connections for Charging Condenser through a Resistance.

Curves for charging and discharging should be found for different capacities, potentials, and resistances, and the effect of a change in any one of these quantities found by a comparison

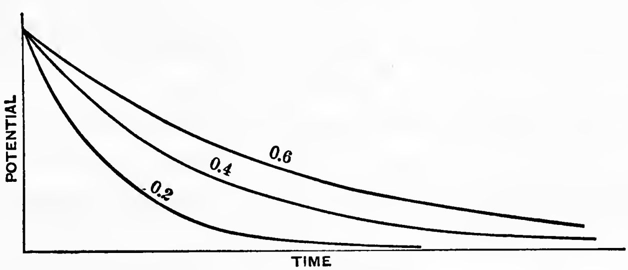


Fig. 89.—Curves of Discharge of Condensers with Capacities of 0.6, 0.4, and 0.2 Microfarad.

of the curves. Exponential curves plotted according to the equation $v = V\epsilon^{-\frac{t}{RC}}$ for discharging, and $v = V(1 - \epsilon^{-\frac{t}{RC}})$ for charging, should be compared with the observed curves. In Fig. 89 are given curves of discharge for different capacities

through the same resistance. Note that the curves for capacities 0.2, 0.4, and 0.6, are equidistant horizontally. The time-constant for the different curves should be computed.

EXPERIMENT 30. **Curves of condenser discharge. Deflection method.**

In this method the condenser is charged, and then discharged through a galvanometer with high resistance in series. The connections are given in Fig. 90. The galvanometer meas-

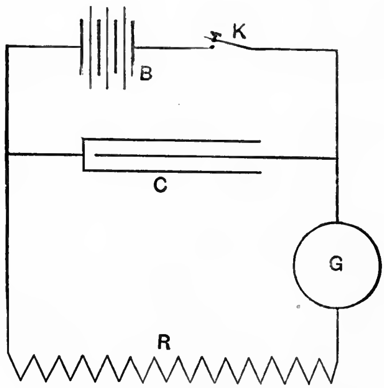


Fig. 90.—Deflection Method

ures the current flowing during discharge, and this is proportional to the potential of the condenser. The remarks of the preceding experiments apply to this method, which is sometimes known as Siemens' loss of charge deflection method.*

EXPERIMENT 31. **Measurement of the capacity of a condenser by the rate of discharge.**

The three preceding experiments give methods for obtaining curves of condenser discharge, according to the relation $q=Q\epsilon^{-\frac{t}{RC}}$ and $v=V\epsilon^{-\frac{t}{RC}}$, where q and v are the values of the charge and potential, respectively, at the time t, and Q and V

* Kempe, p. 333.

represent the initial value of these quantities. If the resistance through which the condenser is discharged is known, all quantities are known but the capacity C. This gives, therefore, a method for measuring capacity in terms of a standard resistance. Solving for C, we have

$$C = \frac{t}{R \log_\epsilon \frac{Q}{q}} = \frac{t}{2.303\, R \text{ com. } \log \frac{Q}{q}};$$

$$C = \frac{t}{R \log_\epsilon \frac{V}{v}} = \frac{t}{2.303\, R \text{ com. } \log \frac{V}{v}}.$$

The units in which the values of potential or charge are given need not be considered, as it is the ratio of potential or charges which enters into the formula. The time t is in seconds. If R is in ohms, the capacity C is in farads. If R is expressed in megohms, the value of C will be given in microfarads. The resistance R should be as large as convenient, at least a megohm.

If the insulation resistance R_c of the condenser is low compared with the resistance R through which it is discharged, as would often occur in the case of cables, for R in the above formulæ we should write $\frac{RR_c}{R+R_c}$. In the Potential Method, if the resistance of the electrostatic voltmeter be not high, a similar correction should be made.

EXPERIMENT 32. **Measurement of resistance by condenser discharge.**

From the foregoing experiments, it is evident that, inasmuch as the rate of discharge of a condenser through a resistance depends upon the value of the resistance, any of the methods for obtaining curves of condenser discharge may be applied to the measurement of a high resistance. From the

expressions already given for charge and potential at any time, we obtain expressions for the resistance. Thus,

$$R = \frac{t}{C \log_\epsilon \frac{Q}{q}} = \frac{t}{2.303\, C \text{ com. } \log \frac{Q}{q}};$$

$$R = \frac{t}{C \log_\epsilon \frac{V}{v}} = \frac{t}{2.303\, C \text{ com. } \log \frac{V}{v}}.$$

The resistance R is in ohms when C is in farads, and is in megohms when C is in microfarads.

Insulation resistance. This method is one of the best for obtaining the insulation resistance of the condenser itself. The

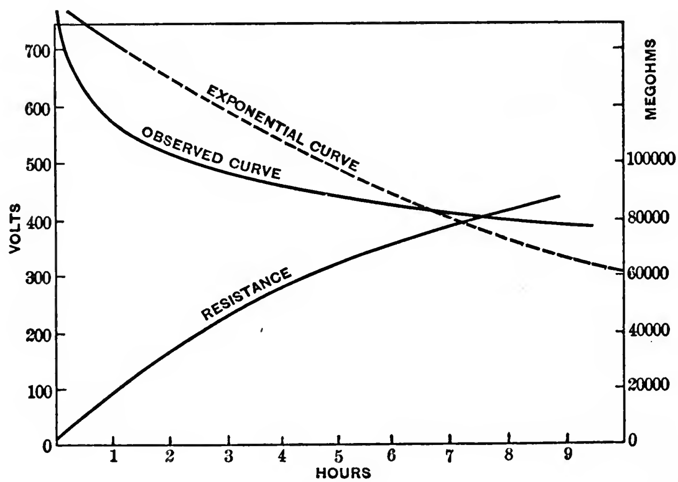


Fig. 91.

condenser is charged, and is then allowed to stand insulated, discharging slowly through its own dielectric. Owing to the soaking in of the charge and other causes, the curve of discharge may deviate widely from an exponential curve. In Fig. 91 is plotted an observed curve for the discharge, by internal

leakage, of a condenser with a capacity of 1.5 microfarads. In the same figure an exponential curve is drawn for the discharge of a *perfect* condenser of the same capacity through a resistance of 25,000 megohms. If the discharge curve were exponential, the computed resistance calculated from different observations would be the same. The values of resistance computed from different observation of the actual discharge curve change as the condenser discharges. These values are shown in Fig. 91, for the condenser above referred to.

This deviation from an exponential curve is most marked in condensers with solid dielectrics with great electric absorption. The absorption is diminished by increase of temperature.

EXPERIMENT 33. **Comparison of capacities. Direct deflection method.**

By this method, the electrostatic capacity of a condenser or cable is determined by direct comparison with the capacity of a standard condenser. The condenser is charged to any convenient potential, and then discharged through a ballistic galvanometer. The throw of the galvanometer is a measure of the quantity of electricity which passes through it, and is, therefore, a measure of the capacity of the condenser, for $Q = CV$; that is, the charge of a condenser is equal to the product of its capacity and the difference of potential of the two terminals. The condenser is again charged to the same potential as before, and the observations repeated until a series of readings are obtained. Between successive observations, it is well to take the precaution of completely discharging the condenser by short-circuiting its terminals, in order to avoid any effects of residual charge.* The condenser is now removed and the standard condenser substituted and charged to the same potential. A series of readings is obtained as before.

---

* If a condenser has been subjected for a long time to a high potential, the dielectric becomes strained and it may take days or even weeks for it to 'recover' and to assume a neutral condition.

If C_1 and C_2 be the capacities of the two condensers, and D_1 and D_2 the corresponding throws of the ballistic galvanometer,

$$C_1 : C_2 :: D_1 : D_2.$$

The unknown capacity is thus readily obtained in terms of the standard.

No correction for damping need be used, inasmuch as this would enter as a factor affecting D_1 and D_2 in the same ratio. For greatest accuracy, the two capacities should be about equal.

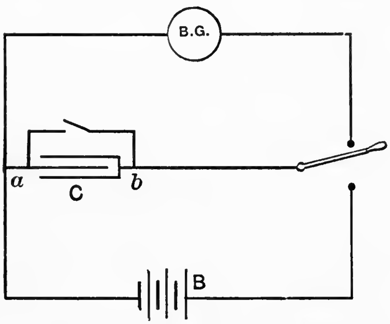


Fig. 92. — Direct Deflection Method.

The connections are shown in Fig. 92. High insulation of the battery is not essential. The rest of the apparatus should be well insulated, inasmuch as any leakage of the charge will cause an apparent diminution of the capacity. When a cable is tested and "ground" is used, the "ground" end should be at *a*, otherwise faulty battery insulation might charge the cable when connected to the galvanometer.

A modification of the direct deflection method, in which a variable shunt is employed for the comparison of widely different capacities by equal galvanometer deflections, is given in Stewart and Gee, Vol. 2, p. 403.

Experiment 34. **Comparison of capacities by the method of mixtures.**

This is often known as "Thomson's Method," and is commonly used in cable testing. The two condensers to be compared are subjected to such differences of potential that each receives the same quantity of charge. That the two charges are equal is ascertained by connecting the condensers together in opposite sense; that is, in such a way that the charges mingle and

neutralize each other. If the charges are equal, no charge will remain in either condenser after mixing, and no deflection will be obtained by connecting them with a galvanometer. The method consists in altering the potentials to which the condensers are charged until the relation between them is such that the quantities of charge are equal. When this condition is reached, we have

$$Q_1 = C_1 V_1 = Q_2 = C_2 V_2;$$

whence $$C_1 : C_2 :: V_2 : V_1.$$

In practice, the measurement is made by making the connections as shown in Fig. 93. The condensers are charged by connecting c_1 and b_1, c_2 and b_2. (When long cables are being tested, five minutes should be allowed for this.) The charges are then mixed by simultaneously disconnecting c_1 and c_2 from b_1 and b_2, and connecting them with a_1 and a_2, respectively. Jamieson recommends ten seconds for mixing. The condensers are now connected with the galvanometer by the key k; a throw of the galvanometer indicates the presence of a charge after the mixture. The relative potentials to which the condensers are charged depend upon the relative values of the resistances R_1 and R_2. By adjusting the ratio of these resistances and making repeated trials, the potentials to which the condensers are charged may be so proportioned that the two charges are equal, and no throw is obtained when the galvanometer is connected.

When this condition is reached, the capacities of the condensers are inversely proportional to the resistances; that is,

$$C_1 = C_2 \frac{R_2}{R_1}.$$

The unknown capacity is thus readily calculated from the value of the known capacity and the ratio of the two resistances.

This test is more accurate when R_1 and R_2 are large, and the capacities are of about the same value. The method is accurate when one capacity is not more than five or six times the other. When a greater difference than this exists, the disparity of potential causes an unequal "soaking-in" effect in the two condensers. A standard resistance box or a slide wire may be used for R_1R_2. The electromotive force of the battery should be as large as the condensers and coils will allow, in order to make the galvanometer sensitive to slight departures from the conditions of equilibrium.

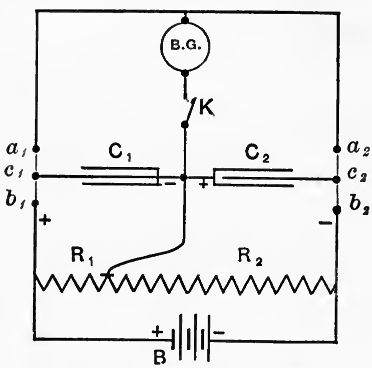


Fig. 93.—Method of Mixtures.

A switch for changing the connections from "charging" to "mixing" is easily constructed by making the terminals a_1, b_1, c_1, and a_2, b_2, c_2 of mercury cups. Wires can be bent so as to make M-shaped rockers which pivot on c_1 and c_2. If these are rigidly connected, they may be moved simultaneously. Two switches of the form shown in Fig. 81 may be used by throwing the handles simultaneously. The parts lettered *a, b, c,* in Fig. 81 and 93 correspond.

A leak in either condenser would cause its apparent capacity to be less than its real capacity, for it would take a greater difference of potential in charging. If the leakage resistance is measured, a proper correction may be applied by considering this resistance to be in parallel with R_1 or R_2. Leakage during mixing lessens any remaining charge, and so decreases the sensitiveness of the measurement.

In cable testing, or tests in which a ground is used, high insulation of the battery and all apparatus is essential.

EXPERIMENT 35. **Gott's method for comparison of capacities.**

In this method the connections are as shown in Fig. 94. The high resistances, R_1 and R_2, should be so adjusted that the ratio between their values is the same as the ratio between C_1 and C_2. In this case the galvanometer shows no deflection when connected. This condition is attained by trial as follows: The values of R_1 and R_2 are first adjusted in approximate proportion to the supposed values of the capacities C_1 and C_2. The battery connection is then made, and after five or ten seconds (five minutes for long cables) for charging, the galvanometer circuit is closed with the battery still in circuit. If the galvanometer is deflected, a readjustment of the resistances must be made. The condensers must be first discharged by opening the battery circuit and closing the galvanometer circuit, or otherwise short-circuiting the condensers. In the final observations time must be allowed for the condensers to *completely* discharge. This operation is repeated until no deflection is obtained when the galvanometer is connected. The battery is in circuit all the time except during discharge.

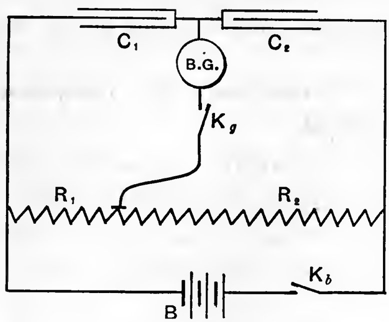


Fig. 94. — Gott's Method.

When this condition of equilibrium is reached, the capacities are inversely proportional to the resistances, as the student should prove; that is,

$$C_1 : C_2 : : R_2 : R_1.$$

The test is most accurate when the capacities compared have approximately the same values and the resistances are high.

Leakage in either of the condensers, or its leads, will cause its potential to fall during charging, and will thus apparently increase the value of its capacity; for $C \propto \frac{1}{V}$. This error

increases with the time of charging; for, in time, the leaky condenser would completely discharge itself, and the difference of potential between its terminals would be zero, if the other condenser did not leak. It is evident that leakage between any of the condenser or galvanometer connections will introduce error; the galvanometer key must therefore be well insulated. If the rest of the apparatus is well insulated, it is not essential that the battery should be; for leaks between two portions of the battery circuit introduce no error. In cable testing, the core should be connected to the galvanometer, and the ground to one of the battery connections.

EXPERIMENT 36. **Comparison of capacities by the bridge method.**

This method is similar to that used in the comparison of resistances by the Wheatstone bridge, and is the same as Gott's method already described, with the battery and galvanometer interchanged. The connections are as shown in Fig. 95. The resistances R_1 and R_2 are adjusted until no deflection is obtained as the key K is thrown back and forth from a to b. This condition of equilibrium indicates that there is no difference of potential between the points m and n; the condensers are charged, therefore, to the same potential, and the quantities of electricity, q_1 and q_2, which flow into each through the

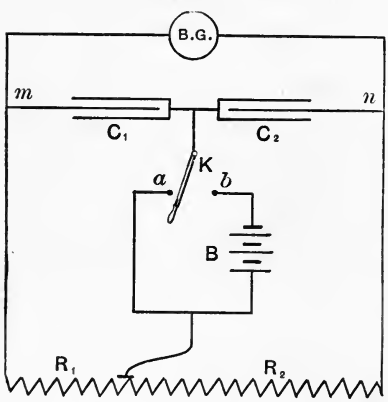


Fig. 95.—Bridge Method.

resistances R_1 and R_2, are proportional to the condenser capacities C_1 and C_2. Since the potential difference around R_1 is equal to that around R_2, it follows that $R_1 i_1 = R_2 i_2$. The currents flowing, and hence the quantities which flow in any time,

are accordingly inversely proportional to the resistances. We have, therefore,

$$\frac{C_1}{C_2}=\frac{q_1}{q_2}=\frac{R_2}{R_1}.$$

The capacity of a condenser is thus determined in terms of a standard capacity and the ratio of known resistances. For accuracy, R_1 and R_2 should be large, and C_1 and C_2 nearly equal. Contacts with a and b should be sufficiently long to allow complete discharge and charge. Self-induction in the resistances and connections must be carefully avoided.

EXPERIMENT 37. **Comparison of capacities by the method of divided charge.**

A standard condenser, with capacity C_1, is subjected to a certain difference of potential, and is charged with a quantity, Q_0, which is measured by discharging through a ballistic galvanometer. It is again subjected to the same difference of potential, and, therefore, again receives a charge Q_0. The condenser or cable, with capacity C_2, to be measured is connected for a brief interval in parallel with the standard condenser, so that the initial charge Q_0 divides into two parts, Q_1 and Q_2, in proportion to the two capacities C_1 and C_2. The condensers are separated, and the charge Q_1 remaining in the standard condenser is measured by discharging through the ballistic galvanometer. It is evident that the charge on each condenser and the sum of the charges are in proportion to the respective separate capacities and the sum of the capacities; that is,

$$\frac{Q_1}{C_1}=\frac{Q_2}{C_2}=\frac{Q_1+Q_2}{C_1+C_2}=\frac{Q_0}{C_1+C_2}.$$

Hence $$C_2=C_1\frac{Q_0-Q_1}{Q_1}.$$

It may be shown (see Kempe, p. 343) that for highest accuracy C_1 should be about $\frac{1}{2}$ C_2. The condenser C_2 should be completely discharged between successive operations.

When the capacity of a condenser is being measured, C_1 and C_2 may be interchanged, so that the unknown condenser is the one initially charged. This should be done when the ratios of the capacities requires it. In cable testing, it is not well to have the cable initially charged, for any leakage would diminish the charge according to the time occupied in the necessary manipulations.

EXPERIMENT 38. **Comparison of capacities by Siemens' diminished charge method.***

The condenser or cable, C_1, to be tested, is charged to a definite potential. A standard condenser, C_2, is charged to the same potential, and its charge, Q_0, measured by discharge through a ballistic galvanometer. The charged cable is disconnected from the battery and gradually discharged by repeated applications to its terminals of the standard condenser, which is completely discharged between successive applications. After a number of such discharges, the quantity Q_n taken off by the standard condenser the nth time the operation is repeated is measured by the galvanometer.

$$Q_n = Q_0\left(\frac{C_1}{C_1+C_2}\right)^n;$$

for the potential is diminished each time by the factor $\left(\frac{C_1}{C_1+C_2}\right)$.

Hence $$C_1 = \frac{C_2}{\left(\frac{Q_0}{Q_n}\right)^{\frac{1}{n}} - 1}.$$

This method has the advantage that large capacities may be thus compared with a standard, but necessitates very high insulation in the condenser tested. For greatest accuracy

$$n = \frac{0.555}{\text{com. log}\left(\frac{C_1+C_2}{C_1}\right)}.$$

* Kempe.

EXPERIMENT 39. **Capacities in parallel and in series.**

If condensers be arranged in series or cascade, as in Fig. 96, the capacity of the combination * is the reciprocal of the sum of the reciprocals of the several capacities; that is,

$$C' = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} = \Sigma \frac{1}{\frac{1}{C}}.$$

If condensers be arranged in parallel or multiple arc, as in Fig. 97, the capacity of the combination is the sum of the several capacities; that is, $C' = \Sigma C$.

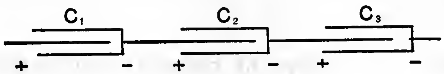


Fig. 96.—Condensers in Series.

The student should establish the truth of the above statements theoretically, and verify by experiment. Condensers should be arranged in various combinations,—such as three in series and two in parallel,—their separate capacities meas-

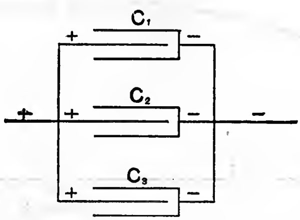


Fig. 97.—Condensers in Parallel.

ured, and their combined capacities determined by calculation and by measurement.

EXPERIMENT 40. **Comparison of electromotive forces by a condenser.**

The charge of a condenser is directly proportional to the difference of potential at its terminals, and it therefore follows

* Stewart and Gee, Vol. 2, p. 404; Alternating Currents, p. 279.

directly that the throw of a ballistic galvanometer through which it is discharged is a measure of this potential difference. Two electromotive forces may be readily compared by the corresponding throws of a galvanometer, obtained by successively discharging a condenser charged first by one and then by the other electromotive force. In this as in other experiments with condensers, it is well to take the precaution of removing the residual charge by short-circuiting the terminals. Measurements with the lower potential should be made first; for if the higher potential were first used, its residual charge might introduce considerable error in the succeeding measurements with the lower potential.

EXPERIMENT 41. **Study of residual discharges.**

When a condenser is discharged, the whole quantity of electricity does not discharge at once; if the condenser be allowed to stand insulated for some time, there is a residual

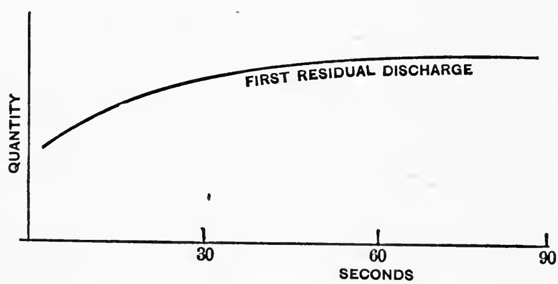


Fig. 98.—Increase of Residual Discharge with Time.

charge which may be measured by again discharging the condenser through the ballistic galvanometer.

To investigate this phenomenon, note the value of this residual charge by charging the condenser to the full, discharging it through a low resistance, and then discharging it through the galvanometer after it has stood insulated for five or ten seconds. Recharge for the same length of time as before, and measure the residual charge in the same way after

the condenser has remained insulated for longer intervals of time. This residual charge increases with the length of time the condenser stands insulated, as shown in Fig. 98. Second, third, and subsequent residual discharges are studied in the same way, the same interval of time intervening between the first and second, second and third discharges, and so on. In the plotted results, abscissas represent the value of this interval of time between successive discharges.

Time Element of Charging. — A condenser continues to receive charge for some time after it is subjected to a difference

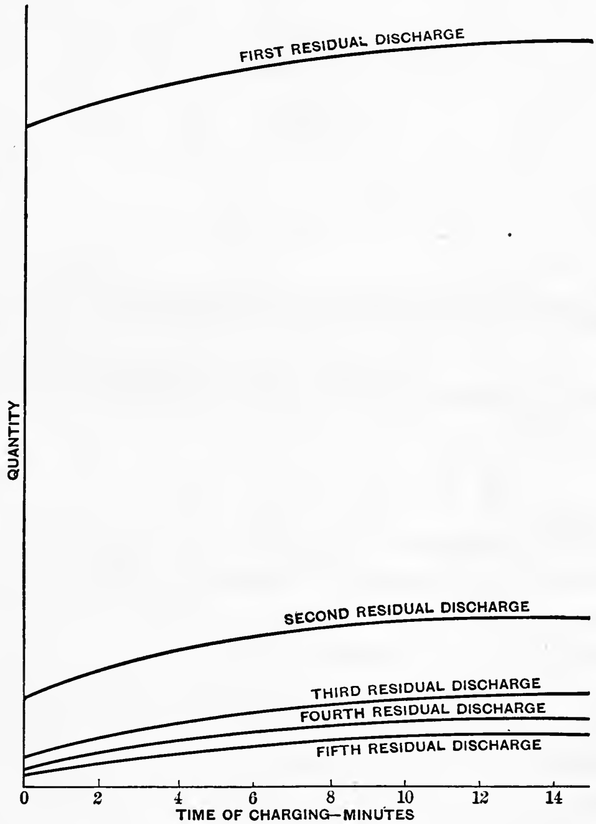


Fig. 99. — Effect of Time of Charging upon Residual Discharges. Intervals between Discharges equal 45 Seconds.

of potential. To investigate this "soaking in" effect, first subject the condenser for a mere instant to a certain difference of potential, and measure its charge by immediately discharging through a ballistic galvanometer. The successive residual discharges, after definite intervals of time, are similarly obtained. Charge the condenser to the same potential for some definite length of time, and measure the charge and residual charges in the same way, as before. Repeat the operation, each time increasing the time of charging. The intervals between successive discharges are to be the same throughout the experiment. The effect of the time element in charging upon residual discharges is shown in Fig. 99. All the residual discharges are much smaller than the first discharge, not given in the figure, which, for the case there shown, did not depend upon the time of charging. The mechanical analogies of these phenomena may be found in *Modern Views of Electricity* by Lodge.

Other phenomena connected with residual charge should be investigated by the student, as the effect of reversing the sign of the charge, charging to different potentials, etc.

EXPERIMENT 42. **Measurement of capacity by means of an alternating current.**

This is a practical method for measuring the capacity of alternate current condensers, and is substantially the same as the impedance method for measuring self-induction. (See Exp. 7.) The condenser whose capacity is to be measured is connected to an alternating current circuit. Measurements are made of the electromotive force to which the condenser is subjected, the current which flows in the condenser, and the frequency. There being no resistance in the circuit, the value* of the current is

$$I = CE\omega.$$

The capacity is computed from this formula.

---

* Alternating Currents, p. 79.

EXPERIMENT 43. **Effects of the variation of the resistance in a series circuit containing a condenser.**

A condenser is connected in series with a non-inductive resistance in a circuit in which there is no self-induction. The measurements are made in the same way as in Exp. 11, and the results graphically shown by curves corresponding to those in Figs. 64 and 65. It is to be borne in mind, however, that the diagrams are to be drawn with the current in *advance* of the impressed electromotive force by an angle θ, and not lagging behind the electromotive force, as in the case of circuits containing self-induction. No correction need be applied for the electromotive force used in overcoming the ohmic resistance of a condenser, which is negligible. Hysteresis losses in the condenser will, in most cases, be inappreciable, otherwise the results will practically be similar throughout to those in the corresponding experiment referred to with self-induction. Theoretical curves, with polar* and rectangular coördinates, should be plotted by the student and compared with those obtained from observation.

EXPERIMENT 44. **Effects of the variation of the capacity in a series circuit.**

The effects of the variation of the capacity of a condenser in a series circuit containing a resistance but no self-induction, as shown in *Alternating Currents*, p. 275, may be found according to the methods of Exp. 12.

EXPERIMENT 45. **Effects of the variation of frequency in a circuit containing capacity, but no self-induction.**

The effects upon the impedance and upon the position of the current with reference to the electromotive force in a circuit containing a condenser and resistance but no self-induction, may be obtained according to the methods of Exps. 17

* Alternating Currents, p. 275.

and 18. Although the method of experiment is the same, the results will be quite different; for a change of frequency which would increase the effects of self-induction would decrease the effects of capacity, and *vice versa.* The theoretical study of the subject is left to the student, who should compare the theoretical and experimental results obtained for a condenser circuit with the corresponding results for a circuit containing self-induction.

EXPERIMENT 46. **Neutralization of self-induction and capacity in series.***

Self-induction in a circuit tends to make the current lag, and capacity in a similar way tends to make the current advance ahead of the electromotive force. These opposing effects become balanced when the reactance is zero, and $L\omega = \frac{1}{C\omega}$. The current is then in phase with the electromotive force.

The value of a current in a circuit containing self-induction and capacity is

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}};$$

that is, the electromotive force divided by the impedance. Under the condition that $L\omega$ is equal to $\frac{1}{C\omega}$, the impedance becomes a minimum, being equal to the ohmic resistance. The current is then equal to the electromotive force divided by the resistance, and has the same value as if the circuit contained neither self-induction nor capacity. It is to be noted that this condition for the neutralization of self-induction and capacity, which gives the current its greatest value, is the same as that

* This experiment should not be performed by the student until he has made a careful study of the subject and understands the principles governing the action of a condenser in a circuit with self-induction. See "Practical Aspects of Low Frequency Electrical Resonance," M. I. Pupin, Transactions American Institute of Electrical Engineers, Vol. 10, p. 370. See Alternating Currents, Chapters IX. and XX.

given above, which causes the current to be in phase with the electromotive force. It is evident that the values of the self-induction and capacity which annul each other are closely

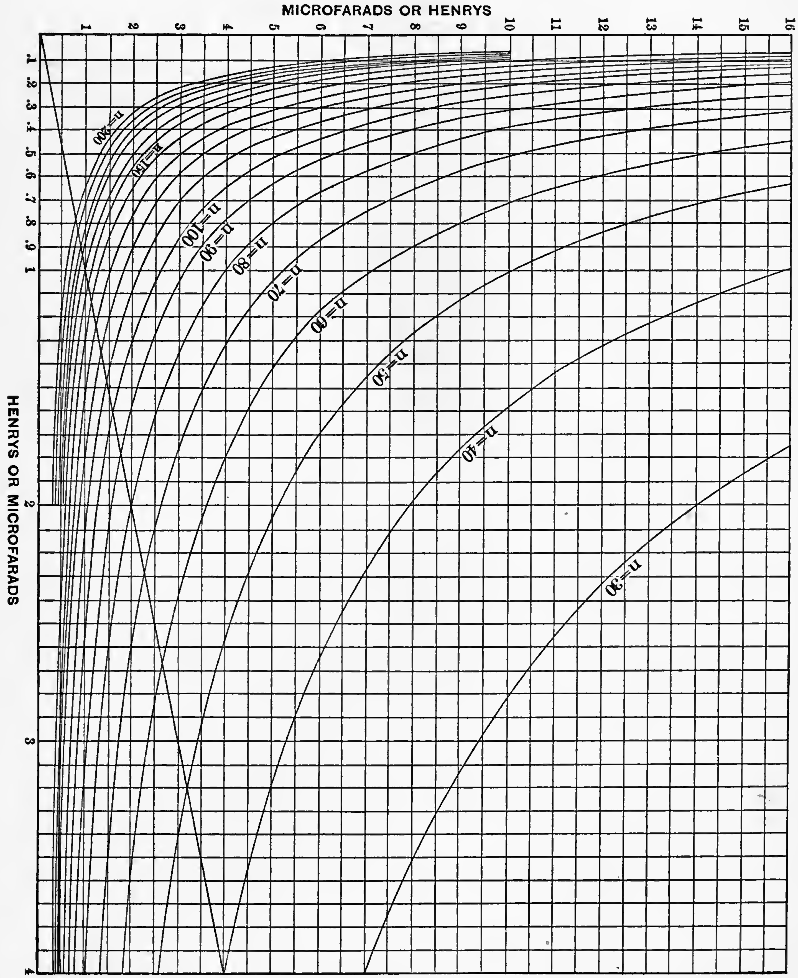


Fig. 100.

dependent upon the frequency. The critical relation between these quantities is shown by a set of curves plotted by

Dr. Crehore, given in Figs. 100 and 101. When this critical relation exists, the phenomenon known as resonance occurs, which is accompanied by a rise of potential at the terminals of the self-

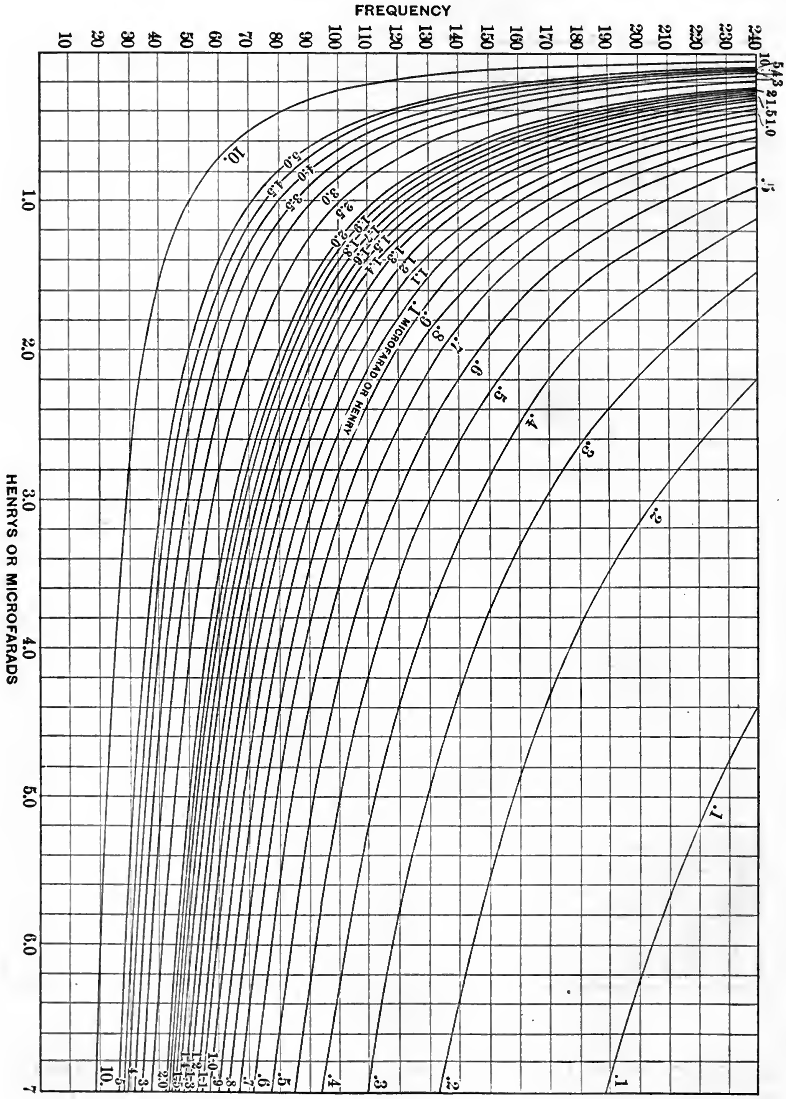


Fig. 101.

induction and at the terminals of the condenser. Care should be taken that this rise of potential does not exceed that which the apparatus can stand. The difference of potential at the terminals of the condenser when resonance is reached may be computed as follows: The current is equal to the total impressed

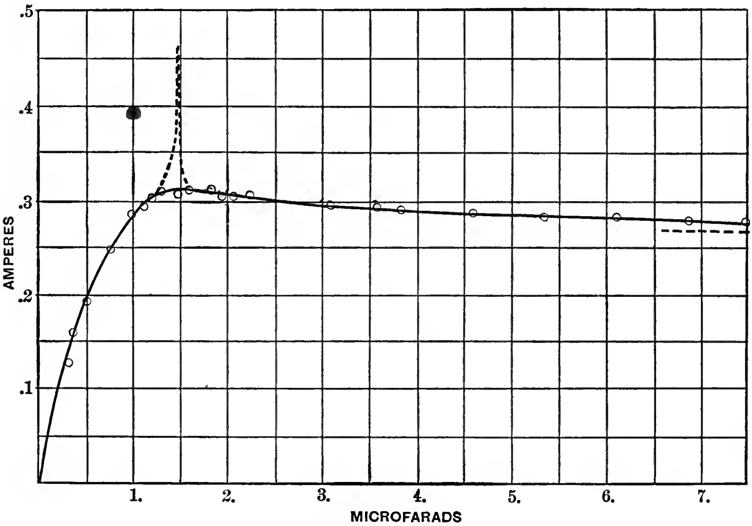


Fig. 102.—Variation of Current with Capacity in a Circuit with Resistance of 862 Ohms, Self-Induction of 0.662 Henry with an Impressed Electromotive Force of 388 Volts, and $\omega = 1691$.

electromotive force E divided by the resistance. It is also equal to $CE_c\omega$, where E_c is the difference of potential at the terminals of the condenser; hence,

$$E = \frac{I}{C\omega} = \frac{1}{CR\omega} E.$$

The difference of potential at the terminals of the self-induction is similarly computed, and is

$$E_L = L\omega I = \frac{L\omega}{R} E.$$

Curves, such as those plotted in *Alternating Currents*, Chapter IX., may be obtained so as to show the effects of a

variation of the resistance, self-induction, capacity, or frequency. The critical points will not be so marked in the experimental curves, being considerably modified on account of magnetic and dielectric hysteresis and on account of the deviation of the current from a sinusoidal form. Figure 102 shows the variation of the current as the capacity in the circuit is changed. The conditions were purposely taken so that the current would not reach a very large value; that is, the resistance was large. The effects of self-induction and capacity were accordingly not so marked. The dotted curve is plotted from computed values under the assumption of a sinusoidal current and the absence of any hysteresis. These effects are fully discussed by Dr. Pupin in the paper referred to, which should be carefully read by the student before performing this experiment.

Experiment 47. **Self-induction and capacity in parallel.**

In the case of direct currents flowing in two or more parallel circuits, the total current is equal to the algebraic sum of the branch currents; with alternating currents, the total current is the geometric or vector sum of the branch currents, which is usually much less than the algebraic sum on account of phase differences. If the parallel circuits contain self-induction only or capacity only, together with ohmic resistance, the total current is greater than the current in any one branch; this is not necessarily so, however, if one of two branches contains self-induction and the other contains capacity.

A divided circuit with self-induction in one branch and a condenser in the other is shown in Fig. 103.

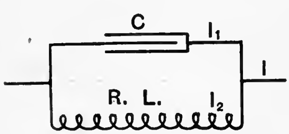


Fig. 103.

The total current, I, will be a minimum for a particular relation between the capacity and the self-induction. The effect of the combination of the self-induction and capacity in parallel is investigated by subjecting the circuits shown in Fig. 103 to a known impressed electromotive force, E, and meas-

uring the total current and the current in each branch. The results are graphically shown by a polar diagram, Fig. 104, in which the magnitude and direction of the currents are shown. The condenser current, I_1 equal to $CE\omega$, is 90° in advance of the electromotive force. The current, I_2, lags behind the electromotive force by an angle, less than 90°, with a tangent equal to $\frac{L\omega}{R}$.

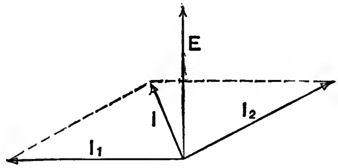


Fig. 104.

Such a diagram should be constructed from measurements made with different relations between the self-induction and

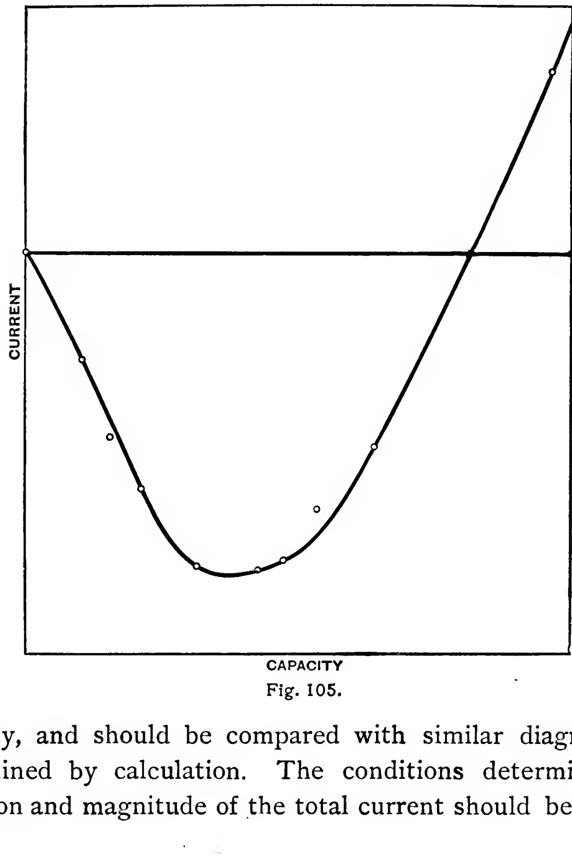


Fig. 105.

capacity, and should be compared with similar diagrams predetermined by calculation. The conditions determining the direction and magnitude of the total current should be carefully

studied. Figure 105 shows the values of the total current, I, for different capacities, obtained by measurement upon a divided circuit as that shown in Fig. 103. The current is a minimum when in phase with the electromotive force. On one side of the minimum it lags behind the electromotive force; on the other side, it is in advance.

The effects of a change of frequency in this case may be similarly investigated.

EXPERIMENT 48. **Instantaneous measurements with a revolving contact-maker and electrostatic voltmeter.**

One of the most accurate, as well as convenient, methods of obtaining the instantaneous values of electromotive force or current by the method of instantaneous contact is the one here described, in which a condenser is kept charged to a certain potential by being periodically connected by means of a contact-maker with the alternating current circuit upon which the measurements are to be made. The potential to which the condenser is charged depends upon the time at which the contact-maker closes the circuit, and represents the electromotive force of the alternating current circuit at some particular point of its period. This potential is measured by an electrostatic voltmeter connected with the condenser. For this purpose a Thompson multicellular voltmeter is convenient. In such an instrument, readings cannot be taken below a certain potential. Potentials lower than this may be measured, however, by connecting in series with the voltmeter an auxiliary condenser which is kept charged to some constant potential, which is accordingly added to the potential to be measured, thus bringing the potential measured by the voltmeter to an available part of the scale.

The arrangement of apparatus for measurement by this method is shown in Fig. 106. It is required to measure the difference of potential between a and b, which are points connected with an alternating current circuit. The condenser

C_1 is kept charged to the potential which is to be measured by being connected with the points *a* and *b* through the contact-maker. The zero of the voltmeter is displaced by the auxiliary condenser, C_2. Suppose the voltmeter be one which reads from 40 to 120 volts with the best portion of its scale above 80 volts; C_2 is a condenser charged to 80 volts; the voltmeter reads the sum of the potentials of the condensers C_1 and C_2; that is, if the voltmeter reads 87 volts, we know that C_1 is charged to 7 volts, which is, therefore, the difference of potential between *a* and *b*. The potential of the condenser C_2 will gradually fall from its initial value. A correction for this fall may be made either by previously determining the rate at which its potential falls, or by measuring its potential from time to time by connecting it directly with the voltmeter by the switch *k*.

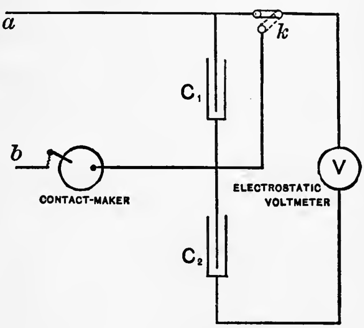


Fig. 106.

To measure differences of potential beyond the upper range of the instrument, the apparatus should be arranged as in Fig. 107, in which it is necessary to measure the difference of potential between *A* and *B*. The points *A* and *B* are connected by a high, non-inductive resistance, and the difference of potential measured around a known portion *a*, *b*, of this resistance, and the difference of potential between *A* and *B* calculated.

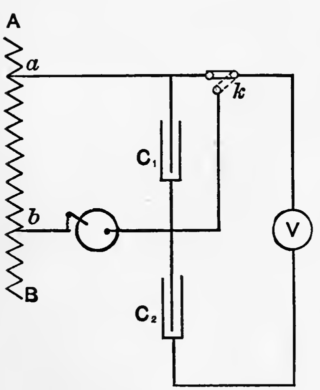


Fig. 107.

Current is measured by means of the apparatus thus arranged for measuring potential, by measuring the fall of potential

around a non-inductive resistance in the circuit the current in which is to be measured. The current may be directly calculated from these measurements of potential when the value of the non-inductive resistance is known. The resistance is likely to vary according to the value of the current, and it must, therefore, be calibrated by previous measurement with a direct current equal to the square root of the mean square value of the alternating current.

One of the first investigations * employing the method of instantaneous contact was made by Professors Ryan and Merritt, who employed a novel form of electrometer to measure the potential difference of the condenser C_1. The terminals of the condenser were connected to two opposite quadrants; the other pair of quadrants were connected together. The suspension carried a magnet needle, the direction of which was controlled by a direct current flowing through a coil of wire surrounding the instrument. The Ryan electrometer is used differentially as a zero instrument, the direct current being adjusted until there is no deflection. This electrometer was designed in order to enable small differences of potential to be measured with accuracy. The same result is accomplished by employing the auxiliary condenser † described above.

EXPERIMENT 49. **Instantaneous measurement with a revolving contact-maker. Telephone method.**

In this method the alternating current circuit is connected periodically with a telephone by means of a revolving contact-maker. In the circuit with the telephone is inserted a difference of potential from some constant source which is adjusted until it is equal and opposite to the difference of potential to be measured, as told by silence in the telephone. The appa-

---

* Ryan on Transformers, Transaction American Institute of Electrical Engineers, Vol. 7, p. 1. The construction and use of the Ryan electrometer is there fully described, p. 6, *loc. cit.*

† Bedell, Miller, and Wagner, Transactions American Institute of Electrical Engineers, Vol. 10, p. 504.

ratus is arranged as in Fig. 108, in which a and b are points connected with the alternating circuit upon which measurements are to be made. The difference of potential between c and d may be controlled by the adjustable resistance R connected with the battery B. The circuit through the telephone is closed once during each revolution of the contact-maker, and in the circuit there are the electromotive forces ab and cd. The resistance R is adjusted until no sound is heard in the telephone, or the sound is a minimum. Evidently the electromotive forces in the circuit are now equal and opposite, and

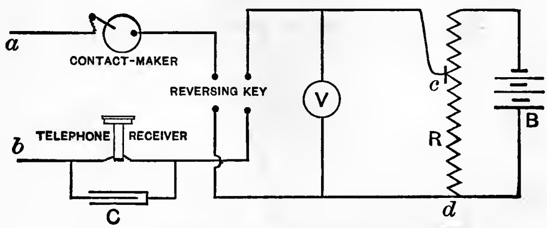


Fig. 108.—Telephone Method.

the difference of potential between c and d, as read by a direct current voltmeter, is equal to the difference of potential between a and b. A condenser C is placed in parallel with the telephone receiver to augment the sound. By a series of readings, corresponding to successive adjustments of the resistance, the errors of observation may be reduced.

EXPERIMENT 50. **Instantaneous measurement with a revolving contact-maker. Ballistic method.**

Potential is measured in this method by charging a condenser through a revolving contact-maker and discharging through a ballistic galvanometer. In Fig. 109, a, b represent points connected with the alternating current circuit to be measured. When the switch s is in the position g, the condenser C becomes charged to the same difference of potential as exists between the points a, b at the time of alternation

corresponding to the time at which connection is made through the contact-maker. If the switch s is thrown to h, the condenser is discharged through the ballistic galvanometer, the throw being proportional to the charge and therefore to the potential of the condenser. Thus

$$Q=k\theta=CV; \text{ or, } V=\frac{k\theta}{C},$$

where k is the constant of the galvanometer; θ, its throw; C the capacity of the condenser; and V, the potential to which it is charged. For low potentials, the capacity, C, should be

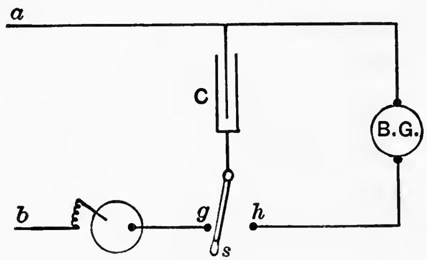


Fig. 109.

large. A reversing key should be placed in the galvanometer circuit and double deflection observed. A D'Arsonval galvanometer, as described in Exp. 61, proves satisfactory for this method, inasmuch as it may be used near the generator without being affected by magnetic influences and is quite sensitive. The galvanometer may be calibrated according to the method given in the experiment referred to.

To avoid jar, the instrument may be placed upon a table with legs set in jars of sawdust.

Experiment 51. **Irregularities in alternating current curves.**

In this experiment a curve, either of electromotive force or current, for an alternating current generator, is to be found for a complete revolution of the armature by one of the methods of instantaneous contact already described. The successive loops

should be compared with reference to the irregularities which occur, and the separate areas should be determined. The sum of the positive and of the negative areas is to be obtained, and the average positive and negative area compared with the separate areas. These results should be explained by consider-

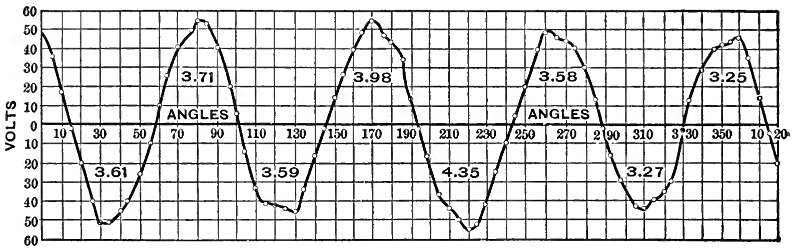


Fig. 110.—Irregularities in Alternating Current Curves.

ing what it is that is represented by each area, and the sum of the areas, whether the curve be one of current or electromotive force.*

In Fig. 110 is shown a curve, taken from a small eight-pole generator, in which the differences between successive loops is very marked.

The positive and negative areas in this case are as follows:

Positive Areas.	Negative Areas.
3.71	3.61
3.98	3.59
3.58	4.35
3.25	3.27
+ 14.52	− 14.82
Total, − 0.30	

The sum of the positive and negative areas are seen to be nearly equal. Such great differences between the separate areas would not usually be found, not occurring at all in the curves from a perfectly symmetrical generator.

* See "Irregularities in Alternate-Current Curves," Bedell, Miller, and Wagner, Physical Review, Vol. 1, No. 3, p. 218.

EXPERIMENT 52. **Measurement of power by the method of instantaneous contact.**

The power expended in any circuit may be calculated from the instantaneous values of the current and electromotive force. The product of the current and electromotive force *at any instant* is the rate at which energy is being supplied to the circuit at that instant. The mean rate of expenditure of energy is found by averaging the value thus found from instant to

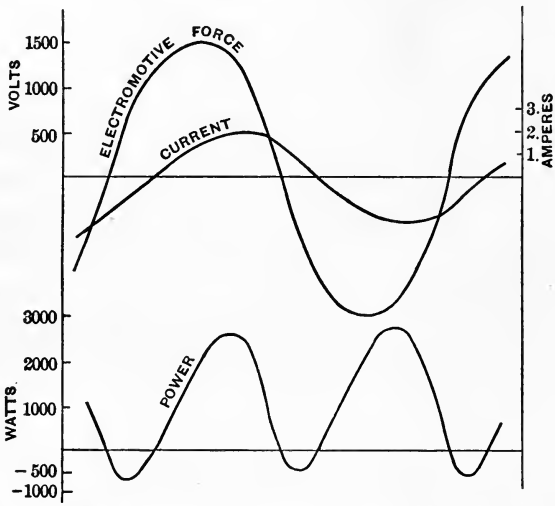


Fig. 111.—Instantaneous Curves of Electromotive Force, Current, and Power.

instant. A non-inductive resistance is placed in series with the inductive circuit, the power of which is to be measured and the instantaneous values of electromotive force and current found by one of the methods of instantaneous contact already described. Such curves from actual measurement are shown in Fig. 111. In the same figure is plotted a *power* curve, the ordinate of any point of which is equal to the product of the corresponding ordinates of the electromotive force and current curve above. When current and electromotive force are both positive or both negative, the power is positive; but when either the

current or electromotive force is negative and the other positive, the power is negative. The power is zero when either the current or the electromotive force is zero, which occurs four times in each period. For a complete period, the power curve consists, then, of two positive and two negative loops. The positive areas represent the energy given by the line to the circuit in one period, and the negative areas represent energy returned to the line from the circuit; the difference between positive and negative areas represents the actual expenditure of energy during each period. The mean ordinate of the power curve gives accordingly the average value of the power expended in the circuit. It is to be noted that the relative values of the positive and negative areas depend upon the position of the current curve with respect to the curve of electromotive force; that is, the *lag* of the current behind the electromotive force. If the two were in phase, which would be the case were there no self-induction in the circuit, there would be no negative areas. As the current lags, the negative areas increase; and where the current and electromotive force curves are symmetrical and 90° apart, the negative and positive areas become equal, indicating no expenditure of energy.* The relative positions of the curves and their irregularities should be interpreted.

By plotting curves representing the square of the instantaneous values of the electromotive force and current, the mean square values of these quantities may be found by the planimeter, and so the square root of the mean square values.

If we consider the current to be equivalent to an harmonic current of the same square root of the mean square value, and the electromotive force similarly equivalent to a harmonic electromotive force, we may represent them by vectors as is usual in the treatment of harmonic currents. In so doing, we have answered the question: to what harmonic current is a

---

* See Fleming, The Alternate Current Transformer, Vol. 1, p. 121.

current equivalent which is not harmonic? It still remains to determine the angle of lag between this current and electromotive force. In Fig. 44, let $\overline{E}$ and $\overline{I}$ represent the square root of the mean square values of the electromotive force and current respectively. θ, the angle between, is determined by the condition that the power is equal to $\overline{EI} \cos \theta$. If the current be measured by an alternating current ammeter, placed in the circuit, and three voltmeter readings be taken around the non-inductive resistance, the inductive portion of the circuit, and the two together, the angle of lag may be determined by the three-voltmeter method. The results thus found can be compared in this way with those found above.

EXPERIMENT 53. **Transformer test by the method of instantaneous contact.**

In transformer tests by this method, instantaneous values are obtained for the primary current and electromotive force, and for the secondary electromotive force. The connections for such a test, employing an electrostatic voltmeter for the measuring instrument, is shown in Fig. 112. The measurements may, however, be made according to any of the methods of instantaneous contact already described. The primary electromotive force is obtained by measuring the fall in potential around a known portion of the non-inductive resistance R_1, which consists of a series of incandescent lamps arranged between the mains. The secondary electromotive force is similarly found by the fall of potential around a certain portion of the lamp resistance R_2. The fall of potential around the lamps R_3 in series with the primary gives the primary current when these lamps are properly calibrated. The arrangement of the resistances R_1, R_2, and R_3 depends upon the values of the quantities measured and the range of the measuring instrument. The difference of potential between the terminals of the primary proper is less than that obtained from measurements upon R_1 by an amount equal to the fall in potential in the

lamps R_3. To obtain the value of the electromotive force impressed upon the primary, we must subtract the instantaneous values of the fall of potential in the lamps R_3 from the instantaneous values of the electromotive force supplied from the mains as measured from the resistance R_1. The

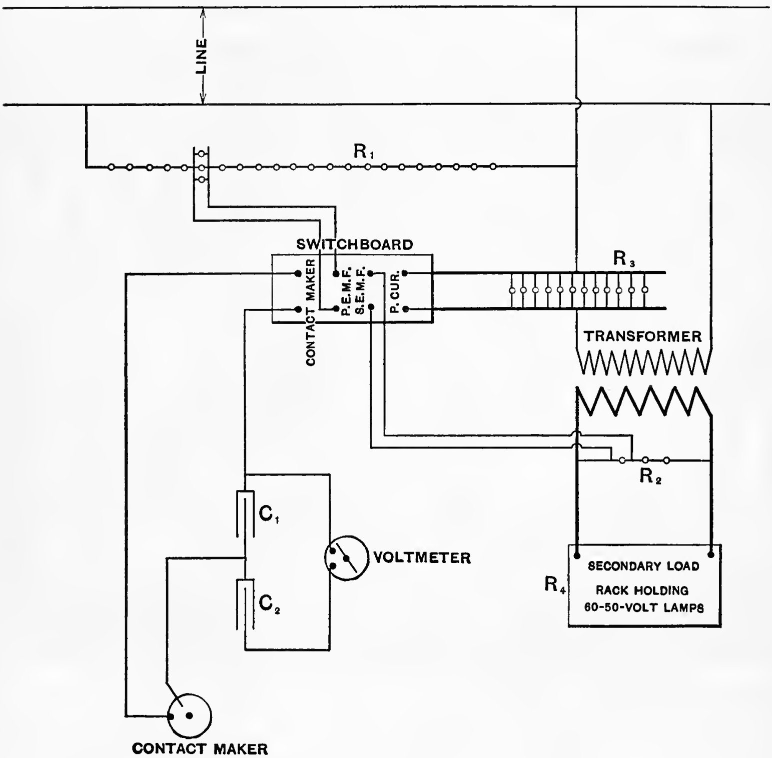


Fig. 112.—Diagram of Connections. Transformer Test.

lamps must be calibrated for the particular current used. This calibration should be made both before and after the experiment. Curves obtained in this way from a sixty-light open-magnetic-circuit transformer are shown in Fig. 113.

The power expended in the primary is found according to the method of Exp. 52. The power expended in the

secondary is best found, not from instantaneous curves, but from the product of the electromotive force and current obtained directly by the alternating current voltmeter and ammeter. Runs may be made at full load, no load, and intermediate points, and the efficiency obtained by taking the ratio of the secondary to the primary power. The efficiency for various loads, however, may be computed from the iron losses obtained from the run at no load. This necessitates

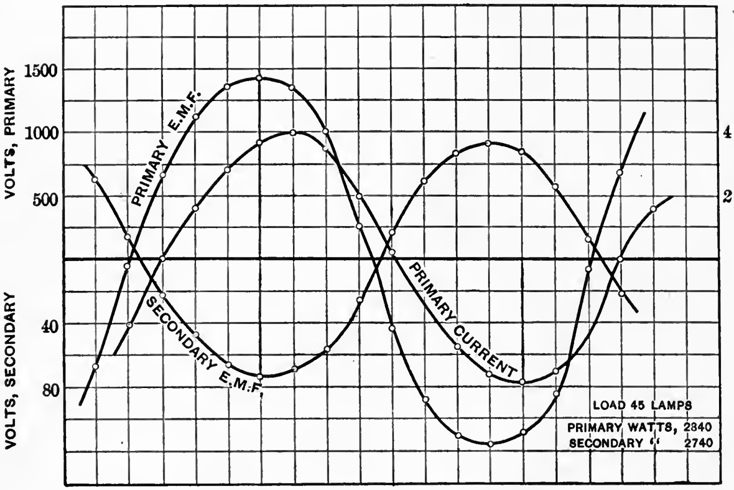


Fig. 113.—Transformer Curves.

merely the taking of the instantaneous curves for primary current and electromotive force at no load, and a determination of the primary current for various loads. The iron losses are determined by subtracting from the power given the transformer at no load, the power expended in the primary resistance. The total losses at any load are found by adding to the iron losses the loss in the primary, $R_1I_1^2$, and the loss in the secondary, $R_2I_2^2$. The efficiency is readily obtained when the total losses at various loads are known.

EXPERIMENT 54. **Study of the effects of capacity by method of instantaneous contact.**

In this experiment a condenser is connected in series with a non-inductive resistance. By means of the method of instantaneous contact, instantaneous values are obtained of the total impressed electromotive force, of the electromotive force at the terminals of the condenser, and at the terminals of the non-

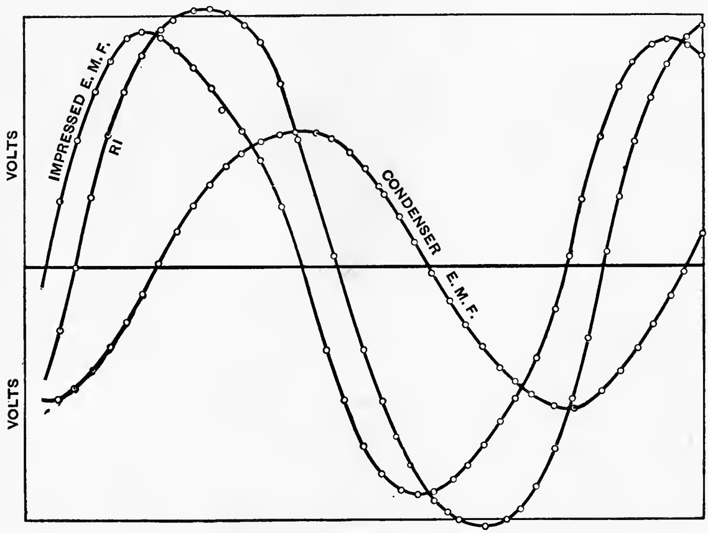


Fig. 114.—Effect of Capacity.

inductive resistance. This last is proportional to the current. These readings are plotted as in Fig. 114.

The results of this experiment are similar to those obtained in Exp. 52 for inductive circuits, and are to be similarly interpreted. The power expended in the condenser is obtained in the same way as for an inductive circuit, by multiplying together the instantaneous values of current and electromotive force. A curve for power is given in Fig. 115. The current is so nearly 90° ahead of the electromotive force that the power expended is small. The positive areas represent the power supplied to the condenser; the negative areas, the power returned from it.

The power consumed by the condenser is represented by the difference of the positive and negative areas, which are seen to be nearly equal in the case. The loss in the condenser and condenser efficiency should be determined.

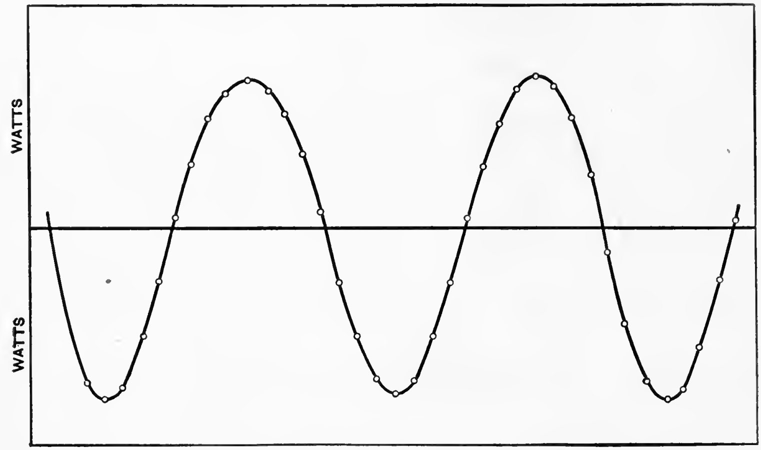


Fig. 115.—Power Consumed in a Condenser.

EXPERIMENT 55. **Determination of dielectric hysteresis.**

From the curves for instantaneous values of the current in a condenser and the electromotive force impressed, a curve may be plotted showing the value of the charge at any instant. The quantity of electricity which flows into the condenser in the time dt is $dq = idt$. The change in the quantity of electricity between the times t_1 and t_2 is, therefore, $Q = \int_{t_1}^{t_2} idt$. The charge is a maximum when the current is zero. As the current increases, the charge diminishes and becomes a maximum with the opposite sign when the current is again zero. Starting from one of the zero points on the current curve the changes in the charge from one maximum to another may be found by obtaining, with a planimeter, the areas between the current-curve, X-axis, and successive ordinates. It is to be observed that areas below the X-axis are negative. The values of the charge of the condenser at each instant of time are thus

obtained. Plotting charge as ordinates, and corresponding potential as abscissas, as in Fig. 116, a loop is obtained which represents hysteresis * in the dielectric of the same nature as the

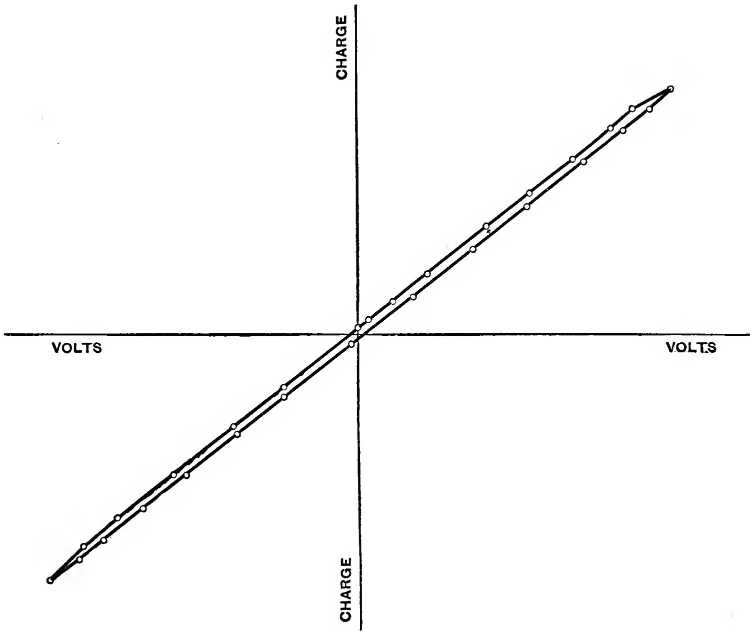


Fig. 116.—Curve for Dielectric Hysteresis.

hysteresis in iron.† The area of this loop, $\int vdq$, represents the amount of energy dissipated in the condenser each cycle, and may be determined by a planimeter.

EXPERIMENT 56. **Test of a non-inductive resistance by method of instantaneous contact.**

Whether a certain resistance is truly non-inductive or not may be ascertained by obtaining curves for instantaneous values of current and electromotive force by one of the methods

* "Alternate-Current Condensers and Dielectric Hysteresis," Bedell, Ballantyne, and Williamson, Physical Review, vol. 1, No. 2.

† See magnetization of iron, Exp. 62.

of instantaneous contact already described. If the current and electromotive force are found to be in phase, the resistance is non-inductive. A lagging of the current indicates the presence of self-induction; an advance indicates capacity. This method, strictly speaking, affords a means of comparing resistances, inasmuch as it requires a non-inductive resistance for the measurement of the current, and does not give an absolute indication of the presence of self-induction. If this resistance is not entirely non-inductive, the results are merely relative. By this method incandescent lamps, liquid resistances, and resistances made of tin strips with little space between are found to be practically non-inductive.

Experiment 57. **Investigation of liquid resistance.**

In many experiments with alternating currents, a non-inductive resistance is necessary, and it is often useful to construct a liquid resistance for this purpose. The object of this experiment is to ascertain the comparative advantages of different materials employed for such a resistance, and to determine the variation of the resistance with the strength of solution, which can best be done by arranging two parallel circular electrodes at a convenient distance apart in a vertical glass jar containing a solution, the resistance of which can be measured by the fall of potential method. Solutions of different strengths are obtained by first using a saturated solution and afterwards pouring off known portions of it and adding distilled water. Solutions of various substances, such as table salt, sulphuric acid, copper sulphate, and sodium carbonate or sulphate can be tested in this way

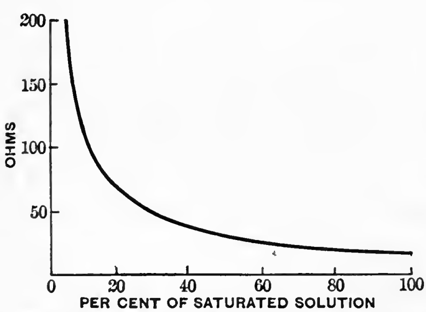


Fig. 117.—Resistance of Sodium Carbonate Solution.

and compared. In Fig. 117 is given a curve for the resistance of a solution of sodium carbonate between two electrodes four inches in diameter and eight inches apart. Such curves should be obtained for different solutions and compared.

The solutions should be further tested as to their carrying capacities and change of resistance with temperature. The solution in which the electrodes deteriorate the least rapidly is the most desirable, and the selection of a solution and the material for electrodes for permanent use should be largely determined by this consideration.

For alternating current work, a non-inductive resistance should manifest no electromotive force due to self-induction, capacity, or other cause. A liquid resistance practically meets this requirement, although no resistance can be absolutely free from such disturbing effects. This may be investigated by the method of instantaneous contact. (See Exp. 56.) For dynamo testing or experimental work in which large currents are used and a wide range of resistance is necessary, a convenient resistance may be made by constructing a long rectangular tank with two vertical iron plates for electrodes. One plate is stationary at one end of the tank; the other plate is movable and may be set by a hand wheel and endless chain at any desired distance from the fixed plate. The tank may be filled with any suitable solution, such as a weak solution of sodium sulphate. The desired resistance is obtained by adjusting the position of the movable plate. To obtain a high resistance, portions of the front or inner surfaces of the iron plates may be covered by glass, thus giving a great range to the resistance which may be obtained. A resistance arranged in this way possesses the advantage that it may be increased or decreased continuously without breaking the circuit, a point of considerable importance in certain lines of work.

EXPERIMENT 58. **Calibration of a hot-wire voltmeter.***

Adjust the instrument and calibrate up to the carrying capacity of the wire; measure the resistance of the voltmeter and construct additional non-inductive resistances (preferably of the same kind of wire) which are multiples of the voltmeter resistance so as to multiply the range of reading; measure the current taken by the voltmeter for each reading and compute the power consumed in the instrument, showing all results by curves. Observe the creeping up of the reading with time, plotting the reading as ordinate, and the time as abscissa; plot a similar curve for cooling, showing the return to zero. Unless the instrument be finely adjusted, the zero will not be constant. The instrument may be calibrated and used by referring all readings to the same zero during any series of observations, or by referring each reading to a zero taken just before and after the observation. Determine what you consider to be the most accurate method of calibrating and using the instrument, with particular reference to the zero, and the time at which the readings are taken.

EXPERIMENT 59. **Calibration of a hot-wire ammeter.†**

Adjust the instrument and calibrate with direct and with alternating current; plot the two calibration curves on the same sheet and interpret ‡ the results. Ascertain the potential

---

* See note under the following experiment.

† A convenient form of hot-wire ammeter can be readily constructed in the laboratory for this experiment by suspending 1 or 2 meters of German silver wire with a heavy weight attached. A small mirror can be so adjusted as to be rocked vertically as the wire is elongated and its deviation read with the telescope and scale. A hot-wire voltmeter may be similarly constructed. A German silver wire, 1 meter long, with a resistance of 270 ohms, will measure 50 volts, with no additional resistance, and will consume about 9 watts. These instruments can by no means be considered instruments of precision, but will serve the purpose of student work.

‡ It is to be borne in mind that the standard instrument used for calibrating may be one that does not read the same for direct and alternating currents.

difference at the terminals of the ammeter, for different currents and the consumption of power. Investigate the time element, as in the preceding experiment, and the constancy of the zero, and determine the most accurate method of using the instrument. Show all results by curves.

EXPERIMENT 60. **Determination of the constant of a ballistic galvanometer.**

If a transient current is sent through a ballistic galvanometer, the throw of the needle is approximately proportional to the quantity of electricity which passes through the galvanometer, provided the throw is small and the flow of electricity is for so short a time that the needle practically does not start from rest until the flow has ceased. The first throw is accordingly taken as a measure of the quantity of electricity which passes through the galvanometer; or,

$$Q = k\theta.$$

For relative measurement it is not necessary that the value of k be known; for absolute determinations the value of k, the quantity per scale division, should first be determined. The constant should be found by at least two of the following methods and the results compared.

(1) **By Discharge of Standard Condenser.** — The most obvious method is to discharge a known quantity of electricity from a standard condenser through the galvanometer and observe the throw. If a condenser with capacity C is subjected to a difference of potential E, its charge is $Q = CE$. If θ is the throw corresponding to this charge, we have $Q = CE = k\theta$; whence $k = \frac{CE}{\theta}$. The constant k is thus determined in coulombs per division when C is expressed in farads, and E in volts.

(2) **By Measurement of Deflection and Time of Vibration.** — By the theory of the ballistic galvanometer, $Q = \sqrt{\frac{HK}{MG^2}} \cdot \theta$, where

K is the moment of inertia of the needle, H the strength of field in which it swings, M its magnetic moment, and G the galvanometer constant. (More strictly, for θ we should write $2 \sin \frac{1}{2} \theta$.) The time for a complete vibration is $T = 2\pi\sqrt{\frac{K}{MH}}$; hence, $\frac{K}{M} = \frac{HT^2}{4\pi^2}$; and $k = \sqrt{\frac{HK}{MG^2}} = \frac{T}{2\pi} \cdot \frac{H}{G}$. A correction for damping when appreciable should be introduced; thus,

$$k = \frac{T}{2\pi} \cdot \frac{H}{G}(1 + \tfrac{1}{2}\lambda),$$

where λ is the logarithmic decrement. The value of k is found by determining separately the values of $\frac{T}{2\pi}$, $\frac{H}{G}$, and λ as follows.

The value of T is found by observing the time of vibration of the needle. To find $\frac{H}{G}$, pass a steady current I_0 through the galvanometer and observe the deflection θ_0. Since

$$I_0 = \frac{H}{G} \tan \theta_0,$$

it follows that $\frac{H}{G} = \frac{I_0}{\tan \theta_0}$, or for small deflections, $\frac{I_0}{\theta_0}$ approximately.

The ratio of damping is the ratio of successive amplitudes of oscillation and is constant. The logarithmic decrement is the logarithm to the base ϵ of the ratio of damping. If a_m is the amplitude of the mth, and a_n the amplitude of the nth oscillation, the ratio of damping is $\left(\frac{a_m}{a_n}\right)^{\frac{1}{n-m}}$. The logarithmic decrement is $\lambda = \frac{\log_\epsilon a_m - \log_\epsilon a_n}{n - m}$. When the observed amplitudes a_m and a_n have a ratio equal to the Napierian base ϵ ($=2.71828$), the error is a minimum.

For accurate methods for using the ballistic galvanometer, determining the period of oscillation, logarithmic decrement, correction for infinitely small arc, etc., the student is referred to Kohlrausch, and to Stewart and Gee.

(3) **By an Earth Inductor.** — When the value of the vertical component V of the earth's field is known, the value of k may be determined by an earth coil connected with the galvanometer. If the coil has S turns and a mean area A, it incloses AV lines when placed in a horizontal position. When reversed, it includes the same number of lines in the opposite sense. The quantity of electricity set in motion by the reversal is

$$Q = \frac{S}{R}(N_2 - N_1) = 2\frac{SAV}{R},$$

where N_1 and N_2 represent the initial and final number of lines passing through the coil, and R is the resistance of the whole circuit through which the induced current passes. The flow of this quantity of electricity through the galvanometer gives the needle a throw θ. Since $Q = k\theta$, it follows that $k = \frac{2\,SAV}{R\theta}$. C. G. S. units are here used.

It does not matter in what manner the reversal of the coil takes place, provided that its plane is horizontal at the beginning and end of operation, and the reversal takes place quickly. The method is somewhat unreliable in most laboratories on account of the liability to variation in the vertical component.

(4) **By a Standard Solenoid.** — A current is sent through a long solenoid (without iron) of known dimensions. A small secondary coil of S_2 turns is wound on the solenoid, and the current induced in it by making or breaking the primary current is measured by the throw of the ballistic galvanometer. If l_1, S_1, and r_1 are the length, number of turns, and radius, respectively, of the solenoid, the number of lines produced by a current i_1 is

$$N_1 = \frac{4\,\pi S_1 i_1}{l_1}\pi r_1^2.$$

The quantity of electricity which flows in the secondary on breaking the current in the solenoid is therefore

$$Q = \frac{4\,\pi^2 r_1^2 S_1 S_2 i_1}{l_1 R_2}.$$

If this produces a throw of θ in the galvanometer, we have

$$k=\frac{4\pi^2 r_1{}^2 S_1 S_2 i_1}{l_1 R_2 \theta}.$$

C. G. S. units are here used.

If the solenoid consists of one layer of wire, and if the secondary is wound outside of this layer, it is the inside diameter of the solenoid that should be used. If there are several layers, the average of the inside diameters should be used. The solenoid should be long (4 or 5 feet); to avoid the use of a long solenoid, a coil in the form of a closed ring may be used. In this case, l_1 in the above formula is the mean circumference of the ring.

EXPERIMENT 61. **Calibration of D'Arsonval or ballistic galvanometer for potential.***

When measurements of potential are to be made upon a circuit with which connection is made periodically by means of a contact-maker, the connection with the instruments used is not continuous, and it is usual to charge a condenser through

---

* A D'Arsonval galvanometer with a very strong artificial field was constructed by Messrs. Danforth and Wood, under the direction of Professor Harris J. Ryan for use as a sensitive ballistic galvanometer. [See "Synchronous Motors," thesis in Cornell University Library, 1891, by R. E. Danforth and E. M. Wood.] An instrument of this kind with an artificial field is independent of the earth's field and surrounding magnetic influences, and may be used with accuracy in the vicinity of dynamos or near wires carrying heavy currents. To make the instrument sensitive and to avoid the instability of permanent magnets, an electromagnet was used with the high magnetic density of 14,000 lines per square centimeter. The instrument resembles in general the galvanometer used in the Morse siphon recorder for use on submarine cables. The field magnets are wound with No. 17 B. and S. wire with a resistance of 50 ohms. The instrument is designed to be excited by any current up to 2 amperes, requiring, therefore, an electromotive force of not more than 100 volts at the terminals of the field magnets. It is often used with the magnet coils subjected to 40 volts. The suspended coil is rectangular, and consists of thirty turns of No. 40 B. and S. insulated copper wire. The coil is supported by a fiber and spiral spring above and below; the springs serve also to convey the current to the coil. A short-circuiting key enables the observer to bring the swinging coil immediately to rest. The coil carries a light pointer, and the throw is read directly from a graduated scale.

the contact-maker and to measure the potential of the condenser by discharging through a suitable galvanometer (see Exp. 50). This experiment consists in adjusting the galvanometer and calibrating by charging the condenser to different potentials and observing the corresponding throws of the galvanometer. A calibration curve, with potentials as ordinates and throw as abscissas, should be plotted for different capacities; that is, the condenser should be arranged for 0.2 microfarad, 0.4 microfarad, etc., in order to give a wide range of reading. A reversing switch should be used with the galvanometer, and double deflections observed.

When an instrument is employed such as that described in the foot-note, in which a magnetizing current is used, calibration curves should be taken for different magnetizing currents. When the calibration curves are approximately straight lines, as would usually be the case if the scale is properly arranged, the student should ascertain the magnetizing current (or the potential to which the magnetizing coil should be subjected) to cause the instrument to be direct reading under certain conditions, so that, for instance, the double deflection represents volts when the capacity of the condenser is 0.2 microfarad.

EXPERIMENT 62. **Magnetic qualities of iron. Ring method.***

The object of this experiment is to determine the magnetization† B, produced in the test-piece when subjected to various

---

* See Ewing, Magnetism of Iron and Other Metals; S. P. Thompson, Electromagnet and Dynamo Electric Machinery. Also Trans. A. I. E. E.; January, June, September, 1892; Lond. Elect., Sept. 30, Oct. 10, 1892.

† The American Institute of Electrical Engineers, March 21, 1894, adopted provisionally the following magnetic units: —

The term **gilbert** for the C. G. S. unit of magnetomotive force, the same being produced by 0.7958 ampere turn approximately.

The term **weber** for the C. G. S. magnetic unit of flux, sometimes described as the C. G. S. line of flux.

The term **oersted** for the C. G. S. unit of reluctance.

The term **gauss** for the C. G. S. unit of the flux density, or one weber per normal square centimeter.

magnetizing forces H, and to ascertain the permeability of the iron for different magnetizations. The results are shown by a "curve of magnetization," Fig. 118, the ordinate at any point of which represents the value of B (lines of induction per square centimeter), corresponding to the abscissa H, which represents

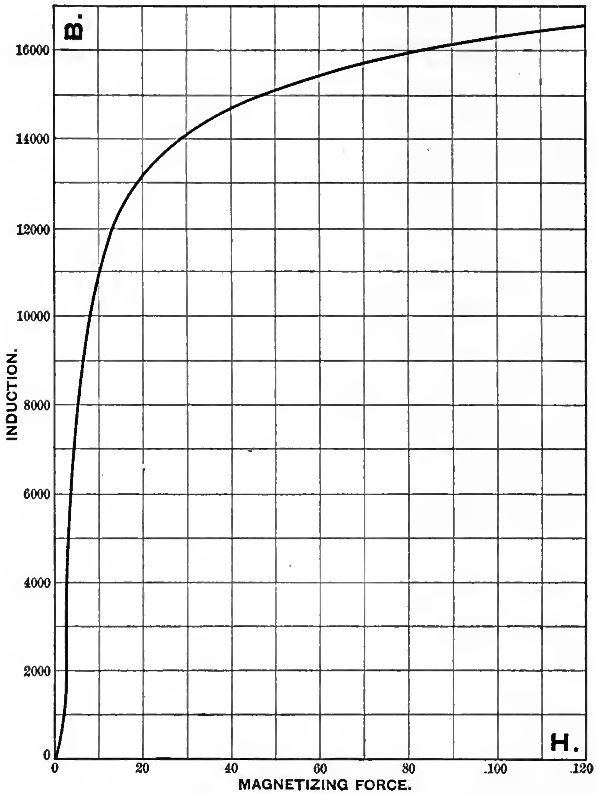


Fig. 118.—Curve of Magnetization. Steel.

the magnetizing force. A permeability curve is shown in Fig. 119.

The iron, usually in the shape of a laminated ring, should first be entirely demagnetized, either by annealing or by being "demagnetized by reversals." This latter process consists in

subjecting the iron to a continually decreasing magnetizing force, which is repeatedly reversed. This reversal of the magnetizing force may be accomplished by passing a direct current through the magnetizing coil, and reversing the direction of this current by a suitable commutator, or by using an alternating current. In either case the magnetizing current is gradually

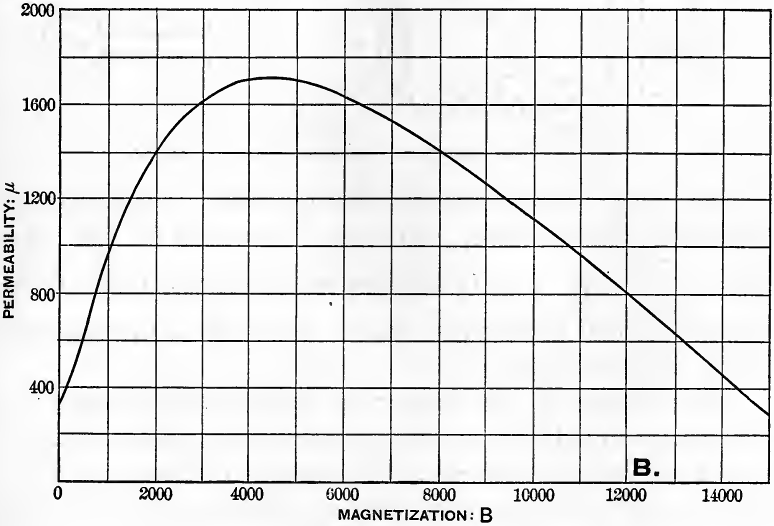


Fig. 119.—Permeability Curve. Steel.

reduced by increasing an adjustable resistance connected in the circuit.

The dimensions of the core are to be accurately determined before the coils are wound upon it. A primary coil and a secondary coil, with S_1 and S_2 turns respectively, are wound uniformly about the ring, the size of the wire and the number of turns depending upon the size of the ring, the galvanometer used, etc. The primary coil is connected with a constant source of electromotive force, and has connected in it an ammeter, a reversing switch, and a resistance which may be adjusted without breaking the primary current. It is important that the resistance should be arranged with this in view, so that the

current can be varied continuously. The secondary is connected directly with the ballistic galvanometer. A resistance may be inserted if necessary. The connections are shown in Fig. 120, in which the two coils are shown apart; but preferably each is wound uniformly over the entire ring.

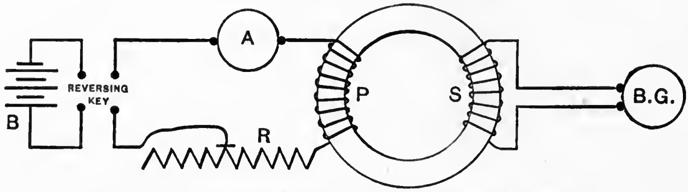


Fig. 120.—Arrangement of Apparatus for Ballistic Method.

The magnetizing force is calculated from the magnetizing current, primary turns, and the dimensions of the ring, $H=\frac{4\pi S_1 I_1}{10 l}$. Here S_1 is the number of primary turns, I_1 the primary current in amperes, and l the length of the magnetic circuit in centimeters.

Any change in the number of lines passing through the coil causes an induced current in the secondary coil, and a throw of the ballistic galvanometer. The quantity of electricity which flows, due to a change in the number of lines from N' to N'', is

$$Q=\frac{S_2}{R_2}(N''-N');$$

that is, it is proportional to the change in the number of lines through the coil. This causes a proportional throw of the galvanometer; for $Q=k\theta$, where θ is the throw of the galvanometer, and k the galvanometer constant. This constant should be determined according to the method given in Exp. 60. The galvanometer should have a long period; five or ten seconds. The flow of secondary current is almost instantaneous, and will take place before the needle begins to move. This is true if the iron is laminated so that the change in induction is not retarded by eddy currents. If the iron is not laminated, a longer period may be used.

From the throw of the galvanometer, the value of the quantity of electricity which flows is determined when k is known, and so the change in the induction, $N''-N'$, when S_2 and R_2 are known. The induction, B, per square centimeter is equal to the total induction divided by the cross-section of the core.

Step by Step Method. — In this method, the magnetization is increased by a series of "steps" caused by a sudden increase in the magnetizing current. The change in the magnetization due to each increment of magnetizing force is measured by the throw of the galvanometer. One disadvantage of this method is that the errors are cumulative.

By obtaining a descending curve of magnetization and taking the iron through a complete cycle of magnetization, a cyclic curve of magnetization or hysteresis loop, Fig. 121, is obtained, and from its area the hysteresis loss for each cycle. Preliminary demagnetization is not necessary in this case. The iron should, however, be carried through several complete cycles before measurements are taken.

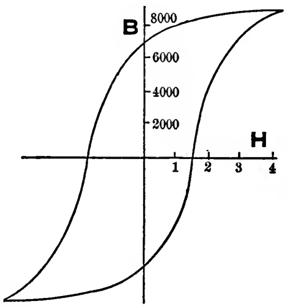


Fig. 121. — Hysteresis Loop.

Method of Reversals. — In this method the value of $2N$ is found by a sudden reversal of the current. In many ways this is more convenient than the former method, and is quite accurate for soft iron; with hard iron it is not so accurate.

EXPERIMENT 63. **Magnetic qualities of iron. Instantaneous contact method.**

The relation between the magnetizing force and the magnetic induction in an iron ring may be found by a method due to Drs. J. and B. Hopkinson,* in which the method of instantaneous

* London Electrician, Vol. 29, p. 511; September 9, 1892. Also, Gray's Absolute Measurements in Electricity and Magnetism, Vol. 2, Part 2, p. 752.

contact is employed to obtain the instantaneous values of an alternating current passing through the magnetizing coil, and of the impressed electromotive force. The iron to be tested is in the form of a ring, and is well laminated to prevent eddy currents. The lamination is best obtained if the ring is formed of iron wire. The ring is wound uniformly with the magnetizing coil, and is connected in series with the non-inductive resistance R, as shown in Fig. 122. An alternating current is passed through the resistance and the magnetizing coil. By one of the methods of instantaneous contact already described, the instantaneous values are found of the potential differences between the points a and b at the extremities of the resistance, and between the points b and c at the terminals of the magnetizing coil.

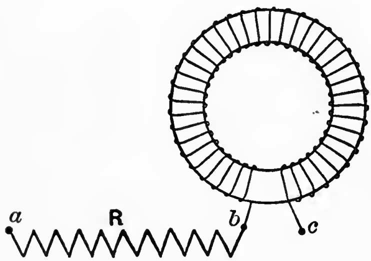


Fig. 122.

From the readings between a and b a curve is plotted to represent the current. This curve, which we will call curve I., is not here shown. A second curve, II., is plotted from the readings between b and c, to represent the value of the electromotive force at the terminals of the coil; that is,

$$R_c i + \frac{dN}{dt},$$

where $R_c i$ is the electromotive force to overcome the ohmic resistance of the coil. If a third curve, III., be plotted by subtracting the instantaneous values of $R_c i$ from curve II., the ordinates of curve III. are equal to $\frac{dN}{dt}$, the time rate of change of the total induction of the coil. By integrating this curve with a planimeter, — that is, ascertaining the area between the curve and the horizontal axis, — we may find the values of the total induction of the coil, and thus plot curve IV. The magnetizing

force may be obtained from curve I., and the corresponding magnetic induction from curve IV.; by plotting the one as ordinate and the other as abscissa, a curve of magnetization may be drawn for a complete cycle, as in Fig. 121.

This method corresponds to that given in Exp. 55 above for the determination of dielectric hysteresis.

EXPERIMENT 64. **Illustrative experiments with alternating current magnets.**

These experiments are to illustrate the effects of eddy currents in a piece of copper placed in the neighborhood of an alternating current magnet.

If a copper disk be placed opposite to the pole of an alternating current magnet, an electromotive force will be induced in the copper on account of the lines of force which cut in and out as the magnetic field is reversed, and currents will be set up in the copper as in the secondary of a transformer. The electromotive force induced, being proportional to the rate of change of the magnetic field, and so also to the rate of change of the current flowing in the coil of the magnet, is 90° in phase behind the current in the coil.

If the induced eddy currents were in phase with this electromotive force, and so 90° behind the current in the coil, there could be no resultant attraction or repulsion between the disk and the coil, for the force between them at any instant is proportional to the products of the currents in each, and would be positive for half the time, indicating an attraction, and negative for half the time, indicating a repulsion. However, these eddy currents lag somewhat, and the repulsion is therefore greater than the attraction, and the disk is repelled. This repulsion is shown by

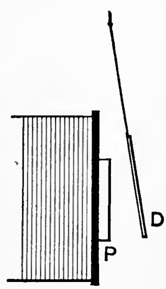


Fig. 123.

suspending a disk of copper *D*, Fig 123, in a vertical position before the pole *P*, of an alternating current magnet. The

repulsion may be shown and measured by suspending the disk from the arm of a balance in a horizontal position over a vertical magnet.* Note the effect of a slit in the disk from edge to center.

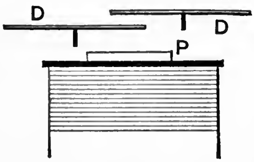


Fig. 124.

Some of the phenomena with alternating current magnets may be better understood after the following experiments with a *direct* current magnet. Hold a sheet of copper over the pole of a strong direct current magnet. Repeatedly make and break the magnetizing current, and note the effect. Strike the magnet with the copper. A repulsion is felt which cushions the blow. This cushioning is well shown by dropping a copper disk upon the magnet. If the copper is suddenly withdrawn, its removal is retarded. These and other effects are to be explained in accordance with Lenz's law.

If two pivoted disks, *D*, *D*, be held over the pole *P*, of an alternating current magnet, as in Fig. 124, so that one partially shields the other, the disks will slowly revolve in opposite directions. If one disk is held still, the other will revolve. The force producing revolution is on account of the unsymmetrical distribution of eddy-currents due to the shielding. If the disks are placed so that there is no shielding, there is no tendency for them to revolve.

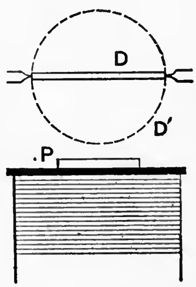


Fig. 125.

A piece of copper in an alternating field tends to place itself in a position such that as few lines of force pass through it as is possible.

This may be illustrated in various ways. A copper disk *D*,

* With a *very strong* alternating current magnet, the force of repulsion exerted between it and a stout copper ring placed above it may be sufficient to support the weight of the ring, so that it will remain suspended in position when *held down* by cords to the table. Note the effect of cutting the ring. This and other experiments were first shown by Professor Elihu Thomson.

pivoted horizontally, and placed unsymmetrically over the pole *P*, of an alternating current magnet, tends to place itself in a vertical position, as indicated by the dotted lines *D'* in Fig. 125. There is a tendency for a piece of copper to move from a stronger to a weaker field.

These phenomena should be investigated by various experiments performed under different conditions and their causes explained. Professor Thomson has described a number of such experiments,* to which the reader is referred.

* "Novel Phenomena of Alternating Currents," Professor Elihu Thomson, Elect. World, May, 1887, p. 258. See also Lond. Elect., May 16, 1890, p. 35, *et seq.*

Volume II. Part III.

SENIOR COURSE IN PHOTOMETRY AND HEAT.

By Charles P. Matthews.

CHAPTER I. — ELECTRIC LIGHT PHOTOMETRY.

CHAPTER II. — HEAT.

CHAPTER I.

EXPERIMENT I. **Standardization of instruments.**

The instruments used in photometric work should, so far as possible, be carefully standardized as a preliminary experiment. Thus if a mirror galvanometer is used for measuring the potential difference between the terminals of an incandescent lamp

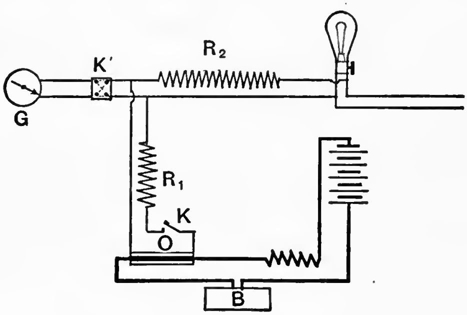


Fig. 126.

whose candle-power is being determined, it is expedient to determine a constant factor which, multiplied into the double deflection in scale divisions, will give the potential difference directly in volts. In Fig. 126, a permanent arrangement is shown by means of which this determination may be made with quickness and accuracy. The galvanometer G, reversing key K' and high compensated resistance R_2 are contained in a shunt circuit from the lamp terminals. An auxiliary or calibrating circuit is arranged as follows: Current, preferably from storage cells, is allowed to flow through a standard carbon *ohm*, in series with a Thomson milli-ampere balance and a suitable resistance. By closing key K, the galvanometer in series with a compensated

resistance R_1 is shunted around O. R_1 should be much smaller than R_2. From observations of current and deflections, and from known resistances, the value of the desired multiplier may be readily calculated. The computation should be made, taking account of the current flowing in the galvanometer circuit, and also when this current is neglected.

* * * * * * *

When a Methven* screen and lamp are used as a standard, a comparison should be made with the standard English candle. For this purpose a special form of balance is provided. The arms of the balance are in the ratio 1 : 2, and two candles are mounted at the extremity of the short arm. The candle-holders are capable of vertical adjustment.

The balance should be placed at the end of the photometer bar, and the plane of the wicks carefully adjusted so as to be coincident with the standard plane determined by the plumb lines. When the candles have burned sufficiently, the pointer on the balance will swing past the zero mark. The time of passage should be noted. A gram weight should now be removed from the pan. The candles will soon lose an equal amount of spermaceti, and the pointer will again swing. The time of passage should be noted as before. From these readings the student will readily compute the consumption of spermaceti in grains per hour, and the per cent deviation from the prescribed standard. Great care should be taken to prevent draughts of air, as the results will be vitiated should such occur. The Methven slit may be compared with the candles by making frequent settings of the photometer during the trial.

The standard British candle is made of spermaceti, burns 120 grains per hour, and weighs $\frac{1}{6}$ pound. In this experiment the intensity is assumed to be proportional to the rate of consumption of material.

---

* For description of Methven screen, see Dibdin's Practical Photometry.

Many investigations of the efficiency of candles as a standard light source for photometric work have shown that the consumption of wax is not the only factor which determines the luminous intensity. The height of the flame is also a very important element. Moreover, the nature and form of the wick and its degree of torsion exert a marked influence. Kruss, who has made exhaustive comparisons of the different candles with respect to the heights of flame, has shown that the least variation occurs in the British candle, the limits of the height of the flame being 44 mm. and 52 mm., the mean height from a number of observations being 47.67 mm., while the prescribed height is 44 mm. His experiments seem to indicate that it is necessary to snuff the candle repeatedly in order to maintain the normal height. The student is referred for a discussion of these matters to the work of Palaz.*

EXPERIMENT 2. **Distribution of candle-power about an incandescent lamp.**

For a description of the Bunsen photometer and method of use see Vol. I. The flame of the Argand burner should be adjusted until it burns freely at the proper height. The lamp to be tested should be mounted on a revolving lamp-holder, care being taken that the center of the filament is in line with the center of the photometer disk and the plumb line which determines the axis of the photometer bar. The lamp should be revolved to see that its axis of figure remains in one plane. In lamps with twisted or drooping filaments this adjustment should be made with reference to the filament itself, and not to the lamp bulb. The lamp is in the *standard position* when the plane of the shanks of its filament is normal to the photometer bar; and when in this position the side of the lamp towards the bar should be marked, so that if the lamp is removed from the holder it may be correctly replaced.

* Traité de Photométrie Industrielle, Paris, 1892.

Since what is desired is the intensity of illumination at a series of equidistant points on a sphere of unit radius described about the luminous source, some conventional method of designating these points must be adopted. In the Franklin Institute tests, carried on during the International Exhibition of 1884,* the unit sphere was considered to be crossed by parallels and meridians, formed by the intersections of the sphere with the chosen planes of rotation. The points were accordingly located through their latitude and longitude as reckoned from the standard positions. The lamp-holder is usually provided with a vertical and a horizontal circle graduated in degrees. The foregoing method is consequently liable to introduce confusion, from the fact that latitude and longitude are not reckoned continuously from 0° to 360°. It would seem better, therefore, to speak merely of horizontal and vertical circles, graduated continuously in degrees, the positive direction on the vertical circles being counted always from the horizontal circle towards the vertex (north pole) of the lamp. Any point on the unit sphere would therefore, by this method, be determined first, by the azimuth of its vertical circle, and second, by its angular position on this circle.

After bringing the potential difference at the terminals of the lamp to the value indicated by the makers, readings of candle-power should be taken on four vertical and one horizontal circles as follows:

13	readings	30°	apart	on the	horizontal circle.				
13	"	"	"	"	"	vertical circle,		0°	azimuth.
13	"	"	"	"	"	"	"	45°	"
13	"	"	"	"	"	"	"	90°	"
13	"	"	"	"	"	"	"	135°	"

Total, 65 readings.

---

* See supplement to Jour. of Franklin Institute, September, 1885.

The following 38 points on the unit sphere are nearly equidistant; their mean intensity of illumination may be taken as the *mean spherical candle-power* of the lamp.

Mean of 4 measurements at the intersection of the vertical circles, 90° V. (north pole)	1
Four measurements on each of the vertical circles, 0° and 90° azimuth, at 60°, 120°, 240°, and 300° V.	8
Four measurements on each of the vertical circles 0°, 45°, 90°, and 135° azimuth, at 30°, 150°, 210°, and 330° V . .	16
Twelve measurements 30° apart on the equator	12
One (zero) reading at the intersection of the vertical circles, 270° V. (south pole)	1
	38

By means of polar co-ordinates the student should plot the distribution of luminous intensity in the different planes. On the plot of horizontal distribution, circles may be drawn corresponding to rated candle-power and mean horizontal candle-power. The *standard reading* is the mean of the four readings in the standard position. The spherical distribution of intensity differs but slightly in the lamps of any given type, and the mean spherical candle-power bears a constant ratio to the mean horizontal candle-power, which latter bears a constant ratio to the standard reading. Hence in most commercial work, the horizontal and spherical distribution being known from many careful determinations, it is customary to compare lamps of the same type through their standard readings; and to make use of *reduction factors* connecting the intensity in the standard position with the mean horizontal and spherical intensities.

$$\text{Horizontal reduction factor} = \frac{\text{mean hor. c. p.}}{\text{standard reading}}.$$

$$\text{Spherical reduction factor} = \frac{\text{mean sph. c. p.}}{\text{standard reading}}.$$

In this experiment it is necessary to keep the potential difference at the terminals of the lamp rigorously constant.

Readings of current should be taken occasionally. The following form of log will be found convenient for exhibiting the results of incandescent lamp tests. The log is filled with the

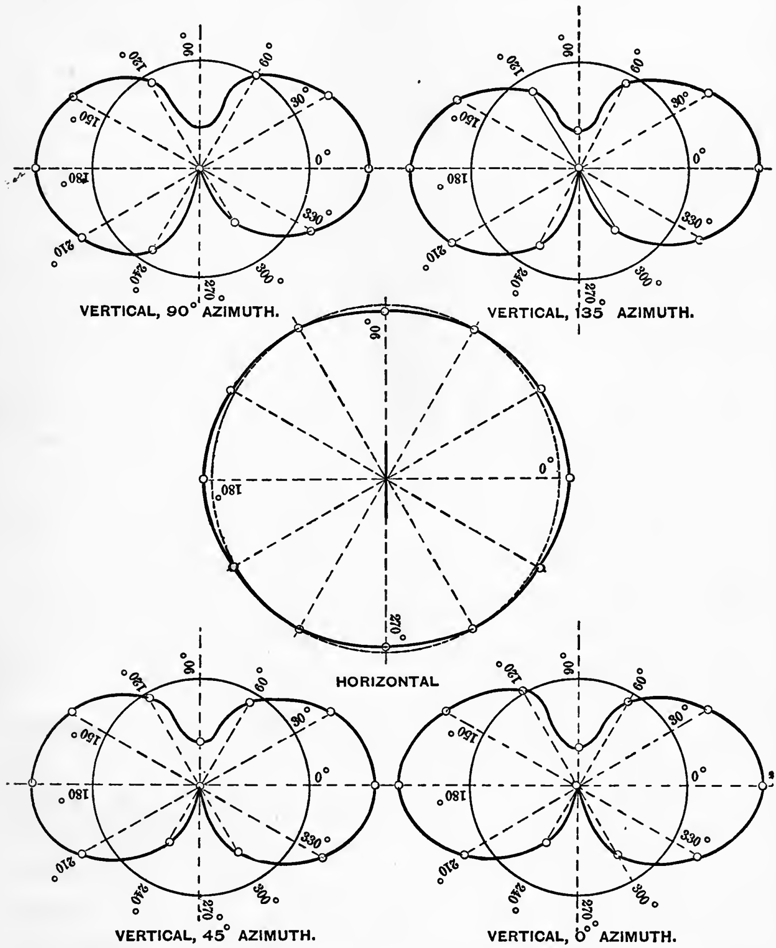


Fig. 127.—Distribution of Candle-Power, Incandescent Lamp.

results of an illustrative case. Fig. 127 shows graphically the distribution of luminous intensity as obtained in the above-mentioned horizontal and vertical planes.

Test of .... *Lamp No.* 26 *Rated* { *C. P.* 10 / *Volts* 100 }

Standard Reading 16 *Mean Horizontal C. P.* 15·28 *Mean Sph. C. P.* 9·25 *Hor. Red. Factor* ·955 *Sph. Red. Factor* ·578 *Volts* 110 *Amperes* ·659 *Watts* 72·49 *Resistance (Hot)* 166.9 *Watts Per Mean Hor. C. P.* 4·74 *Watts Per Mean Sph. C. P.* 7·836 *Lamps Per E. H. P.* 10 + *Mean Hor. C. P. Per E. P. H.* 157·3.

Observers ....

Remarks ....

				Azimuth.	0°	45°	90°	135°
Horizontal Circle.	0°	16	Vertical Circle.	0°	16	15	14.5	15.6
	30	15.3		30	15.4	13	13.1	13.2
	60	15		60	8.6	8.6	9.5	8.5
	90	14.5		90	3.6	4	3.8	3.4
	120	15.1		120	9.7	9	8.8	8.1
	150	15.6		150	13.4	13.1	12.9	12.3
	180	16		180	15.7	15	14.6	14.9
	210	15.6		210	11.3	11.9	12	12.9
	240	15.1		240	5.5	5.4	8	7.5
	270	14.6		270	0	0	0	0
	300	15.1		300	7.1	6.5	6.6	6.3
	330	15.5		330	12.1	11.2	10.8	12
	360			360	16	15	14.5	15.6

Experiment 3. **Characteristic curves of an incandescent lamp.**

By analogy with the graphics of the dynamo, those curves which show the variation in luminous intensity of an incandescent lamp as a function of such variables as current, potential difference, energy supplied, etc., may be called the *characteristics* of the lamp. The directions for performing this experiment are briefly these: Employ a standardized incandescent lamp as a secondary standard. This lamp should be of such intensity as to bring the settings of the photometer in a convenient part of the bar. As the lamp under test becomes

very brilliant under high potential differences, it may be found convenient to employ two standard lamps, one of low candle-power for the first part of the run, and one of higher candle-power for replacing the first as the intensity of the lamp under test becomes high. By means of a resistance in series with the lamp, the potential difference at its terminals should be gradually increased from that which just produces a readable intensity to that which ruptures the filament. The increments of potential should be about 10 volts in the first part of the test, and later about 3 volts. Make simultaneous readings of the photometer setting, the current and the potential difference of both test lamp and standard.

In working up the results, the readings of the 1000-part bar must be reduced to candle-power. This may be done conveniently by means of a curve plotted with bar readings as abscissas, and the ratio $\frac{(l-r)^2}{r^2}$ as ordinates, where r is the reading reckoned from zero at the standard lamp, and l is the length of the bar in scale divisions. This gives the candle-power corresponding to a given setting computed on the basis of a standard light of unit intensity. The ordinate of the curve must therefore be multiplied by the candle-power of the standard as obtained from a calibration curve giving the relation between potential difference and candle-power. For making this computation it may be preferable to many to use a four-place logarithm table showing at a glance the logarithm of the foregoing ratio, to which, of course, must be added the logarithm of the candle-power of the standard. Both curves and logarithm tables for this purpose are given in Appendixes A and B. The student should plot the following curves:

Potential	(abscissas)	and	candle-power	(ordinates).	
Current	"	"	"	"	"
Watts consumed	"	"	"	"	"
Watts per candle	"	"	"	"	"
Resistance	"	"	"	"	"

Fig. 128 shows the behavior of a 10 candle-power lamp. When this lamp was at a very high temperature, it was cut out of circuit in order to change from the storage cells to a source of higher potential. During this interval of time the lamp became

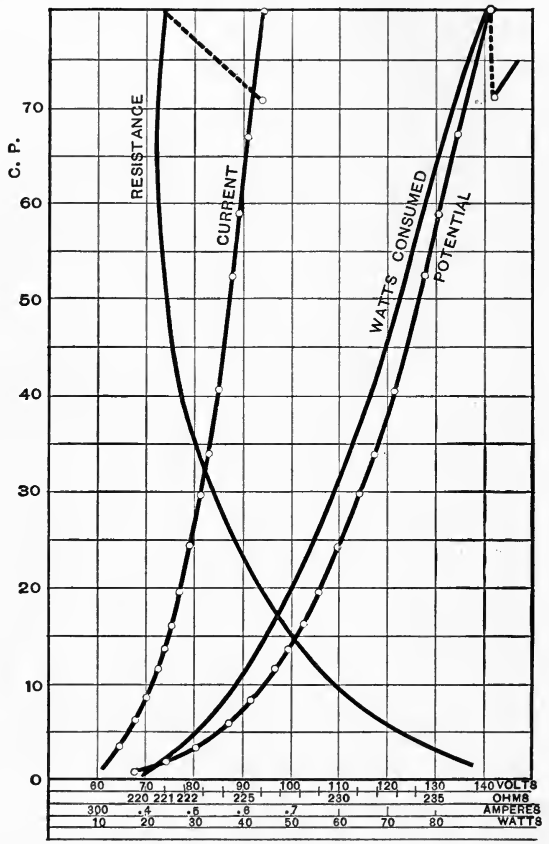


Fig. 128.—Characteristic Curves of an Incandescent Lamp.

quite cool, and, as indicated in the figure, when the potential was raised to the highest value previously attained, the lamp was of markedly lower candle-power, the resistance of the filament having changed to a considerable extent. This peculiarity has been observed by a number of experimenters, and it is claimed

by some* that the lamp returns to its former efficiency after a period of about fifteen minutes.

The resistance of the filament falls off very rapidly at first, with increasing temperature. The rate of decrease, however, becomes smaller and smaller, and a minimum value, such as is here shown, has been reported in many cases. It must be remembered, however, that at these extreme temperatures disintegration of the filament occurs quite rapidly. This fact renders it difficult to obtain accurate readings in this portion of the curve. Thus if the lamp be allowed to remain any length of time at so high a temperature, the change in resistance is shown by a slow creeping of the needles of the measuring instruments.

A number of investigators have found equations for the various curves. Throughout the working range, the potential curve may be represented by an equation of the exponential form, $y=ax^n$, although Götz† has shown that the relation is more rigorously given by $y=ax+bx^n$.

EXPERIMENT 4. **Photometry of the arc-light.**

With slight modifications, the methods employed in the foregoing experiment on the distribution of intensity about an incandescent lamp may be employed in connection with the arc-light. It is customary in the latter case to obtain the distribution in a single vertical plane; and, as it is impracticable to set the lamp at different inclinations to the vertical, a small crane is used for giving the lamp different vertical positions while the desired ray is received on a mirror, so adjusted as to reflect the incident ray to the photometer disk. This mirror is inclined to a horizontal axis by an angle of 45°; it is capable of rotation about this horizontal axis, and is provided with a pointer and graduated disk. The process consists, then, in setting the mirror at the desired angle and then raising or lowering the

* Ferguson and Center, London Electrician, xxviii. 112.
† Centralblatt für Electrotecknik, v. 720.

lamp by means of the ropes of the crane until the illuminated ellipse produced by the mirror is concentric with the Bunsen disk. The distance of the arc from the mirror must of course be maintained constant. Thus if the mirror is set at 800 divisions of the bar, the distance from the mirror to the arc must be 200 divisions when using a 1000-part bar. If the crane is of

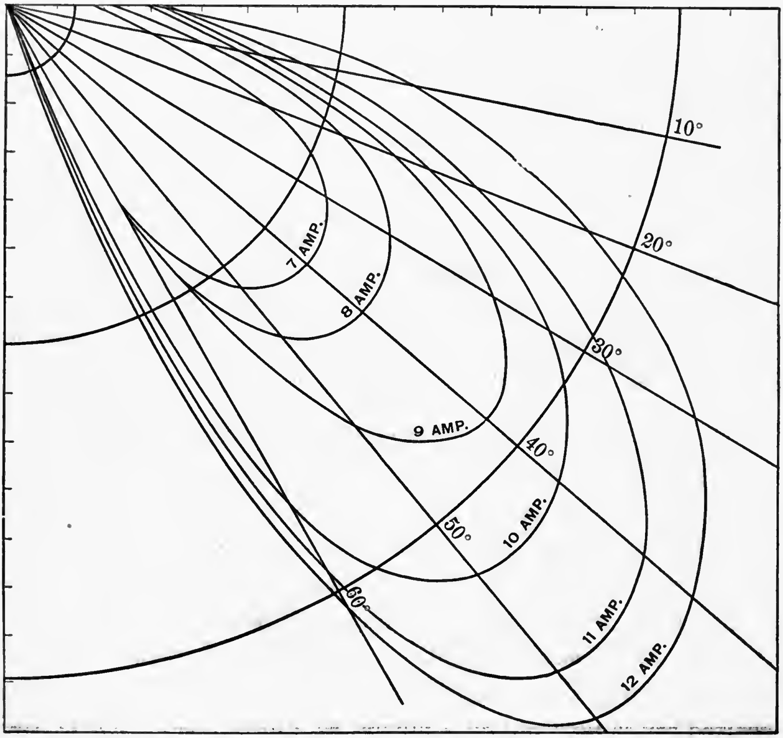


Fig. 129. — Distribution of Candle-Power, Arc Lamp.

the ordinary type, that is, one provided with an adjustable boom, this may be accomplished, within certain limits, by suitable adjustment of the ropes, which regulate not only the angle of the boom, but also the vertical position of the lamp.

If an incandescent lamp is used as a light standard, it is evident that the intensity of the light from the arc must be

reduced a known amount in order to bring the photometer settings near the middle of the bar. This may be accomplished by the use of rotating disk provided with adjustable sectoral openings. As usually arranged, one-half the incident light is allowed to pass when the sectors are fully opened. Moreover, a certain amount of light is lost by the absorption of the mirror; the constant for correcting for this must be known.

The arc is continually shifting from one side to another of the carbons, causing large fluctuations in the intensity. The accepted reading for each position should therefore be the mean of a number of settings. The energy consumed by the lamp should be determined by making frequent readings of current and of potential difference at the terminals of the lamp.

Figure 129 shows a set of curves which are representative of what is obtained in a vertical plane in the case of an arc-light. It shows also the intensity as a function of the current.

The chief difficulty in arc-light photometry arises from the great difference between the quality of the light from the arc and that from all of the common standards. The recent attempt of Blondel* to use an isolated portion of the crater of the positive carbon as a standard has not been entirely successful, and it is probable that this scheme will not fulfil all the requirements of a satisfactory standard for arc-light photometry.

---

* La Lumière Électrique, xlvii. 573, and London Electrician, xxx. 658.

CHAPTER II.

EXPERIMENT 5. **Specific heat by the method of mixtures.**

It is assumed that the student has become familiar with the principle of the method of mixtures through the performance of such experiments as I_3, Vol. I. The accurate determination of the specific heats of solids and liquids requires close attention to a number of sources of error, and it is important that the experimenter should acquire a working knowledge of the experiment in its simpler form as a preliminary step. Among the chief sources of error are the following:

(1) In addition to the heat which goes to raise the temperature of the cold body and of the calorimeter, a certain amount is absorbed by the thermometer and the stirrer which are necessarily present in the calorimeter. The water equivalent of these objects must therefore be found.

(2) If the experiment be performed by transferring the hot body from a bath to the calorimeter, the body will cool during the transfer. Also a certain amount of hot water will be carried over with the hot body. In any case, heat will be lost or gained during the actual process of making the mixture unless special precautions are taken. The apparatus described below is designed to minimize errors from this cause.

(3) The final temperature of the mixture is not attained immediately. During the interval between the instant of mixing and the attainment of the final temperature, radiation or absorption of heat occurs, according as the temperature of the calorimeter is above or below that of the surrounding air. The error arising from this cannot be neglected in exact determinations, and a number of methods of evaluating the

loss by radiation have been proposed and used by different physicists. A summary of these methods has been given by Berthelot.* When the mass of the refrigerating water is such that the total temperature change is small, producing but a slight excess over the air temperature, the correction for radiation is very small. It may be said in general that a correction is unnecessary in the case of a calorimeter containing more than 500 grams of water, in which the final temperature is established in less than two minutes, and without exceeding the temperature of the air by more than 2°. These conditions cannot always be met. Too small a rise in temperature diminishes the sensitiveness of the method, and necessitates the use of a very delicate thermometer. Furthermore, the thermometer in the mixture may not indicate a maximum until 5, 10, or, in the case of substances which are poor conductors, even 15 minutes have elapsed. The cooling or warming due to the influence of the surrounding air cannot, in such cases, be neglected.

Sometimes the error arising from this cause is eliminated by a method of compensation due to Rumford. This method consists in beginning the experiment with the temperature of the calorimeter and cold water as many degrees below that of the air as the temperature of the mixture will be above it. The heat gained by absorption is then assumed to be equal to that lost by radiation. This would be quite approximately true provided the periods of warming and cooling were of the same duration. This, however, is far from being the case, since the immersed body parts with its heat rapidly at first, thereby making the first period considerably shorter than the second. As remarked by Regnault, the compensation is more perfect when the experiment is so arranged that the excess of the air temperature over that of the refrigerating water is equal to

* Journal de Physique, ii. 345 (1873). See also method recommended by M. N. Hesehus, *Ibid.* vii. 489 (1888).

two-thirds the total rise of temperature. This method assumes at least an approximate knowledge of the specific heat of the substance which is to be examined.

Another method of correction for radiation consists in making an experimental determination of the *radiation constant* of the calorimeter, on the assumption that the law of Newton is rigorously true for the excess of temperature liable to occur in the specific heat experiment. (See Chap. III., Vol. 1.)

The chief objection to either of the foregoing methods of procedure lies in the fact that they assume an accurate knowledge of the temperature of the air surrounding the calorimeter. It has been shown by experiment that this is not a well-defined quantity, and that it varies at different points in the immediate vicinity of the calorimeter. A method of correction for radiation which involves the temperature of the air, but which necessitates no measurement of it, was used by Regnault. The details of this method have been published by Pfaundler.* The reader is also referred to a method, due to Berthelot,† which accomplishes this same result, with perhaps greater rigor, although with less simplicity. A description of the former method will be deferred until after the process of the experiment has been described.

The apparatus used by Regnault is shown in section in Fig. 130. It provides a means of heating the substance without wetting it, and of quickly transferring it to the calorimeter. The chamber A is heated by steam present in the annular space B, the steam being generated in the boiler F, and condensed in G. A third space C acts as an air jacket, and prevents condensation. The upper end of the heating chamber is closed by a cork, through which the stem of a thermometer passes, while the lower end is closed by a slide, which may be instantly opened by releasing a spring. A vertical sliding par-

* Annales de Chimie et de Physique, 4th series, xi. 248 (1867).

† Journal de Physique, ii. 345 (1873).

tition protects the calorimeter from the heat of the apparatus after the mixture has been made. The calorimeter is made of thin brass, highly polished externally, and surrounded by a similar brass dish, of which the interior surface is polished. The calorimeter is sometimes suspended in the outer dish by

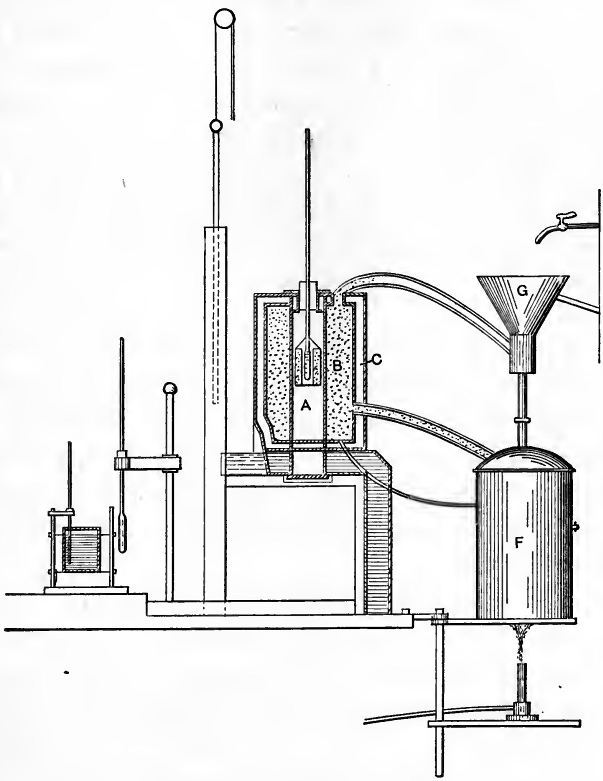


Fig. 130.—Regnault's Apparatus.

silk threads, but it is more stable if allowed to rest on light wood supports, which raise it by an amount equal to the width of the annular space between the two dishes. The substance is lowered into the heating chamber in a wire-gauze basket, of small mass as compared with the mass of its contents. Baskets of any desired lightness may be obtained by forming them of

the wire gauze and then allowing them to remain for a short time in nitric acid. On the interior of the basket a gauze cylinder is provided for the reception of the thermometer bulb.

The material to be examined should be reduced to a granular form, in order that it may quickly yield up its heat to the water in the calorimeter. When the substance is pulverulent, it may sometimes be formed into pellets by first moistening it with water. When this is not possible, the powder must be placed in a cylindrical vase with extremely thin walls. As remarked by Regnault, this method should be used only as a last resort, since the escape of heat to the refrigerating water is greatly retarded by the presence of the retaining walls. In fact, if the substance be a bad conductor, the final temperature may be reached only after 10 or 15 minutes, in which case the method loses much of its rigor. The loss of heat by radiation during this period may be quite large; and it is evident that the substance is not at the temperature of the water at the instant the thermometer indicates a maximum, but is at a somewhat higher temperature, since the thermometer will indicate a maximum at the instant when the calorimeter is losing heat at the same rate that heat is being given to by the substance in virtue of its higher temperature.

The experiment may be performed as follows: Start the heating apparatus. Suspend the basket and material in the heating chamber by means of silk threads. The thermometer will indicate a rapid rise of temperature at first, but a maximum (usually about 98°) will obtain only after $1\frac{1}{2}$ to 2 hours. The water equivalent of the calorimeter may be determined during this period. The heating should be continued for 30 minutes after the reading becomes constant to insure that all parts of the substance have come to a common temperature. Make a reading of the thermometer in the calorimeter; raise the slide; move the carriage containing the calorimeter under the heater, and lower the substance quickly but smoothly into the water; return the carriage to its former position; lower the slide, and

begin readings of the thermometer in the mixture immediately. Stir the water in the calorimeter gently, and continue the readings at intervals of 20 seconds until the final temperature is obtained. This temperature may be taken as that at which the mercury just begins to fall. When employing the compensation method of eliminating the radiation error, no further readings are necessary. It is best to follow the indications of the thermometer in the mixture by means of a telescope, and also to make arrangements for stirring the contents of the calorimeter from a short distance.

When it is desired to evaluate the radiation or absorption, the Regnault-Pfaundler method previously referred to may be used. It involves reading the thermometer at 20-second intervals, not only during the time elapsing between the immersion of the hot body and the attainment of the final temperature, but also during a certain time previous to, and a certain time after this period. Thus the whole operation covers three periods. The first period includes five or ten intervals previous to the immersion of the hot body; the second period begins with the immersion of the hot body and ends when the maximum temperature has been reached; the third period includes five or ten intervals further. During the first period we will assume that the calorimeter is slightly warmer than the air; it is slowly falling in temperature. Let T' be the mean temperature during the first period, and let v' equal the mean fall in temperature *per interval*. During the second period there is a rapid gain in heat from the immersed body, and also increased loss by radiation. Let θ_1, θ_2, θ_3, $\cdots$ θ_n be the temperatures at the end of the first, second, $\cdots$, and nth intervals of this period. During the final period heat is lost by radiation. Let T'' and v'' be respectively the mean temperature and the mean fall per interval for this period. For the second period write

$$t_1=\frac{\theta_0+\theta_1}{2},\ t_2=\frac{\theta_1+\theta_2}{2},\ \ldots,\ t_n=\frac{\theta_{n-1}+\theta_n}{2}$$

as mean temperatures for each interval.

Now, the final temperature observed θ_n, is too low because of the radiation which has taken place during the second period. The true maximum is $\theta_n + C$. This correction may be obtained by a graphic method. On the axis of abscissæ $\overline{OX}$, lay off $\overline{OA} = T'$, and erect the ordinate $\overline{AV} = v'$; similarly for the final period lay off $\overline{OA'} = T''$, and erect $\overline{A'V'} = v''$. Draw $\overline{VV'}$. Then

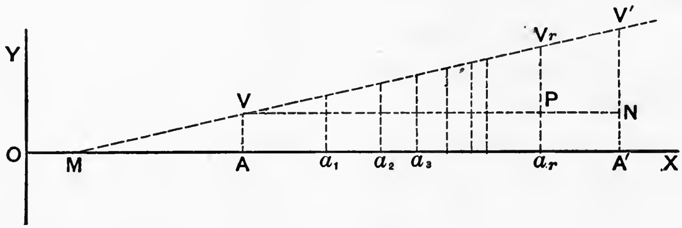


Fig. 131.

this method assumes that for any temperature t_r of the second period (represented by $\overline{Oa}$), the fall for the corresponding interval is the ordinate $a\,V$, and that consequently the total correction C is equal to the sum of all these ordinates.

The summation may also be made by a formula readily deduced from the figure. From similar triangles, we have any ordinate,

$$a_r V_r = v' + \frac{v'' - v'}{T'' - T'}(t_r - T'),$$

and the sum of all similar ordinates is

$$C = nv' + \frac{v'' - v'}{T'' - T'}[t_1 + t_2 + t_3 + \cdots + t_n - nT']$$

$$= nv' + \frac{v'' - v'}{T'' - T'}[\theta_1 + \theta_2 + \theta_3 + \cdots + \theta_{n-1} + \frac{\theta_0 + \theta}{2} - nT'].$$

The graphic method will in general be found more convenient of application. It is evident that in case the average temperature of the initial period is below that of the air, v' is negative, and it may happen that the correction C comes out as a subtractive term.

The foregoing method is not strictly rigorous, but it yields excellent results. The student may show that it assumes (1) Newton's law of cooling, the temperature of the air being given by $\overline{OM}$, (2) that the heat capacity of the calorimeter and contents remains unchanged during the three periods, and (3) that the temperature of the air is constant.

SPECIFIC HEATS (REGNAULT).

Aluminium	.20566	Bismuth	.03084
Cobalt	.10696	Antimony	.05077
Nickel	.11095	Pewter	.05623
Iron	.11379	Platinum	.03243
Zinc	.09555	Gold	.03244
Copper	.09515	Mercury	.03332
Silver	.05701	Brass	.09391
Lead	.03140	Glass	.19768

EXPERIMENT 6. **Specific heat of a liquid; method of mixtures.**

The heat capacities of different liquids may be obtained by sealing them in glass or metallic tubes and employing the method and apparatus of the preceding experiment.

In this way, Regnault obtained the values 1.00709 and 1.00890 for the mean specific heat of water between 15° and 100°, referred to the same liquid between 10° and 15°, thus showing the remarkable accuracy of which the method is capable.

When the liquid is a poor conductor, however, this method introduces errors of the same nature as those mentioned in the case of poorly conducting solids.

An apparatus designed especially for the examination of liquids is shown in Fig. 132. The liquid is heated or cooled in a brass cylinder E immersed in a bath, the nature of which depends upon the range of temperature through which the determination is to be made. The temperature of the liquid is given by a thermometer placed directly in it. Air pressure serves to quickly force so much of the liquid as is desired

into the calorimeter when the cock R' is turned. The calorimeter is of special design. The chamber Q contains the liquid, while any vapor which may rise from it is condensed in the upper chamber U. A wood screen protects the calorimeter from the heat of the apparatus.

For ordinary temperatures a bath of water may be used, while for temperatures below 0° it is necessary to surround the cylin-

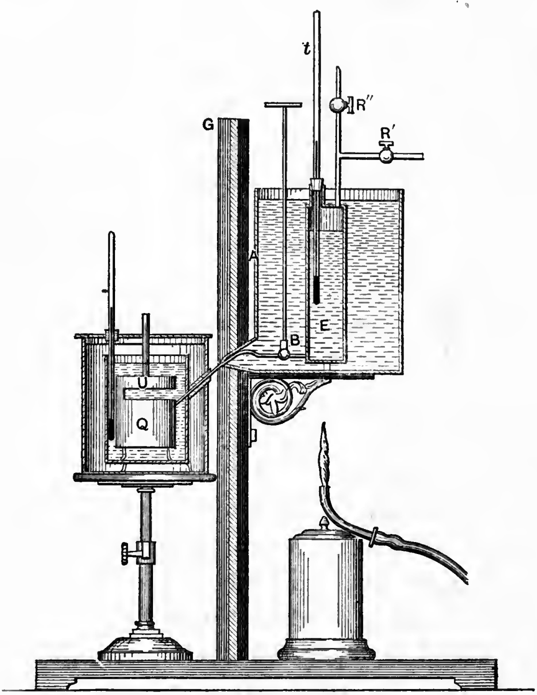


Fig. 132.—Apparatus for Specific Heat of Liquids.

der with a freezing mixture. The general directions for performing the experiment are similar to those for the preceding experiment. Make a careful determination of the water equivalent of the calorimeter as a whole. This may be done while the liquid is heating in the bath. The desired temperature is reached by filling the heater with water which is near this temperature, and then carrying the heating further by means of

a gas flame. The bath should be continually stirred. When the thermometer indicates a temperature which remains rigorously constant, the cock which communicates with the reservoir of compressed air may be turned, and at the same instant B may be opened, thus allowing a certain amount of the liquid to flow into the calorimeter. Readings of the thermometer in the calorimeter are then begun and continued at intervals of 20 seconds until the final temperature is attained. It only remains to weigh the calorimeter and contents.

The correction for radiation may in most cases be neglected without serious error. When it is necessary to evaluate the fall in temperature due to this cause, it is best to employ the method described at length in the last experiment.

The specific heats of most liquids vary to a marked extent with the temperature. Water is an exception. The variation may be represented by expressions of the form

$$Q = At + Bt^2 + Ct^3,$$

where Q is the quantity of heat which the substance gives out in falling from a temperature t to zero. Evidently the specific heat at any temperature t is

$$S = \frac{dQ}{dt} = A + 2\,Bt + 3\,Ct^2.$$

The third term in these expressions is for many substances unnecessary. The values of three constants are determined by making three successive determinations through widely different temperature ranges. From the results obtained three equations may be formed, from which, by elimination, A, B, and C may be evaluated.

The constancy of the specific heat of water renders it a valuable substance for calorimetric purposes. Determinations by the method above described show the specific heat of water at 230° to be 1.0568, water at 0° being taken as unity.

The following figures show the variation in the specific heats of alcohol and turpentine: —

	Turpentine.	Alcohol.
$-20°$	0.505315	0.38421
$0°$	0.547541	0.41058
20	0.595062	0.43376
40	0.647877	0.45376
60	0.705987	0.47056
80	0.769381	0.48419

Experiment 7. **Specific heat by the method of cooling.**

This method is based on the supposition that the rate of loss of heat of a body in a given inclosure depends only on the excess of the temperature of the body over that of the inclosure, and upon the extent and nature of its surface.* It is thus possible to determine the ratio of the specific heats of two substances, by successively inclosing them in a small vessel of constant emissive power, and noting their times of cooling through a fixed range of temperature under precisely identical conditions; that is, in the same inclosure maintained at a constant temperature. In this case we have

$$\frac{mc}{m'c'} = \frac{t}{t'},$$

where mc and $m'c'$ are the heat capacities of the substances, and t and t' their respective cooling periods. Since the containing dish and thermometer bulb cool through the same range of temperature as the substance, their thermal capacity must be determined. The modified equation is

$$\frac{mc+k}{m'c'+k} = \frac{t}{t'},$$

where k is the heat capacity of the accessories. To evaluate k,

* See Preston, Theory of Heat, 1894.

the experiment may be performed with two substances whose specific heats are known, giving

$$k = \frac{mct' - m'c't}{t - t'}.$$

The quantity k may also be computed by estimating the masses of the material of the retaining vessel, and of the glass and mercury of the thermometer bulb. The former method is preferable.

An apparatus used by Regnault* is shown in Fig. 133. The substance to be examined should be reduced to the form of a powder and then closely packed into the annular space in the small silver vessel shown at D. The inner cylinder is occupied by the thermometer bulb. When the vessel has been filled, the base should be carefully fitted in place, in such a way that the material lies smoothly in contact with it. The chamber in which the cooling takes place is coated with lampblack on the interior, while its walls are kept at zero temperature by the presence of melting ice in the space F. A tube is provided at B for producing a vacuum, it having been found by Regnault that the time of cooling was nearly twice as long in vacuum as in air. The surface of the silver vessel is very brilliantly polished to further prolong the time of cooling.

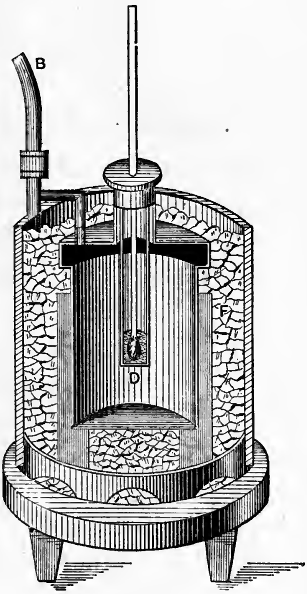


Fig. 133.

The exact determination of the time of cooling is a matter of some difficulty, unless special measures are taken. Witz †

* Annales de Chimie et de Physique, 3rd series, ix. 327 (1843).

† Annales de Chimie et de Physique, 5th series xviii. 208 (1879), and xxiii. 315 (1880).

recommends the use of a Morse telegraph receiver in connection with a clock, the latter being arranged to close the receiver circuit every second. The observer may follow the cooling of the thermometer by means of a telescope, and may record upon the receiver paper the instant the mercury reaches any particular graduation by pressing a conveniently placed key. If the distance between dots on the paper is 2 cm., the time may be estimated within $\frac{1}{20}$ second.

When liquids are to be examined, the silver vessel is replaced by one of glass (Fig. 134), in which equal volumes are placed, the densities being used in the formula. The glass stem is cemented into the metallic cork which closes the cooling chamber; the liquid should come to a level *m* at a temperature determined at the beginning of the experiment. With these modifications, the experiment is performed in the same way as in the case of solids. For a substance with which to make the comparison, the essence of turpentine has been largely used. Its specific heat at 15° is 0.414.

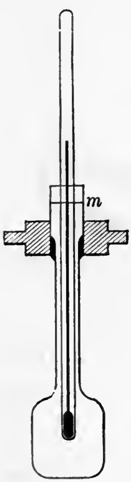


Fig. 134.

Following is the process of the experiment. Thoroughly clean and dry the silver vessel and the thermometer bulb, and determine the tare, preferably by a process of double weighing. Then fill the cylinder with the substance, place the base in position, and weigh again. Introduce the cylinder into the blackened chamber, place the whole apparatus in a bath of water at about 60,° and exhaust the air. Allow the chamber to fill again with air which has passed through tubes filled with pumice stone moistened with sulphuric acid. Repeat this operation several times. When the thermometer in the substance indicates a temperature of about 50°, remove the apparatus from the bath and surround the cooling chamber with melting ice. Begin observations of the thermometer, and at the instant the mercury reaches 35° indicate the beginning of the initial period. Record the times corre-

sponding to each fall of 5 degrees until the thermometer indicates 5°. Plot a curve of cooling. These results may be used to obtain the times corresponding to the fall from 35° to 20°, 30° to 15°, 25° to 10°, and 20° to 5°—four periods in all. It only remains to repeat the experiment with the substance used as a standard of comparison.

The method of cooling was first used Dulong and Petit,* and it was from results obtained by the application of this method to a series of simple substances that they were able to enunciate the law of equal atomic thermal capacities. The method was put to rigorous test by Regnault † at the beginning of his classic experiments to examine the range of application of Dulong and Petit's law. His results show that the method is well adapted to the examination of liquids, but that the results obtained with solids are not sufficiently concordant. The reader is referred to the original memoir for a critical analysis of the sources of error, chief among which are: (1) variation in the absorptive power of the lampblack coating due to the presence of layers of moisture; (2) variation in the contact of the powdered solid with the walls of the vessel, depending upon how tightly the substance is packed; (3) conduction of heat through the mercury of the thermometer stem. As a result of (2) the temperature is not uniform throughout the substance, and it cannot be assumed that all parts of the substance have fallen through the range of temperature indicated by the thermometer. As previously remarked, these errors have but slight importance in the case of liquids, and the method is accordingly particularly adapted to their examination.

EXPERIMENT 8. **Specific heat by the Bunsen ice calorimeter.**

In this method the substance under examination cools from a temperature θ to 0 in an enclosure surrounded by ice. The

* Annales de Chimie et de Physique, 2d series, x. 395 (1819).

† Annales de Chimie et de Physique, 3d series, iv. 327, and 2d series, lxxiii. 5.

heat surrendered by the body melts some of the ice, and the amount of water formed is measured, not by a direct process of weighing, as in the earlier forms of the ice calorimeter,* but by the diminution in the volume of the mass of ice and water.

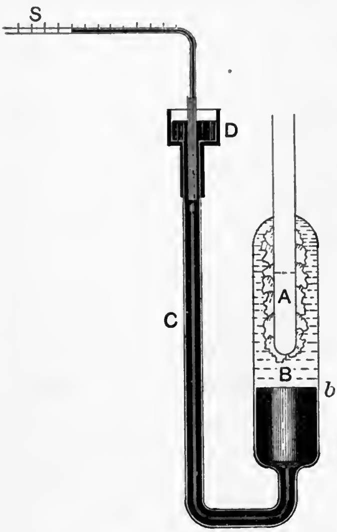


Fig. 135.—Bunsen Ice Calorimeter.

The apparatus designed and used by Bunsen† is shown in Fig. 135. With the exception of the iron collar *D*, it is made entirely of glass. It consists of the following essential parts: A test-tube *A* for holding the substance to be studied; a larger tube *B*, containing the water and ice, and into which *A* is fused; and a stem *C*, into which the graduated tube *S* is secured by means of a closely fitting cork. The stem *C* is filled with mercury, which also occupies the tube *B* to a level *b*. By inserting the cork the mercury may be made to flow out to any desired position in the graduated tube *S*.

The accuracy of the indications of this instrument depends largely upon the purity of the water used, and especially upon its freedom from air. The filling of *B* with air-freed water is accomplished as follows: Sufficient mercury to cover the joint between *A* and *B* when the apparatus is inverted is put in. The tube *B* is then half filled with boiled water and clamped with the mouth of *A* downwards over a gas-jet. A small piece of wire gauze should be wrapped around the joint between *A* and *B*. The stem *C* is allowed to dip into a beaker of water,

* Black's Ice Calorimeter, "Elements of Chemistry," Edinburgh (1803). Lavoisier and Laplace, Mem. de l'Académie des Sciences (1780), p. 355. Œuvres de Lavoisier, 2, 283.

† Pogg. Annalen, Vol. 141, p. 1 (1870), and Phil. Mag., Vol. 41, p. 161 (1871).

which is continually boiled over another gas flame. After the boiling has continued for some time, the flame is removed from beneath the bulb, and as the latter cools it fills with the boiled water from the beaker. The calorimeter is then set upright and allowed to cool. Freshly boiled mercury is then introduced until it occupies *B* to a level *b*, and stands at about the same height in *C*. The water remaining in *C* is removed by a siphon, the stem dried by a current of air, and the iron collar secured in place by sealing-wax. The final filling of *C* with mercury is done with a capillary glass tube to avoid the presence of air bubbles on the sides of the stem. The cork carrying the graduated tube must be very carefully fitted to the stem. The freezing of a certain amount of the water in *B* is accomplished by causing cold alcohol to circulate through the test-tube *A*. The process is essentially that used by Bunsen. The calorimeter is placed in a glass jar and cooled to 0° C. by surrounding it with melting ice. When its temperature has become stationary, the ice is removed, in order that the process of freezing may be watched, and the mouth of the jar stopped with wadding. *F* and *G* (Fig. 136) represent two metallic cylinders containing alcohol or methylated spirits. These chambers are surrounded by a well-made freezing mixture (salt and snow or ice). The tubes *f* and *g* are joined by rubber tubing to a four-way tap, which is in turn connected to an aspirating pump. By turning the tap at the proper moments the operator may cause the cold liquid to flow to and fro through *A*. Considerable time elapses before freezing begins, since the air-freed water must

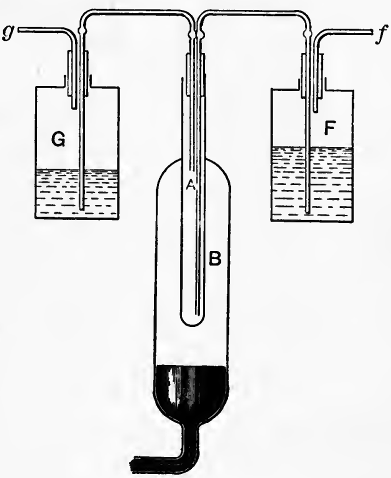


Fig. 136.

be reduced much below 0° C. Once begun, the freezing continues rapidly and in an interesting manner. The mercury which flows from the end of the tube S may be caught in a beaker. The operation should be continued until a cylinder of ice from 6 to 10 mm. thick has formed about the inner tube. The calorimeter, after the freezing process has terminated, should be kept constantly surrounded with snow or well-washed and pounded ice. If the packing be renewed twice daily, one freezing will serve for a large number of determinations.

It generally happens that a slow contracting movement of the mercury is observed for many hours after the freezing process has been discontinued. This is due to a gradual freezing produced by the presence of impurities in the surrounding snow which lower its melting-point. This creeping of the mercury was carefully studied by Bunsen. He recommends that only the purest snow be used for packing, that observations be delayed until the movement of the mercury does not exceed a few divisions per hour, and that above all a small layer of water should be visible between the sides of the test-tube and the adjacent cylinder of ice. It is also advantageous to work in a room whose temperature is not much above 0° C.

The instrument may be used in two ways.

(1) By obtaining the actual change in volume corresponding to a movement of one division of the mercury. Thus 1 gram of ice has the volume 1.09082 c.c., while 1 gram of water at 0° has the volume 1.00012 c.c. If the heat of liquefaction be taken at 79.4, the change in volume per unit of heat received is

$$\frac{0.0907}{79.4} = \frac{1}{875.4} \text{ c.c.}$$

The volume v of one division of the tube is needed. Suppose that a thread of mercury n divisions long is found to weigh m grams. If t is the temperature at the time this observation is made, then

$$v = \frac{m(1 + 0.00018\,t)}{13.596\,n} \text{ c.c.}$$

(2) The heat change corresponding to a movement of one division of the mercury may be obtained by using known quantities of water at known temperature in the test-tube A. Thus if q c.c. of water at temperature t be dropped in A, and the contraction produced be δ, then

$$\text{one division} = \frac{qt}{\delta} \text{ calories.}$$

Radiation is overcome by taking t at the temperature of the air.

The mass of the substance examined need not exceed 3 or 4 grams. A small piece of cotton-wool should be placed at the bottom of the tube A to prevent shock, and the tube should be corked as soon as the substance has been put in. In order to heat the substance, a small steam heater made of test-tubes, one within the other, may be used. The substance is placed in the inner tube, which is then corked. Steam is generated in a small boiler and passes through the annular space between the tubes. It escapes through a rubber tube attached to the bottom of the outer tube, an opening being made for this purpose. The heating apparatus is clamped in a holder, and when the steam has passed for about an hour through the tube, the latter is inverted over the calorimeter and the substance allowed to fall quickly into a little distilled water placed in the tube A. The temperature of the body is taken to be that of the steam.

The values of the specific heats of substances examined by Bunsen are slightly lower than those obtained by Regnault. This is undoubtedly due to the fact that they represent the mean specific heat between 0° and 100°, while Regnault's determinations were made between 16° and 100°.

Experiment 9. **Use of the Favre and Silbermann water calorimeter.**

In determinations of the *heat of combustion* of different substances Favre and Silbermann* used for much of their work

* Annales de Chimie et de Physique, 3d series, Vol. 34, p. 357.

a water calorimeter of excellent design, it being an improved form of an apparatus used at an earlier date by Rumford. The instrument is adapted to a wide range of live combustions, such, for example, as the burning of the different forms of carbon in an atmosphere of oxygen. The extent to which it can be used in laboratories for instruction is dependent upon many things, notably the equipment of the laboratory, the time allotted, and the general nature of the course. It is the purpose of this writing to give a brief description of the apparatus, with outlined suggestions for its use. Specific directions for the performance of some one experiment must be given by the instructor.

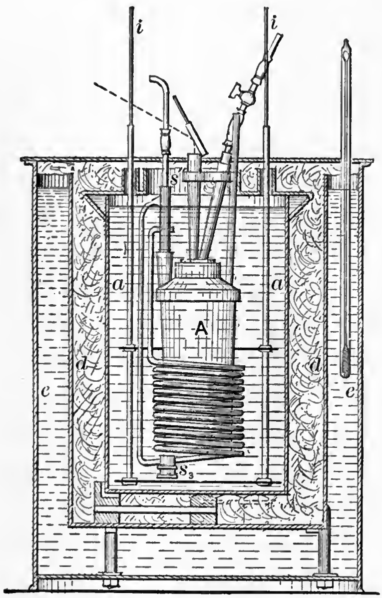


Fig. 137.—Favre and Silbermann Water Calorimeter.

The principal features of the apparatus are (1) a combustion chamber *A*, (2) a calorimeter *aa*, and (3) two concentric chambers *dd* and *ee*, designed to prevent loss or gain of heat from external causes. The calorimeter, of about two liters capacity, is made of thin copper, and is silver-plated externally in order to reduce its emissive power. The rods *ii* control two thin metal rings, constituting the stirrer. A metallic trough is provided at the top to catch any particles of water which might be thrown outward by the agitator. The cover is also nickel-plated, and is pierced by tubulures for the stirrer rods and the thermometer. There is also a large central opening for the manipulation of the combustion chamber. The vessels *dd* and *ee* extend 2.5 cm. higher than the calorimeter. The inner chamber is lined

with a swan's skin, the down being turned inward. This is intended to prevent the circulation of air. The outer chamber is filled with water at the temperature of the atmosphere. The three chambers touch only at such points as are necessary for their proper support. The whole is covered by a plate dividing on a diameter, and lined with down. The apparatus is accompanied by a special base, provided with leveling screws. From

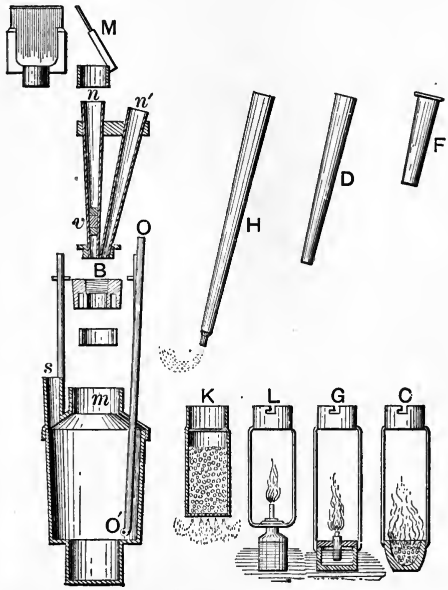


Fig. 138.

this base rise three standards supporting a top plate, from which may be suspended thermometers, tubing, and other accessories. The combustion chamber (Fig. 138) is made of thin copper, and is furnished with a screw top, in which is a large opening, *m*, which may be closed by the stopper *B*. In the bottom of the stopper is cut an annular groove, and into this small brass collars are secured by a pin and groove arrangement. From these collars are suspended various burners and receptacles, *K*, *L*, *G*, *C*, etc., adapted to the different liquids and solids which

are to be burned. It will be noted that the substance to be examined is suspended in the lower and smaller portion of the combustion chamber. In the case of volatile liquids it has been found best to surround the receptacle with a little water. It requires only 2 to 3 grams of water to fill the lower chamber when the receptacle is in place. This water is borrowed from that in the calorimeter. Oxygen is supplied, when needed, through the tube OO', while the products of combustion escape through the tube S into the worm, where they are cooled to the temperature of the bath. The worm is 2 m. in length, and is furnished at its lower end with a small chamber, s_3, provided with a screw cap. The object of this chamber is to collect any liquid formed in the worm by condensation. Two additional tubes lead into the combustion chamber through the stopper B. One of these, n, is an observation tube, and is closed at v by an athermanous window, composed of alum, quartz, and glass. A small mirror, M, enables the observer to note the extent to which the combustion has proceeded. The other tube is for the introduction of gases when it is desired to examine them with reference to heat of combustion.

Some of the modifications which it is necessary to make in the apparatus in order to examine different substances are as follows:

Gases. — The apparatus is perhaps in its simplest form when arranged for the combustion of hydrogen. Since in this case the product is water, the worm is unnecessary, and may be corked at S''. The oxygen enters through OO'. The flow of gas should be steady, and means must be provided for regulating it. A large glass vessel may be filled with the gas, which may then be driven into the combustion chamber at any desired rate by means of water displacement, the water coming from an elevated reservoir. It is well to allow the gas to pass through a flask containing sulphuric acid; the bubbles then indicate the rapidity of flow. For most experiments the gas must be devoid of moisture; this is insured by using drying tubes of the ordi-

nary form. The hydrogen must also be furnished at a steady rate, and must be thoroughly dried. The tube H is used for introducing the hydrogen; it fits closely into n'.

Liquids. — The liquid is burned in a small lamp, L, which is suspended in a manner already described. A wick of earth flax is used. The chamber is submerged to the level of the water and filled with oxygen; the lamp is then lighted and introduced, after which the chamber is completely submerged. In this case the tube n' is closed by F.

Coals. — In determining the heat of combustion of coals of different kinds, the apparatus must be varied somewhat, according to the refractoriness of the sample. The small cylinder, K, of platinum is generally used, its bottom being pierced with holes. The oxygen arrives through the nozzle D, which is inserted in the tube n'. The products of combustion traverse the worm. The chamber having been filled with oxygen, the coal is lighted by removing the nozzle D for an instant and allowing a small particle of lighted coal (4 to 5 mg.) to slide down the tube n'; the nozzle is immediately replaced and the flow of gas continued. It is not necessary to lift the combustion chamber from the water to do this.

The very extended researches of Favre and Silbermann required the use of a number of additional attachments which it is not possible to describe here. Their thermometers were held in a fixed position by the frame which accompanies the apparatus, and readings were made by means of a cathetometer — the whole being mounted on a stone pier. Where the principal object to be gained is a knowledge of the methods involved, these refinements are unnecessary.

To eliminate the radiation correction, the method of compensation may be employed, but this is quite inconvenient. It has been found that the law of Newton holds for the excess of temperature occurring in these experiments, which justifies the use of a radiation constant. Favre and Silbermann found that their calorimeter cooled only $0°.002$ C. per minute and per de-

gree difference of temperature between the water in the calorimeter and that in the jacket. The weights should be made in nearly all cases of the products of combustion rather than of the material to be burned, and accurate determinations require that the loss of weight due to weighing in air should be taken into account.

The quantities that must be determined are:

1. The weight of the body burned, either directly or from the weight of a definite product of the combustion.

2. The weight of the water in the calorimeter.

3. The water equivalent of the combustion chamber, worm, and accessories.

4. The rise of temperature produced by the combustion.

5. The time of each event in the experiment.

Illustrative Data.

EVALUATION OF HEAT PRODUCED.

2012.000	grams	distilled water	. . . .	2012.000
684.603	"	copper × 0.09515	. . . .	65.140
1.961	"	platinum × 0.03243	. . . .	0.064
7.750	"	glass × 0.1980	thermometer . . .	1.540
20.033	"	mercury × .0333	thermometer . . .	0.666
				2079.410

Rise of temperature corrected for radiation and absorption, 8°.7404.
Total heat produced = 2079.41 × 8.7404 = 18174.9.

This number divided by the mass of the material burned gives the heat of combustion.

The correction for the heat gained or lost from the air is made in the usual way. The observer should plot a curve between temperatures as ordinates, and times as abscissas. The total time may then be divided into several periods, for each of which the average temperature difference may be obtained. These values, multiplied into the time and the radiation constant (determined once for all), give the correction for each period. The experiment should be begun with the water in the calorimeter 2° to 3° below the temperature of the water in the

jacket. The final temperature will usually be about the same amount above. This, however, is a matter determined by the nature of the experiment. Some of the corrections will be positive, others negative, and the final correction will be quite small. The maximum temperature does not occur at the instant of the extinction of the combustion. The mercury continues to rise for a short time after this act.

HEAT OF COMBUSTION OF HYDROGEN.

Exp.	Calories per Gram of H.	Water Formed. Grams.	Duration of Experiment. Min.	Rise of Temperature in Calorimeter.
1	34540	2.903	8	5.356°
2	34413	3.266	12	6.003
3	34461	2.539	15	6.514
4	34576	2.987	13	5.517
5	34340	3.013	14	5.527
6	34442	3.761	16	6.917

Mean heat of combustion = 34462.

EXPERIMENT 10. **Use of the Favre and Silbermann mercury calorimeter.**

This instrument is little else than a thermometer provided with an enormous bulb. The position of the mercury thread in the capillary stem is taken to be a measure, not of the temperature, but of the amount of heat which the substance under examination has communicated to the mercury. Its indications, therefore, are not unlike those of the Bunsen ice calorimeter.

Figure 139 shows the apparatus in its original form as used by Favre and Silbermann.* Modern forms for laboratory use are modified but slightly in detail, the essential features remaining unaltered. The large iron globe *m* is completely filled with mercury. The amount of mercury in the usual type is 25 to 30 kilos, although they have been constructed with a capacity of

* Annales de Chimie et de Physique, 3d series, Vol. 36, pp. 5 and 33; Vol. 37, p. 406.

250 kilos. The chamber for receiving the hot body consists of a very thin tube, n, of iron or platinum, extending past the center of the globe. This is called the muffle. The neck o' is stopped by a cork t', through which passes the glass tube t. A piston, o'', operated by a screw, serves to bring the mercury to any desired graduation in the capillary tube. The globe rests on a base of cork, and is surrounded by a wooden box packed with non-conducting material. For many experiments, it is desirable to have more than one muffle. In modern forms the iron globe is mounted on bearings which admit of its being inverted for the purpose of filling with mercury. Mr. Favre

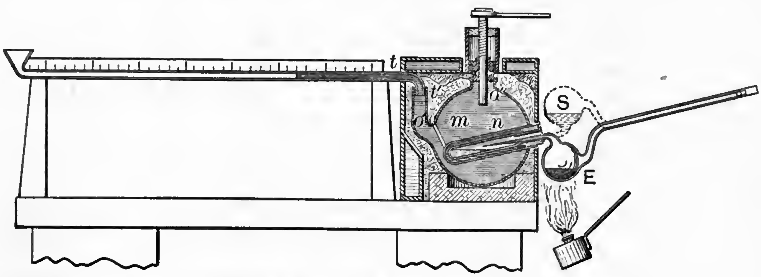


Fig. 139.— Favre and Silbermann Mercury Calorimeter.

recommends that the air be completely expelled from the globe during the process of filling, and in order to do this two cocks are provided at the bottom of the globe; one is connected to an air-pump while the mercury is allowed to enter through the other.

In some cases the hot body is dropped directly into the iron muffle, a quantity of mercury being placed therein to facilitate the communication of heat to the mercury in the globe. This will answer for substances which do not act on mercury or iron. It is customary, however, to place in the muffle a glass tube with a rather narrow neck surrounded by cork, which, fitting into the mouth of the muffle, serves to hold the tube in place against the buoyancy of the mercury into which it dips.

The readings of the instrument should be made by means of

a reading glass, or, preferably, with a telescope and micrometer. The tube must be of uniform bore (or perfectly calibrated), and the movement corresponding to one calorie received must be found. This is accomplished by introducing into the muffle, containing its glass tube, known weights of boiling water, and noting the movement produced as the water cools to the temperature of the calorimeter. For this purpose a special form of pipette, *E*, is provided. The process is as follows: The glass muffle tube is weighed and put in position. The mercury thread is brought to the zero of the scale by manipulating the piston. Into the bulb of the pipette are placed some small pieces of platinum wire, after which 4 or 5 grams of distilled water are drawn into the bulb. The large tube of the pipette is then closed by a cork. The water is heated to the boiling-point by means of an alcohol lamp, care being taken to heat the neck of the pipette at the same time. When boiling begins, the flame should be removed for an instant, the pipette inverted to the position *S*, and the point inserted in the muffle. At the same instant the cork must be removed from the outer end of the pipette. The boiling water flows into the muffle tube, and an immediate advance of the mercury is observed. When the mercury has become stationary, the temperature of the water in the muffle is taken by means of a thermometer, which is plunged into the mercury in the muffle immediately after removing the glass tube. A second weighing gives the amount of water introduced. The temperature of the boiling water may be found either from a preliminary measurement, or from the barometer reading. From these data the movement corresponding to one calorie is readily deduced. This process has been described in detail because it is really that which is employed in the actual use of the calorimeter. A certain amount of practice in manipulating the pipette is necessary in order not to introduce steam into the muffle, which in condensing would cause serious error in the result. The following data on a single trial will illustrate the method:

Time. min.	sec.		Position of Mercury. mm.
0	00	Liquid introduced	0.00
0	30		100.00
1	00		126.00
1	30		134.00
2	00		136.50
2	30	Stationary	137.30
3	00	Stationary	137.30
3	30	Stationary	137.30
4	00	Stationary	137.30
4	30	Receding	137.00
5	00		136.00
5	30		135.80
6	00		135.30

Weight of water introduced,	6.23 grams
Boiling-point,	99.82°
Final temperature,	28.00

$$\therefore \text{ one calorie} = \frac{137.30}{6.23 \times 71.82} = 0.30685 \text{ mm.}$$

From 0.1 to 0.3 mm. is the usual elongation per calorie; hence the necessity of accurate readings of the position of the mercury.

In the actual use of the instrument a correction is necessary for the slow movement of the mercury thread due to outside influences. This may be taken as proportional to the time, and the correction evaluated by following the movement of the mercury during periods previous to, and after the experiment proper. This will be illustrated in data to follow.

Specific Heats. — This instrument is well adapted to the determination of the specific heats of substances obtainable only in small quantities, and is especially valuable in the examination of liquids, in which case the value obtained is the mean specific heat between the temperature of ebullition of the liquid and the temperature of the calorimeter, which is usually not far from that of the atmosphere.

To obtain the specific heat of a liquid, determine the boiling-point of the liquid by means of a thermometer placed in the

vapor of the boiling liquid. The liquid may be boiled in a long test-tube. Then introduce from 2 to 5 grams of the liquid into the pipette, and proceed as in process of calibration. The readings of the mercury thread should be begun at least 15 minutes before the introduction of the hot liquid, and should be continued for an equal time after equilibrium has been established. From these data the law of variation due to external causes may be obtained and the proper correction made. Let

n = total corrected movement of mercury.

λ = length corresponding to one calorie.

t, t' = the initial and final temperatures.

W = the weight of the liquid.

Then the specific heat is

$$C = \frac{n}{\lambda W(t-t')}.$$

The following values were found by Favre and Silbermann for a sample of wood alcohol :

Specific Heat.	Liquid Used.
	grams.
0.6765	3.081
0.6728	3.498
0.6648	3.236
Mean, 0.6713	

Heat of Vaporization.—This is determined in much the same way. The liquid in the pipette is boiled freely, and the neck is inserted in the muffle tube, without, however, allowing the glass of the pipette to come in contact with that of the tube. The amount of liquid vaporized need only be from 1 to 2 grams. If L be the heat of vaporization, and C' the mean specific heat of the liquid for the range of temperature involved, we have

$$L = \frac{n}{\lambda W} - C'(t-t').$$

The following figures are for the heat of vaporization of water:

$$W = 0.753. \qquad t = 99^{\circ}.81.$$
$$n = 136.8 \text{ mm.} \qquad t' = 30^{\circ}.$$
$$\lambda = 0.3 \text{ mm.}$$

From these data, $L = 535.77$.

Measurement of High Temperatures. — By using metals of known specific heats, this method is evidently applicable to the measurement of temperatures. The following method is given by G. Brown.* The attempt was made to estimate the temperature of platinum heated in the Bunsen flame. A piece of platinum wire was coiled and placed on a support about 30 cm. above the mouth of the receiving tube. Through this coil a piece of fine platinum wire was passed vertically down into the muffle (the glass tube being removed), in which its lower end was kept by means of a platinum weight. A screen was placed below the coil, which was then heated to a steady white heat in a Bunsen flame. The screen was then removed, the cork withdrawn and the coil pushed off its support; and, being guided by the fine wire, it fell directly in the muffle. The cork was immediately replaced. Data: $t' = 15^{\circ}.2$; 1.345 scale divisions correspond to 1 calorie; specific heat of platinum, 0.0373 (0° to 1000°), mass of platinum, 4.2 grams.

OBSERVATIONS.

Time.	Reading.	
0	12	Gain per min. = 2.6.
5	25	
10	166	
12	178	
16	193	
20	203.7	Gain per min. = 2.26.
21	207.0	
25	215.0	

* Practical Work at the Cavendish Laboratory, edited by W. N. Shaw, M.A., London, 1886.

Mean gain per min. = 2.43.

Reading at 20	203.7
" " 0	12.0
	191.7
Due to external causes . . .	48.6
" " platinum	143.1

Calories given out, $\frac{143.1}{1.345}$.

$t - 15^\circ.2 = 814^\circ.5$.

t = temperature of platinum = $829^\circ.7$.

When extremely hot bodies are introduced into the muffle tube, it sometimes happens that the mercury rushes suddenly forward and then almost immediately recedes. This is due to the sudden heating and expansion of the muffle, which speedily parts with its heat to the surrounding mercury, causing the subsequent recession.

Heat of Chemical Combination. — The apparatus was largely used by its designers for the determination of the heat of combination of various substances. For these determinations, two or more muffles are desirable. The two substances, for example, which are to be united are kept at first in separate muffles, and are thus at the same temperature; namely, that of the calorimeter. The general method is the same as that already described.

The results obtained with the mercury calorimeter are for the most part remarkably exact. There is, however, one source of error which needs to be considered. The experimenter cannot be sure that the heat communicated to the center of the sphere of mercury may not, during the time of an experiment, reach the walls of the globe and cause expansion, which would, of course, give rise to an effect opposite to that resulting from the expansion of the mercury.

Experiment 11. **Heat of combustion of metals.**

The following method is adapted to the determination of the heat of combustion of such metals as can be obtained in

the form of wire or ribbon. It can be used only in the case of metals which will burn freely in an atmosphere of pure oxygen. The apparatus was first described by F. J. Rogers* (1892), who made use of it in the determination of the heat of combustion of magnesium. It consists of a cylindrical calorimeter (Fig. 140) containing an inner vessel A, which holds about 500 c.c. Through the air-tight cover of the latter, two electrodes, E and E', are inserted, one of which passes through an insulating collar. The outer vessel is large enough to admit a stirrer, S, and a thermometer between its walls and those of the closed vessel.

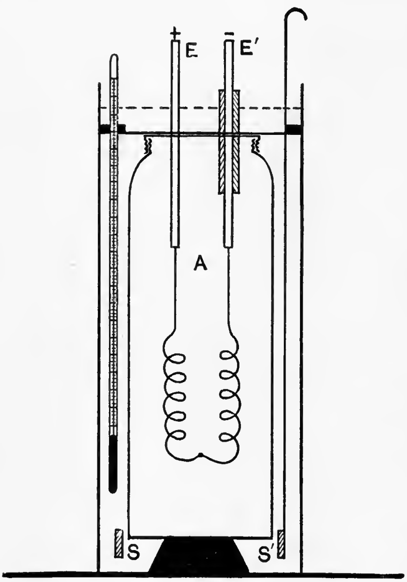


Fig. 140.

In performing the experiment, the piece of wire, the heat of combustion of which is to be determined, is coiled into a spiral, and a small quantity of phosphorus is fastened to it near the middle. The terminals of the spiral are connected to the electrodes. The mass of this coil, which should be carefully weighed, should be such that the metal will be completely consumed without entirely exhausting the oxygen with which it is surrounded. Before placing the wire in the vessel A, the latter is filled with oxygen gas. The cover carrying the wire spiral is then put in place, and the vessel is thus closed. For this purpose the cover is threaded with a screw which fits the metal neck of the vessel A. The chamber having been thus filled and placed within the calorimeter, water is poured into the

* American Journal of Science, Vol. 43, p. 301.

space between the two vessels until it is a centimeter deep over the top of *A*. The ribbon is then ignited by the momentary application of a current of electricity. The water value of the apparatus having been previously determined, the heat of combustion is measured by the rise of temperature, as indicated by reading the thermometer. After the experiment, the vessel *A* is opened, and such portions of the metal as remain unconsumed are collected. Their weight is to be subtracted from the original weight of the wire. Corrections may be applied for the amount of heat developed by the igniting current, and for that liberated in the combustion in the bit of phosphorus. These corrections, however, will be found very small. The precautions to secure uniform results with this apparatus are those common to calorimetric work. They have been described in some detail in previous experiments, and need not be repeated here. As regards the accuracy of the method, it may be stated that Rogers found as the average of eight determinations of the heat of combustion of magnesium 6010 lesser calories per gram of magnesium. These are the only determinations by direct process which have been made in the case of this metal. The value computed indirectly from the heat developed in the formation of the hydrate by Julius Thomsen is 6077. Nearly all determinations of the heat of combustion of the heavier metals thus far recorded have been made by the indirect processes of solution or decomposition. This apparatus affords a means of verifying such results by the direct method of burning in oxygen.

EXPERIMENT 12. **Pressure of saturated vapors at low temperatures.**

The pressure exerted by a saturated vapor in a closed space is a function of the temperature and of the nature of the substance. The variation in the pressure, or, as it is often called, maximum tension, of a saturated vapor has been studied by a

number of experimentalists, notably Dalton, Gay Lussac, and Regnault.*

The method consists in maintaining the vapor at a known temperature, and in measuring the corresponding pressure.

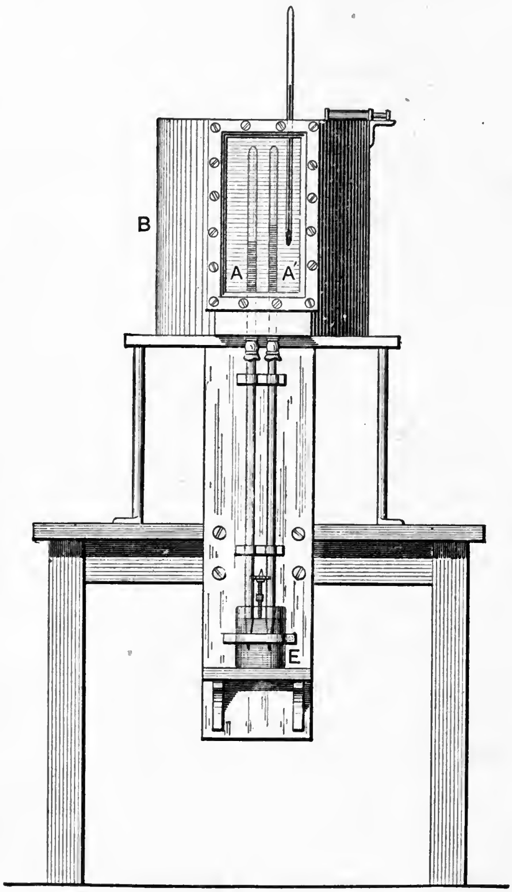


Fig. 141.

Between 0° and 50° C., this has, in nearly all cases, been done by introducing the liquid to be evaporated into the vacuum of

* For a complete set of references to work previous to that of Regnault, see Relation des Expériences, Vol. 1, p. 467. See, also, Preston's Theory of Heat, p. 322.

a barometer tube, and noting the depression produced at the observed temperature. Enough liquid must be put in to insure the presence of a thin layer upon the top of the mercury column.

The best form of apparatus for this purpose is that of Regnault (Fig. 141). Two barometers, A and A', provided with a common cistern E are mounted in an accurately vertical position. Their upper ends are surrounded by a water bath contained in a metallic case B of about 45 liters capacity. The case is provided with a window of plate glass through which the barometer heights may be read by means of a cathetometer. A gas flame or alcohol lamp is used in warming the bath, and a stirrer is provided for maintaining a uniform temperature throughout the water. The temperature is observed from a thermometer suspended near the barometer tubes.

In this form of apparatus the difference in the level of the two columns is at the temperature of the bath, and it is this difference which must be corrected to 0° C. The columns are not, it is true, warmed throughout their length, but the assumption is made that they are under precisely similar temperature conditions. In early types of this apparatus the attempt was made to warm the entire length of the mercury columns. This, however, lead to discordant results because of the tendency of the water in so tall a bath to settle in layers at different temperatures.

Regnault made a careful determination of the error in the readings due to refraction because of the presence of the glass and water. He found an absolute deviation amounting in some cases to 0.5 mm., but the relative deviation, upon which the accuracy of the readings depends, was scarcely appreciable. An error also arises from capillarity. The surface tension in the barometer tube differs from that in the vapor tube in which the mercury is in contact with a liquid. This error was found by Regnault to be 0.12 mm. in the case of water. It is not in general necessary to take this into account.

In performing the experiment, the liquid whose vapor is to be examined should be introduced into one of the tubes in such quantity that the mercury is surmounted by a layer of liquid 3 or 4 mm. in height. The value of this added layer must be reduced to its equivalent in millimeters of mercury, and added to the observed difference of level. This may be done by measuring the distance between the summit of the mercury meniscus and the lowest point of the concave meniscus of the liquid. This height divided by 13.59 gives the needed correction. The water in the bath should be continually stirred, except when a reading is being made. By adjusting the position of the alcohol lamp, and by keeping up a lively stirring, it is possible to keep the temperature of the bath rigorously constant for a considerable time. This admits of a number of cathetometer readings being made at the same temperature. To obtain the next higher temperature it is best to siphon out a portion of the water and replace it by warmer water. Then by further adjustment of the lamp another set of readings may be made. The student may plot a curve, using temperatures as abscissas, and pressures as ordinates.

Regnault found that the results of his extended experiments on the vapor of water were represented very closely by an empirical formula suggested by Biot; viz.

$$\log p = a + b\alpha^{\theta} + c\beta^{\theta},$$

in which the five constants a, b, c, α, and β are found from five equations formed by taking the pressures from the curve which correspond to five equidistant temperatures. Thus, if the chosen temperatures be 0°, 12°, 24°, 36°, and 48°, by putting $\alpha^{12} = \alpha'$ and $\beta^{12} = \beta'$, the equations become

$$\begin{aligned}
\log p_0 &= a + b + c,\\
\log p_1 &= a + b\alpha' + c\beta',\\
\log p_2 &= a + b\alpha'^2 + c\beta'^2,\\
\log p_3 &= a + b\alpha'^3 + c\beta'^3,\\
\log p_4 &= a + b\alpha'^4 + c\beta'^4.
\end{aligned}$$

For the detailed process of elimination, the student is referred to the original memoir.*

EXPERIMENT 13. **Pressure of saturated vapors at high temperatures.**

When the temperature of a saturated vapor exceeds 60, the apparatus described in the preceding experiment is not adapted

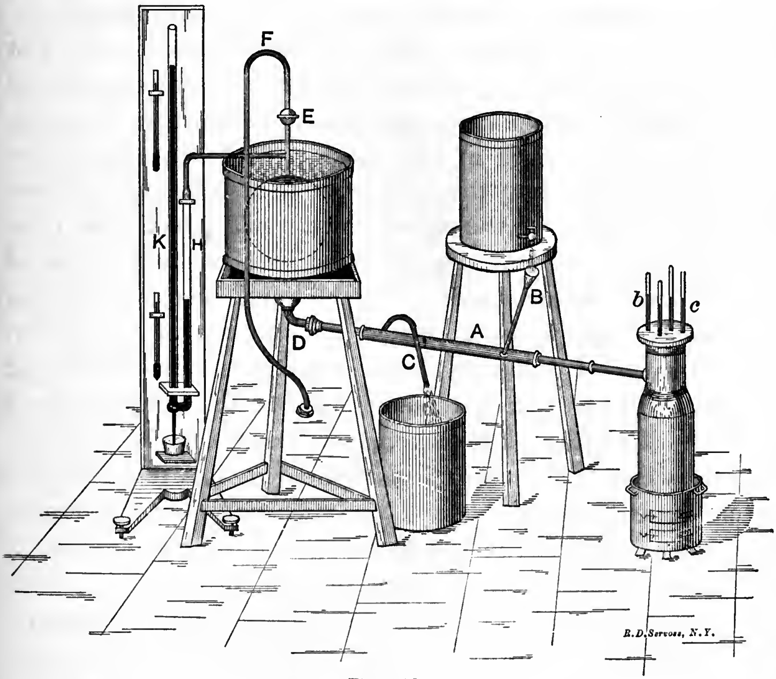


Fig. 142.

to the measurement of its pressure. For temperatures higher than this value the length of the water bath must be increased, which introduces difficulties in the way of maintaining a uniform temperature. The method becomes absolutely impracticable for temperatures above 100°. Fortunately a method of extreme precision is available for high temperatures and pressures. It

* Relation des Expériences, Vol. 1, p. 594.

is based on the law of ebullition that the pressure of the vapor of a boiling liquid is equal to that under which the boiling takes place, and consists in determining the temperature of a liquid boiling under an artificial atmosphere of measured pressure. This was very successfully accomplished by Regnault in his researches upon the elastic force of water vapor. The results of previous experiments by Dulong and Arago, and by a Franklin Institute Committee, were not at all concordant.

The form of apparatus which Regnault used in his first experiments, and which involved the same principles as one of more elaborate design constructed later, is shown in Fig. 142. The liquid is vaporized in a small air-tight boiler. Four thermometers project into the boiler, but their bulbs are relieved from the pressure of the vapor, which would affect their readings, by iron tubulures partially filled with mercury. Two of the thermometer tubes reach into the liquid, and two into the vapor only. The boiler is connected by a tube, *A*, to a spherical chamber of about 24 liters capacity. This chamber is surrounded by a water bath maintained at the temperature of the air. The tube *A* is jacketed, and kept cool by a stream of cold water. The vapor is condensed in this tube, and the liquid returns to the boiler. An atmosphere of the desired pressure is produced in the globe by connecting the tube *F* with an air pump or with a compression pump, according as this pressure is less or greater than that of the external atmosphere. It only remains to describe the means of measuring this pressure. For pressures less than atmospheric, Regnault used two barometers dipping in the same cistern, as in Fig. 141. One of these gave the pressure of the atmosphere, while the other, which was connected at its top with the vapor chamber, gave the depression due to the vapor pressure. The temperature of the mercury columns was obtained by thermometers placed near by, and the difference in level corrected to zero. It is customary to use instead of this arrangement a mercury manometer for pressures both above and below that of the atmosphere. The

pressure is obtained by adding to, or subtracting from, the barometer reading the observed difference of level. The readings should be made with the aid of a cathetometer. When experimenting with pressures producing a difference of level of more than 1 m., Regnault used two cathetometers, one for each meniscus.

The experiment is performed by first starting the boiler, and exhausting the air from the chamber until the pressure at which it is desired to begin is obtained. When ebullition has begun, which will be indicated by the thermometers showing a constant temperature, readings of the manometer and barometer may be made. After several sets of readings have been made, the next pressure may be obtained by cautiously turning the stop-cock, and admitting more air into the chamber.

At low pressures the thermometers in the vapor will indicate a slightly lower temperature than that indicated by those in the liquid. This difference reduces to zero when the boiling takes place under atmospheric pressure. The student may take the mean reading of the thermometers. There is always a slight oscillation of the mercury in the manometer, but the observer has little difficulty in making an accurate reading.

READINGS.

Thermometers in the liquid.			Thermometers in the vapor.			Pressure.		
A	*B*	Mean.	*A'*	*B'*	Mean.	Observed.	Calculated.	Diff.
						mm.	mm.	mm.
47.84	47.56	47.70	47.15	47.17	47.16	80.19	79.74	+0.55
62.40	62.40	62.40	62.06	62.01	62.04	163.44	163.46	−0.02
71.77	71.77	71.77	71.44	71.46	71.45	248.17	248.04	+0.13
79.20	79.21	79.20	78.94	78.95	78.95	340.27	339.79	+0.46
84.34	84.35	84.35	84.15	84.15	84.15	419.64	418.76	+0.88
87.62	87.60	87.61	87.52	87.47	87.49	476.50	477.01	−0.51
91.48	91.34	91.41	91.30	91.32	91.31	550.36	551.79	−1.43
93.74	93.67	93.70	93.62	93.59	93.60	601.98	601.47	+0.51
95.01	94.85	94.93	94.83	94.87	94.85	628.61	930.28	−1.67
96.91	96.76	96.83	96.76	96.77	96.76	676.18	676.08	+0.10

EXPERIMENT 14. **Vapor density (Dumas).**

The density of a vapor is usually defined as the ratio of the weight of a given volume to the weight of an equal volume of dry air at the same pressure and temperature.* This is more correctly the specific gravity of the vapor referred to air as a standard.

The weight of a volume v of air at pressure p and absolute temperature θ is

$$w_a = vw_0 \frac{p}{760} \cdot \frac{273}{\theta}, \tag{1}$$

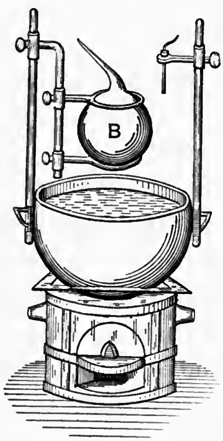


Fig. 143.—Apparatus for Dumas' Method.

where w_0 is the weight of unit volume of air at 0° C., and a pressure of 760 mm. If w is the weight of a volume v of vapor under the same conditions of pressure and temperature, the vapor density is

$$\rho = \frac{w}{w_a} = \frac{w}{vw_0} \cdot \frac{\theta}{273} \cdot \frac{760}{p}; \tag{2}$$

and if the vapor obeys the laws of Boyle and Charles to the same extent that air does, we have

$$\rho = \frac{w}{w_a} = \frac{760}{273\,R} \cdot \frac{w}{w_0}, \tag{3}$$

which is independent of the pressure and temperature. Vapors do not, however, fulfill this condition.

From equation (2) it is evident that to determine a value of ρ, it is necessary to measure the volume, temperature, and pressure of a known weight of vapor. The following method is the classic one used by Dumas: †

The needed apparatus is very simple (Fig. 143). A small quantity of the liquid to be vaporized is placed in a glass globe,

---

* See Preston's Theory of Heat, p. 333; Jamin's Cours de Physique, Vol. 2, p. 200.

† Annales de Chimie et de Physique, 2d series, Vol. 33, p. 337 (1826).

provided with a long neck, of which the opening is about 1 mm. in diameter. The globe is heated in a bath, the nature of which depends upon the temperature that must be attained to completely vaporize the liquid. When this condition is reached, the end of the tube is sealed. A certain quantity of vapor is thus shut in at the temperature of the bath and pressure of the atmosphere. It remains to determine its volume and weight. The volume is found by breaking the point of the neck under water, and then weighing the water that rushes in to fill the globe. A number of corrections are necessary. These and the experimental details may be better taken up after a consideration of the equations. Three weighings are made: (1) globe empty (*the air weighing*), (2) globe filled with vapor (*the vapor weighing*), (3) globe filled with water (*the water weighing*). Let

w = weight of globe filled with air;
W = weight of globe filled with vapor;
w' = weight of globe filled with water;
H = pressure of atmosphere at time of sealing;
T = temperature of atmosphere at time of sealing;

$$\left.\begin{array}{l} h = \text{pressure} \\ t = \text{temperature} \\ f = \text{tension of aqueous vapor} \end{array}\right\} \text{at time of } \textit{vapor weighing};$$

$$\left.\begin{array}{l} h' = \text{pressure} \\ \theta = \text{temperature} \\ f' = \text{tension of aqueous vapor} \end{array}\right\} \text{at time of } \textit{water weighing};$$

w_0 = weight of 1 cc. of air 0° C. and 760 mm.;
β = coefficient of expansion of glass;
α = coefficient of expansion of air;
δ_θ = weight of 1 cc. of water at temperature θ.

Then ρ is obtained from the equation

$$W - w = \rho V_0(1 + \beta T) w_0 \frac{H}{760} \cdot \frac{1}{1 + a T} - V_0(1 + \beta t) w_0 \frac{h - \frac{3}{8} f}{760} \cdot \frac{1}{1 + a t},$$

where V_0 is the volume of the globe at 0° C., determined from the results of the *water weighing* by the equation

$$w' - w = V_0(1 + \beta\theta)\left[\delta^\theta - w_0 \frac{h' - \frac{3}{8}f'}{760} \cdot \frac{1}{1 + a\theta}\right].$$

The student may take $\beta = 0.0000235$, $w_0 = 0.001293$. Values of δ_θ and the expression $1 + 0.03665\,t$ are found from laboratory tables. These formulæ contain all the necessary corrections. They correct for the expansion of the glass, the pressure of aqueous vapor in the atmosphere, the expansion of the water with rise of temperature, and for the losses of weight due to weighings in air. The change of loss in weight with change of temperature and pressure is neglected, as is also the fact that the drop of liquid resulting from the condensation of the vapor has a density differing from that of water. It sometimes happens that the air has not been entirely expelled from the globe, and consequently when the point of the neck is broken under water, the globe does not completely fill. When the amount of air imprisoned is very small, its effect may be neglected; but if this is not the case, it indicates that the vaporization and sealing were not conducted with sufficient care.

The heating bath is contained in an iron kettle, provided with suitable clamps for holding the globe beneath the surface of the bath, and for retaining the thermometers. Various substances are used for the bath, the requirement being that the substance shall not vaporize at a temperature well above that of ebullition for the liquid in the globe. Water may be used for a number of vapors, notably those of chloroform and ether. Sulphuric acid is available for this purpose up to 200° C. In determining the density of mercury vapor, Dumas used the alloy ($8\,Bi + 5\,Pb + 3\,Sn$). When this is used, the globe, on cooling, becomes coated with a thin layer of the alloy, which may be removed by the use of a knife and by solution in mercury.

The directions for performing the experiment are these: Carefully clean and very thoroughly dry the globe, and take its

weight. Then introduce about 10 c.c. of the liquid. To do this, warm the globe in the hands and dip the point beneath the surface of the liquid. When it is not possible to draw in a sufficient amount in this way, the globe must be heated by a gas flame. This must be done with some care. Heat only the part of the globe opposite the neck, and then dip the point in the liquid, not allowing that which is drawn in to come in contact with the heated region. When enough liquid has entered, quickly reverse the globe. Submerge the globe in the bath, allowing the neck to protrude in such a way that the stream of vapor given off when the bath is heated may be plainly visible. Heat the bath rapidly until the thermometers indicate a temperature 10° or 15° below the boiling-point of the liquid; then allow the temperature to rise more slowly, as otherwise the vapor may be formed too rapidly. Continue the heating without allowing the temperature to fall, until a temperature about 20° above the boiling-point of the liquid is reached. When the jet of vapor becomes invisible, quickly seal the end of the neck by means of a blow-pipe. Take at this instant the temperature of the bath, and the atmospheric conditions. When the globe has cooled, it must be washed and weighed, the readings of thermometer, barometer, and hygrometer being again noted. Break the point under water, and weigh the globe for a third time when filled with water, the short piece of the neck broken off being included, of course, in this weighing. The water used should be at the temperature of the atmosphere, and should have been previously freed from air. The atmospheric conditions are noted with this weighing as with the others. It may be desirable in some cases to gauge the water contained in the globe by the use of a graduate. The neck is broken at a point nearer the globe in such cases. The formula requires a few modifications when the volume of the globe is found in this way. The student will readily supply these.

Vapors.	Densities.	Temperatures.
Alcohol	1.61	78°.4
Benzine	2.77	92.0
Cloroform	4.20	60.8
Water	0.64	100.0
do.	0.67	180.3
Ether	2.59	35.5
Iodine	8.72	175.0
Mercury	6.98	350.0

EXPERIMENT 15. **Heat of vaporization (Despretz).**

The heat of vaporization of a liquid is the quantity of heat necessary to convert one gram of the liquid at the boiling-point into saturated vapor at the same temperature. This is to be distinguished from the *total heat* of a vapor, which is the quantity of heat necessary to convert unit weight of a liquid at 0° C. into saturated vapor at a chosen pressure. The latter quantity was that sought by Regnault in his exhaustive experiments on the latent heat of steam.*

Laboratory methods of determining the latent heat of vaporization are based not on measurements of the amount of heat given to a liquid in vaporizing it, but on measurements of the heat surrendered by the vapor in condensing. (See Exp. 1_1, Vol. 1.)

An apparatus used in many laboratories is that of Despretz. The liquid is vaporized in a copper or glass vessel. The vapor passes over into a worm contained in a calorimeter. It is here condensed, and the amount of heat surrendered is determined from the initial and final temperatures of the water in the calorimeter. Either of two methods of procedure may be adopted. The liquid condensed in the worm may be allowed to drip into a vessel outside of the calorimeter. Then, if the worm be of such length that the liquid dripping from it may be taken to be

* Memoires de l'Académie des Sciences, Vol. 21, p. 635.

at all times at the temperature of the water in the calorimeter, the student will readily prove

$$wL + ws\left(\theta - \frac{\theta_1 + \theta_2}{2}\right) = W(\theta_2 - \theta_1) + R.$$

Here

L = heat of vaporization;
w = weight of liquid condensed;
s = specific heat of liquid condensed;
θ = temperature of vapor;
W = mass of water + water equivalent of calorimeter;
θ_1, θ_2 = initial and final temperatures of water;
R = the radiation correction.

A better way is to allow the distillate to collect in a receptacle at the end of the worm and entirely within the calorimeter. In this case, the equation becomes

$$wL + ws(\theta - \theta_2) = W(\theta_2 - \theta_1) + R.$$

In performing the experiment, the first step is to determine the water equivalent and the radiation constant of the calorimeter. This is done by the methods previously described. Determine the mass of the water in the calorimeter. Place the liquid to be vaporized in the flask. To obtain the amount of liquid vaporized it is sometimes best in this form of apparatus to weigh the flask before and after the process of vaporization, but in most cases a cock is provided by means of which the distillate may be drawn off and weighed. In the latter case the vapor may be allowed to escape into the air for a time in order to warm the tube. When ebullition begins, make readings of the thermometer in the vapor and in the calorimeter every half minute. Make several determinations.

There are several sources of error in this method which are difficult to completely eliminate. Condensation of the vapor occurs in the tube connecting the flask with the worm. The

steam is thereby rendered wet, and a certain amount of water is carried over into the worm. The tube is usually inclined towards the boiler, so that the water formed by condensation for the most part returns to it. If the ebullition be too violent, water may be carried over in small drops, *i.e.* by priming. The existence of this error may be detected by operating with salt water, in which case a trace of salt will be detected in the distillate. The condensation of the vapor in the tube may be prevented by the use of a jacket, or other heating device, which will keep the vapor dry without superheating it. In the case of steam this is effected without much difficulty, as the tube may be surrounded by a steam jacket in which steam from the boiler circulates. Or, the steam may be dried by passing through a spiral tube contained in the boiler itself. Another source of error arises from the conduction of heat through the walls of the tube into the calorimeter. This would give too high a value for L, while the error due to above-mentioned causes would be in the opposite direction. It would not, however, be justifiable to assume compensation. The specific heat of the liquid for the range of temperature involved must be known from a previous determination, or taken from tables. With this apparatus Despretz obtained $L=540$.

Experiment 16. **Heat of vaporization (Berthelot).**

An apparatus designed to minimize the errors which are mentioned in the foregoing experiment is illustrated in Fig. 144. It is due to Berthelot.* The liquid is heated in a flask F of about 100 c.c. capacity. The vapor which is formed reaches the calorimeter by way of the tube T, which passes down through the hot liquid and through a hole in the flat-ring burner *ml*. Any condensation of the vapor while on its way to the worm is thus prevented, while the distillate accumulates in the reservoir R, lying wholly within the calorimeter. A slab of wood covered

* Journal de Physique, Vol. 6, p. 337 (1877).

with a screen of wire gauze closes the calorimeter, and protects the water in it from the heat of the burner. A water jacket surrounds the whole.

The amount of liquid distilled need be only 20 to 30 grams, and this is formed in 3 or 4 minutes. The corrections are thus very small, and may be made by a method similar to that described in Exp. 5.

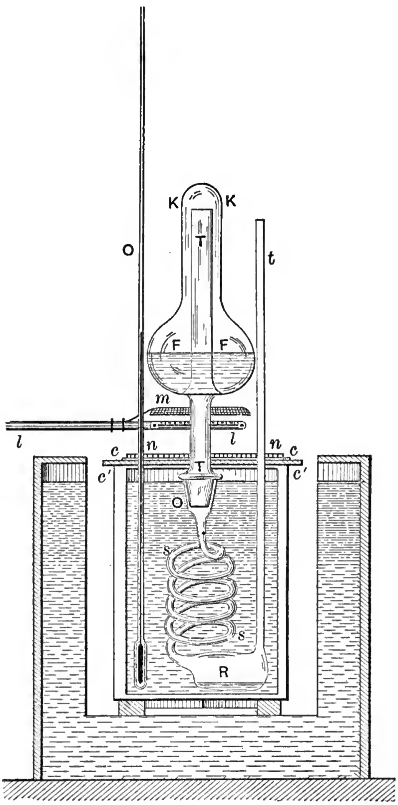


Fig. 144.

The calorimeter is first filled with a known weight of water (800 to 900 grams). The liquid is then put in the flask, and after its weight has been taken the flask is put in position and the gas lighted. The reading of the thermometer in the calorimeter is taken at this moment, and readings are continued until distillation begins. This constitutes the initial period. The second period is that of distillation, and should last until a rise of 3 to 4 degrees is attained in the water in the calorimeter. During this time 20 to 30 grams of vapor will form in most cases. When this result is reached, the flame may be extinguished, the flask removed, corked, and allowed to cool, after which it must be again weighed. During this time the observer follows the indications of the thermometer for a third and final period. From these observations the student will readily

make the necessary corrections. The heating due to the gas flame is obtained from the observations of the first period, when these are considered in connection with those of the last period, in which the calorimeter is under the influence of the air solely.

This apparatus is well adapted to the examinations of vapors of various organic liquids, since only small quantities are necessary. Berthelot obtained for water the value 536.2, while Regnault, by a very elaborate determination, found 536.5.

Experiment 17. **Determination of the mechanical equivalent of heat by means of a current calorimeter.**

The current calorimeter is a piece of apparatus for determining the mechanical equivalent of heat by measuring the quantity of heat generated in a given portion of an electric circuit, and finding the relation between it and the amount of work done in that circuit during the same time. The conductor is suspended in a vessel filled with distilled water, and is so arranged as to readily impart its heat to the water with as little loss as possible.

The calorimeter is constructed as follows: The outer vessel, and all of the other parts except the wire and binding posts, are made of vulcanized rubber. The wire, which is of platinum, is wound upon a notched framework or reel, which is attached to the lid and hangs from it. Inside the reel is a vertical rotary stirrer, and between the reel and the side of the vessel is suspended the thermometer.

In an apparatus of this type, in the physical laboratory of Connell University, the wire which is uncovered is 0.64 mm. in diameter, and has a length of about 36 m. Its resistance, at ordinary temperatures, is about 25 ohms. From the binding-posts are rods projecting down into the calorimeter which afford connection with the wire. There are several of these, so that the whole or only part of the wire may be used.

These rods, to prevent the action of the liquid upon them, are encased in platinum tubes which are soldered with gold.

At all the joints in the wire, and at the points where the rods are attached to it, gold solder was used. To prevent as much as possible the passage of the current from wire to wire through the liquid, distilled water is used. This must never be allowed to become warmer than 40° C., on account of the danger of softening the rubber.

The preliminary steps are necessary in performing the experiment:

1st. The determination of the radiation constant of the calorimeter.

2d. The determination of the water equivalent.

3d. The measurement of the resistance of the wire at various temperatures.

The radiation constant is to be obtained by the method described in the General Directions for Calorimetric Experiments, page 110, Vol. 1.

The water equivalent requires a more extended operation than will be found in those directions. The hard rubber does not become warmed as quickly as would the same thickness of metal, but there is a slow conduction of the heat toward the inner parts of the rubber, which is entirely surrounded by water, and toward the outside of the walls of the vessel itself. Readings of times and temperatures have therefore to be taken, and a curve plotted, from which the final temperature of the mixture is read.

Before putting in the warm water, the calorimeter should be tipped over to such an angle as will cause the water in it to moisten the inner surface up to the level to which it is to be filled. It should be swung around at that angle for several minutes. This is for the purpose of bringing the upper part of the vessel to the same temperature as the water. Then as soon as the warm water is added, a series of thermometer readings should be begun. These should be taken every 20 seconds for about 10 minutes.

The curve, when plotted, shows several points of change of rate of cooling, which are probably due to the change of flow of heat as the different parts are brought up to the temperature of the water. The vessel itself is probably the last to become warmed through, and as soon as this has taken place, radiation begins, and beyond this point the curve will be a smooth one, except for small irregularities caused by errors of observation. The ordinate of this last point indicated is the final temperature of the mixture, and is the value to be used in the computation of the water equivalent.

The resistance of the wire is to be measured several times, changing the temperature of the water 4 or 5° each time, and stirring well; also reversing the connections so as to obtain a pair of readings for each temperature. The mean of each pair is to be taken, and then a curve is to be plotted with temperatures as abscissas and resistances as ordinates. From this curve the resistance for any given temperature within the range can be obtained.

If in making the experiment proper, the connections are so made that the whole wire will be in circuit, the resistance of the calorimeter itself will, without any extra resistance, be sufficient for a 120 to 130 volt supply. The water must cover the upper turns of the wire, and at the beginning it should have a temperature of 13 or 14°. The amount required will be about 21.5 litres.

Each minute until the water has become warmed to 36 or 38°, the current, potential, and temperatures of water and of room should be read. Curves may then be plotted for each of these quantities, using times in each case as abscissas.

The readings should be divided into 10-minute sections, and the result obtained for each section. Then the mean of these should be taken for the final result. This result should be the number of kilogram-meters mechanically equivalent to one calorie of heat energy.

The following notation may be employed :

Let x = Mechanical equivalent of the calorie.
W = Work in kilogram-meters.
H = Heat generated.
M = Weight of water in kilograms.
M' = Water equivalent of the calorimeter.
T = Time in minutes, t in seconds.

To be read from curves.
θ_0 = Temperature at beginning of a 10-minute section.
θ_1 = " " end " " "
θ_n = " of room.
I = Current in amperes.
E = Volts P.D. of terminals of calorimeter.

R = Radiation constant.

One watt-second = 0.101937 kilogram-meter per second. The value of x can be computed from the formula,

$$x=\frac{W}{H}=\frac{EIt\times 0.101937}{(M+M')(\theta_1-\theta_0)+RT\left(\frac{\theta_1+\theta_0}{2}-\theta_n\right)}$$

Other computations may be made by using the average resistance in a section instead of the difference of potential of the terminals. This result may differ somewhat from that by the first formula, and so it may be well to estimate the weight to give to each method and then by combining obtain a mean.

EXPERIMENT 18. **Determination of the cubical expansion of solids by the method of balance transits.**

For the purposes of this experiment an ordinary analytical balance is to be mounted upon a high table or wall shelf, as shown in Fig. 145. The right-hand scale-pan having been removed, small holes are bored through the table or shelf, these being in a direct vertical line with the hook at the hand of the balance arm. A fine platinum wire, the upper end of which is fastened to this hook, passes through these holes, and by means

of the same the substance, the coefficient of expansion of which is to be determined, is suspended.

The material in question should be denser than water, and insoluble in that liquid. It should, if practicable, be cut into some regular form, preferably that of a disk or ring, and should be suspended so as to hang with its axis of figure in the line of the suspension wire. This method will not give good results if applied to porous substances. Previous to mounting the

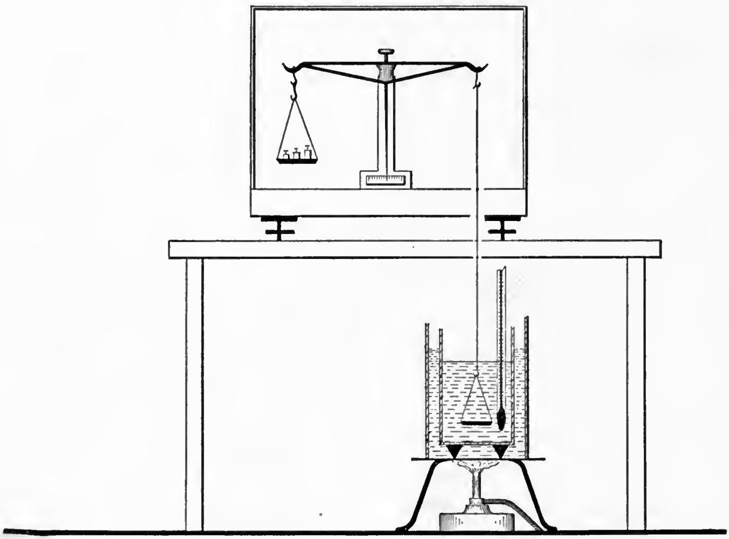


Fig. 145.

specimen in the manner described, the same should have been carefully weighed.

After the test piece has been suspended as indicated in the last paragraph, a copper vessel containing water, or a beaker, the contents of which should not be less than 1 liter, is carefully raised from below until the specimen is entirely submerged. The vessel should be adjusted so that the latter hangs as nearly as possible at its center, and must be supported in that position by means of a stand or tripod to be heated from below in the usual manner with a Bunsen burner.

Before any readings are undertaken, the water surrounding the specimen must be brought to the boiling-point, in order to drive out all occluded air and to prevent the subsequent formation of bubbles upon the surface of the specimen. An accurate thermometer should be placed in the bath, as near the suspended solid as possible without contact, care being taken to have the bulb as nearly as possible at the same level as the latter.

While the liquid is still near the boiling-point, the flame having been extinguished, weights are to be placed upon the scale at the left hand of the balance pan until the submerged mass is counterpoised. Owing to the greater coefficient of expansion of water, the liquid will gain density more rapidly upon cooling than will the solid, and the loss of weight of the latter, due to submergence, will increase continuously. If the weight applied to the left-hand arm of the balance be slightly too small at a given temperature, the difference will diminish as the bath cools, and the pointer of the balance will begin to move across the scale, until after a short interval of time, the length of which will depend upon the rate of cooling, the weight will be in excess. A small weight, say 10 m., may now be taken from the scale-pan, which will bring the pointer back again to a position showing a deficiency of weight in the pan. The drift past the zero point will then reoccur, and by adding small weights from time to time this movement may be repeated over and over again until the bath has cooled almost to the temperature of the surrounding air.

It is upon this process that the method under consideration depends. The determination consists, in a word, in obtaining two time curves. The one gives the times, as indicated by the transit of the balance pointer across its zero reading, at which the successive weights upon the left-hand scale-pan precisely counterbalance the submerged mass. The other is the time curve of cooling of the liquid of the bath as indicated by the thermometer. As a matter of convenience, readings of the

thermometer may be made in alternation with those of the balance transits, and in practice the amount of weight taken each time to the scale-pan should be sufficient to allow an accurate reading of the thermometer and of the time-piece to take place in the interval which elapses between the removal of the weight and the time of the following transit.

From these time curves and from the density of boiled water,* one can plot a curve showing the volume of the submerged solid as a function of the temperature, from which in turn may be deduced the curve of temperature coefficients. It will be found in the case of almost all substances that this coefficient is itself a function of the temperature and is capable of being expressed by means of an empirical equation of the type

$$\alpha_t = \alpha_0(1 + At + Bt^2 + Ct^3),$$

in which A, B, and C are constants. These constants should be determined for the substance in question. The constant C is in most cases very small. Indeed, it may, without introducing errors comparable with those of the errors of observation with this method, be considered as equal to zero.

Experiment 19. **Measurement of temperatures by means of a thermo-element.**

In many cases the measurement of temperatures by ordinary thermometric means is very difficult, and not infrequently quite impossible. The thermo-element furnishes a means of measuring temperatures which is often of extreme utility to the physicist. It is particularly adapted to the measurements that must be made in spaces difficult of access. It is, moreover, when suitably arranged, a very sensitive method. The couples most frequently employed are: bismuth-antimony, iron-german silver, platinum-iron, while sometimes platinum-iridium is a useful combination. The junctions should be extremely light, but may

* See Landolt and Börnstein's Tables, edition of 1894, p. 39.

vary to a considerable extent in form, the arrangement being varied somewhat to suit the problem at hand. Two wires of equal length, but of different material, are soldered together at one end, and at the other to copper wires. The former junction is that used for the temperature to be measured, while the two copper junctions are kept at some constant temperature as, for instance, that of melting ice. For measuring the currents produced, a sensitive mirror galvanometer is used.

For small temperature differences (up to about 20°) the current is very nearly proportional to the difference of temperature, which makes the measurement of temperatures within this range an easy matter. For higher temperatures the temperature must be read from a calibration curve, which is obtained in some such way as the following: Place one junction in a bath of oil at the temperature of the atmosphere, while the other is placed in a similar bath at a temperature considerably above that of the air, and consequently slowly cooling. Take readings of the galvanometer, using a reversing key, and of the temperatures of the two baths at such intervals as are necessary to obtain a smooth curve.

The principal difficulty encountered in the use of this method is the instability of the arrangement. One is not always sure of obtaining the same current when the two junctions are brought at different times to the same temperature conditions. This necessitates frequent calibrations. The existence of a neutral point is also often a source of inconvenience.

VOLUME II.—PART IV.

*OUTLINES OF ADVANCED WORK IN GENERAL PHYSICS.**

By Edward L. Nichols.

INTRODUCTION.

The following chapters are written for the use of students who have completed the routine work of the physical laboratory and desire to enter upon original investigation. At this stage in the development of the physicist there is a critical point. Further success depends upon several matters which have been necessarily somewhat neglected during the earlier periods.

In the first place, the student must acquire independence and self-reliance; he himself must face the experimental difficulties of the problem upon which he may be engaged, and must overcome them by devices of his own. These are the all-important things, the requisites of the successful investigator. In order that they may be acquired, it is necessary that the rather close supervision of the student shall be relaxed; and since it is no less important that he shall cut himself loose from the text-book than from the living instructor, whatever is written for his use must be planned to meet the new condition.

There are other minor matters to be considered in the training of the advanced student, one of which is that he shall turn

* Based chiefly upon researches done in the physical laboratories of Cornell University.

more freely than ever before to original sources for his information, learning to regard all compendiums and treatises as secondary; another is, that he shall have in hand but a single problem at a time, looking upon that broadly, with the expectation of attacking it in as thorough a manner as possible. To accomplish anything worthy of the name of original work in physics abundant leisure must be given, and the student must accustom himself to new plans of work which involve the concentration of his energies upon one subject chiefly for very considerable periods of time.

That there is danger of narrowness in this need not be feared. No scientific problem is so narrow that the contemplation of it in all its bearings does not tax the knowledge and skill of the experimenter throughout a considerable range. Oftentimes, indeed, a greater variety of topics and methods will be found within the limits of the single investigation than in a year of practice work selected for the purpose of giving a diversity of experience.

Bearing in mind the object to be attained in the following pages, theoretical discussion on the one hand, and systematic and explicit directions of procedure on the other, have been avoided, and this portion of the manual has been reduced to a series of suggestions based upon previous experience in the direction of original work. A sufficient number of references to previous work have been furnished under each head to enable the student to get hold of the literature of his topic without undue loss of time. Typical results from the labors of those who have gone before, and upon whose foundations it is assumed that other and more complete structures will be built, have been freely used. In selecting material for these chapters the principle has been followed that in order to be entitled to make serious suggestions for the guidance of others, one must have been over the ground himself, not through the medium of books, but in actual experience. The strict application of this principle to the case in hand is not without drawbacks, the

chief of which arises from the fact that in every laboratory where original work is done the inmates perforce devote themselves largely to certain subjects to the neglect of others; thus it will be found that the following pages deal more fully with the study of radiation than with any other branch of physics, and that they leave many quite as important domains unexplored.

It is assumed that when the student has arrived at this stage of his development, and has selected a theme for further study, he will acquaint himself, first of all, with the literature, leaving nothing which bears upon it unread, so far as the books at his command will permit. To do this, he should learn the method by which the titles of the various papers which he is to consult may be most readily found. To search the vast literature of physics without some guide would be at best a time-consuming occupation.

Fortunately there have been gathered together very complete lists of scientific memoirs, the use of which will save much labor. Of these the most important are the *Royal Society's Catalogue of Scientific Papers*, in which, classified under the names of the authors, will be found nearly everything that was written up to the year 1867 with supplementary volumes covering the earlier portions of the alphabet for the ten years following.

Another important source for the early literature is Poggendorff's *Handwörterbuch*, which comes down to 1863.

In electricity the index of Wiedemann's *Elektricität* will be found most useful.

For literature of the period beginning with 1877 the indexes of the *Beiblätter* of the *Annalen der Physik* should be consulted.

In addition to these, the various lists and indexes contained in the volumes of the *Fortschritte der Physik*, which is complete to 1888, and of the *Fortschritte der Elektrotechnik*, from 1887 to date, will be found useful.

Landolt and Börnstein's *Physikalisch-Chemische Tabellen* (edition of 1894), finally, is an invaluable source of bibliographical as well as of numerical information.

By making use of these works, which should be accessible to the students of every well-equipped laboratory, many titles will be found relating to the subject with which the student desires to make himself familiar, while further references will, of course, be met with in reading the papers themselves.

CHAPTER I.

STUDIES OF THE INFLUENCE OF TEMPERATURE UPON VARIOUS PHYSICAL CONSTANTS AND UPON THE PROPERTIES OF MATTER.

I.

(Preliminary.)

The critical study of thermometers.

In nearly all work of a precise nature in the domain of heat, it is necessary to make a systematic investigation of the thermometers used. In cases in which a carefully calibrated standard instrument is available it is oftentimes sufficient to make a comparison between the thermometers to be actually used in the experiment and the standard, degree for degree, throughout the entire range of temperature to be covered in the subsequent work. Such standards are, however, comparatively rare, and they are subject, moreover, to changes of zero point like ordinary thermometers. It is best, therefore, even in cases where a comparison with a standard instrument is made, to supplement the same by a detailed study of the thermometer to be calibrated.

The study of thermometers falls naturally into two divisions: the determination of freezing and boiling points and the calibration of the stem.

The determination of the freezing-point of a thermometer is a simple operation, but certain points must be observed in order to render it exact.

1. The snow or powdered ice in which the bulb is packed must be at its melting-point. Ice is a poor conductor of heat, and, if taken from a locality which has been subject to low

temperatures for a considerable time, it will frequently be found to be chilled considerably below the melting-point.

2. The ice or snow must be free from admixture with soluble impurities. The presence of small quantities of substances capable of combination with water or of solution merely will convert the mass of powdered ice into a freezing mixture, and a more or less marked fall of temperature will result.

3. A sufficient time must be permitted to elapse before the reading of zero point is taken, to insure the cooling of the mercury and glass parts of the thermometer bulb to the ice temperature. The difference in temperature between the bulb and the surrounding bath does not become inappreciable until an interval of at least twenty minutes has elapsed. In cases of extreme precision it may be necessary to further extend the time.

4. The ice should be in intimate contact with the surface of the thermometer bulb throughout the entire time of exposure, and the mass of powdered ice should be thoroughly drained. Intervening air chambers, such as are likely to form through the melting away of the ice granules nearest the glass, afford effective thermal insulation. In order to bring the thermometer to the actual temperature of the ice, it is necessary to press the powdered ice freshly to the bulb at frequent intervals during the progress of the experiment. The gathering of water beyond that which will cling to the surface of the ice particles in a vessel so constructed as to afford drainage will also tend to impede the cooling of the bulb.

In the determination of the boiling-point of a thermometer there are likewise certain precautions to be observed.

1. The thermometer bulb should be exposed to an atmosphere of steam, and not to the boiling liquid itself. It is well established that the temperatures beneath the surface of boiling water vary considerably from the true boiling-point, which temperature is defined as that of the atmosphere of steam above the surface of the liquid. Variation from the boiling-point

within the mass of the liquid is always in one direction; the temperatures are too high.

2. The steam chamber within which the exposure of the bulb takes place should be jacketed. This condition is very readily fulfilled by making use of an apparatus of the form shown in Fig. 146. This apparatus consists of a reservoir surmounted by a cylindrical tube with double walls, within which there are apertures so arranged as to secure a circulation of steam; first, from the boiling water through the inner cylinder to the top, thence between the inner and outer cylinders to a point of efflux near the base of the outer wall.

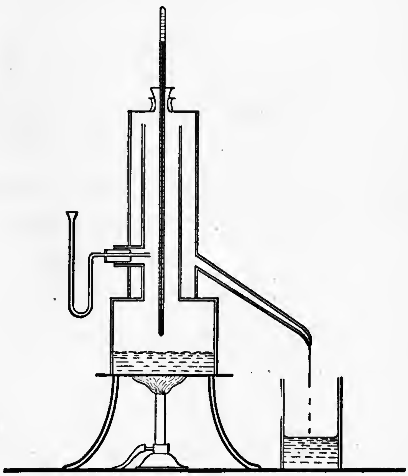


Fig. 146.—Calibration of a Thermometer.

3. Such portions of the stem of the thermometer as are to be exposed to the temperatures to be measured should be within the steam jacket at the time of the determination, the principle being to secure conditions as nearly as possible alike in the two cases.

4. The pressure of the steam within the boiling-point apparatus should be carefully measured by means of an open-tube manometer; the difference of level between the arms of which is to be added or subtracted, as the case may be, from the barometric reading.

5. A sufficient interval of time should elapse before readings are begun, to insure equality of temperature between the steam and the thermometer bulb.

It should be noted in connection with this test that whenever a thermometer is heated to the temperature of boiling water, certain changes in the volume of the bulb are brought

about, which changes last for a very long time. The consequence is that the zero point of the thermometer is changed, and that an error corresponding in size to the change in zero point is introduced, which error will affect all subsequent readings.

In cases in which the thermometer is to be used at temperatures in the neighborhood of the boiling-point, observations of boiling-point must certainly be made; but these should be followed by a redetermination of the freezing-point, in order that a definite estimate may be formed concerning the amount of change which the thermometer has suffered by being subjected to heat.

When the thermometer is to be used only at low temperatures, it is better to avoid subjecting it to any temperature greatly in excess of those to be measured in the actual experiment, and either to substitute for boiling-point determinations, comparisons with the standard throughout the range of temperatures for which the calibration is desired, or to rely upon the boiling-point mark made by the manufacturer of the thermometer. Even without comparison with a standard, a very accurate knowledge of the errors of a thermometer scale can be obtained from the determination of the freezing-point and the calibration of the stem by the method now to be described.

Calibration of the tube of a thermometer.

Since thermometer tubing is made of drawn glass, the cross-section of the bore is never quite uniform. It may be said in general to be conical rather than cylindrical, and it is almost always the case that marked irregularities exist even in the most carefully constructed instruments. The best makers of thermometers attempt to compensate for these irregularities of the tube by adjusting the lengths of the graduated spaces along the stem in such a manner that each degree interval shall contain $\frac{1}{100}$ of the entire volume of the thermometer bore from $0°$ to $100°$.

Even with the exercise of the highest skill and of the most tireless patience, this is an end but imperfectly attained, and it is unquestionably better to have thermometers which are to be used in operations of extreme precision graduated uniformly, in millimeters, rather than to have them divided into unequal intervals which purport to be degrees of the centigrade scale. Whatever be the nature of the scale, however, it is necessary to apply a correction. In the case of the uniform graduation, this correction expresses directly the irregularities of the bore; in the case of the graduated calibration, it expresses the imperfections in the maker's corrections. For a full description of the method by which these corrections are to be determined, see Rowland's paper entitled "The Mechanical Equivalent of Heat."*

The following method will give good results:

(1) A thread of mercury is detached, the length of which corresponds approximately to some short interval (not more than 5° in a stem graduated to 100°). The thermometer is laid upon the bed of a dividing engine, and the length of this thread is measured in a series of positions extending throughout the entire length of the tube. For this purpose the thread is made to travel through the tube by gently tapping the upper end. These positions should overlap so that no portion of the tube remains unmeasured. From the results thus obtained a curve is to be plotted, preferably on a large scale, in which reciprocals of the relative lengths of threads are ordinates and degrees measured along the thermometer stem are abscissas.

(2) Having completed these measurements, the distances between the degree marks corresponding most nearly to the length of the thread (for example, five-degree spaces) are measured upon the dividing engine, and a curve is plotted showing the variations in volume of these spaces in all parts of the tube (*B*, Fig. 147). If the calibration performed by the

---

* Rowland, Proceedings of the American Academy of Arts and Sciences, 1878–79, p. 75.

maker of the thermometer has been a successful one, these curves when drawn to proper scale should agree closely.

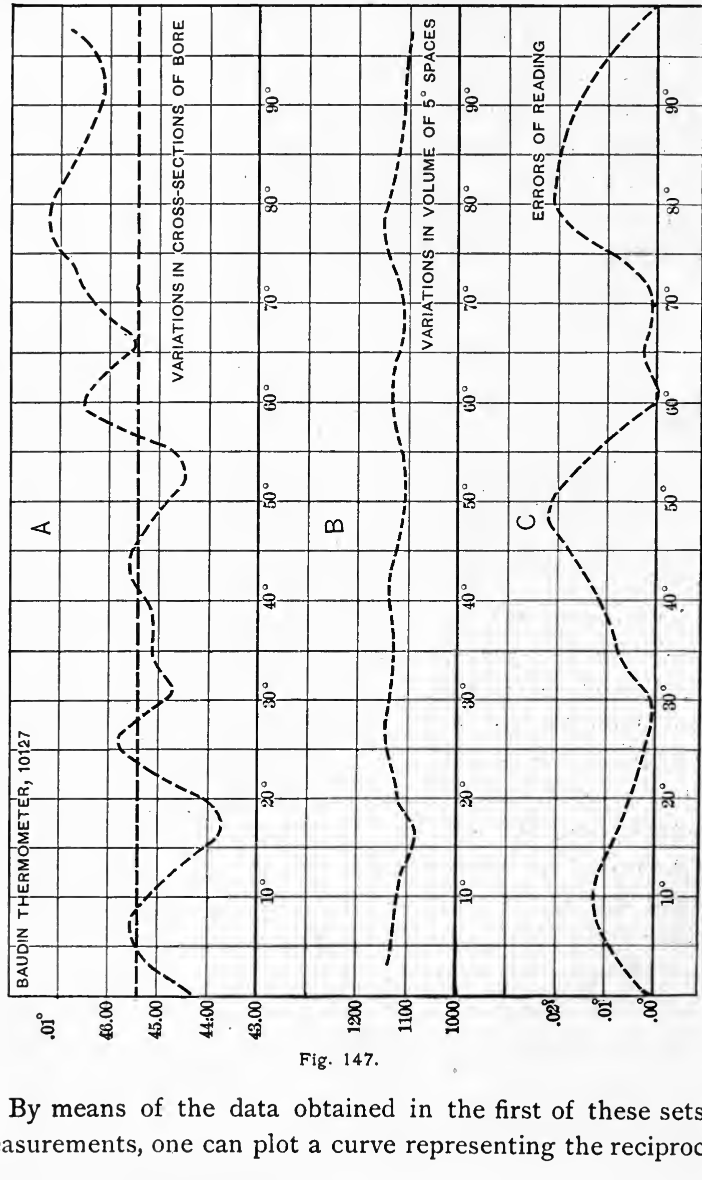


Fig. 147.

By means of the data obtained in the first of these sets of measurements, one can plot a curve representing the reciprocals

of the length of thread for the entire tube. This curve (*A*, Fig. 147) will give a picture of the bore, since these reciprocals represent the relative volumes contained by unit length of tube from end to end.

(3) A curve should be plotted showing the error of each one-degree space throughout the tube; also a curve of total error (C. Fig. 147), assuming the zero point to be correct. By adding to the readings of this last curve the experimentally determined error of freezing-point, a corrected reading may be obtained for any part of the scale.

In using corrections based upon this calibration, the assumption is made that the freezing-point and boiling-point marks were accurately placed when the thermometer was graduated. The error involved in such an assumption is usually less serious than that which is introduced in the attempt to redetermine the boiling-point of the thermometer, excepting in those cases in which the thermometer is subsequently to be used in the determination of high temperatures. The redetermination of the freezing-point, however, does not cause any appreciable change in the contents of the thermometer bulb, and should always be made.

The curves in Fig. 147, to which reference has just been made, were obtained by measurements of the stem of Baudin's thermometer, No. 10,127. The observations were made under the writer's direction by P. J. Darlington.*

In addition to the points covered in this brief discussion, various minor details of procedure might be mentioned. It has been found, for example, that in very precise determinations, the application of a rapid tapping to the thermometer stem ensures uniformity of readings not to be obtained in any other way. This tapping motion, which may be imparted by the use of an electrically driven mallet or hammer, prevents the clinging of the surface film of the mercury column to the

* See Laboratory Reports, Department of Physics, Cornell University, 1891.

glass, — a tendency which leads to noticeable differences in the height of the mercury column where no differences of temperature exist.*

Some references to memoirs on thermometry :

REGNAULT: Relation des Experiences. 1847.
HANSEN: Sächs. Ges. der Wissenschaften. Vol. 15. Gotha, 1874.
MILLS: Phil. Mag. (5). Vol. 6, p. 62. 1878.
MAREK: Carl's Repertorium. Vol. 15, p. 300. 1879.
THIESEN: Carl's Repertorium. Vol. 15, p. 285. 1879.
ROWLAND: Am. Acad. Arts and Sciences. 1879.
CRAFTS: Comptes Rendus. Vol. 91, p. 862. 1880. Travaux et Memoires du Bureau international des Poids et Mesures. Paris, 1881.
LE CHATELIER: Comptes Rendus. 1888. p. 862.
GUILLAUME: Thermometrie de Precision. Paris, 1889.

II.

The influence of temperature upon Young's modulus.

The investigation to be outlined under this heading deals with the elasticity of tension of a steel wire at various temperatures. Nearly all writers on this subject agree that the influence of a rise of temperature is to diminish the modulus, although Wertheim (1845) found the modulus greater at 100° than at higher or at lower temperatures.

It is usual in such experiments to use a vertical wire; but where the temperature effect is to be taken into account, the greater ease with which a horizontal wire can be maintained at a constant temperature renders the following method preferable to those usually employed.

The wire to be stretched is placed over the bed of a comparator. One end of it is fastened while the other passes over a wheel mounted on friction rollers, like the wheel of an Atwood's machine. To this free end is attached a suitable stretching weight, to which, when measurements are to be made, further weights are added. The permanent weight should

* See E. H. Loomis, The Freezing-Point of Dilute Solutions, Physical Review, Vol. 1, p. 199.

remain in place for a considerable time before readings are begun. Marks upon the wire, at points about 1 m. apart, are observed by means of fixed reading microscopes with micrometer eye-pieces (Fig. 148).

The horizontal portion of the wire is to be heated by means of a spiral coil of iron wire which extends beyond the observation points to a distance of about 20 cm. and which surrounds the entire intervening part. Iron should be chosen for this coil rather than German silver, because when the former is heated to a high temperature the surface becomes uniformly oxidized, and acquires everywhere the same emissivity. German silver, however, tends to variations in surface which ultimately lead to marked differences of temperature, amounting sometimes to several degrees between neighboring regions, and which thus introduce serious errors.

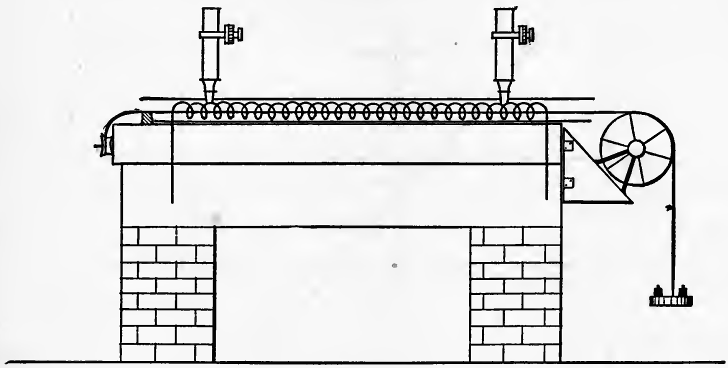


Fig. 148.

The heating coil is placed within a glass tube, the diameter of which should be 4 cm. or 5 cm., and the length of which should be about 140 cm. This tube has holes ground in its walls, through which the microscope objectives may be inserted.

The turns of the heating coil should be as uniformly situated as possible within the glass tube, so that the amount of heat developed per unit of length of the enclosed stretched wire

may be everywhere the same. The temperature of the heating coil is maintained by means of a current from some steady source, preferably from a storage battery. If proper precau-

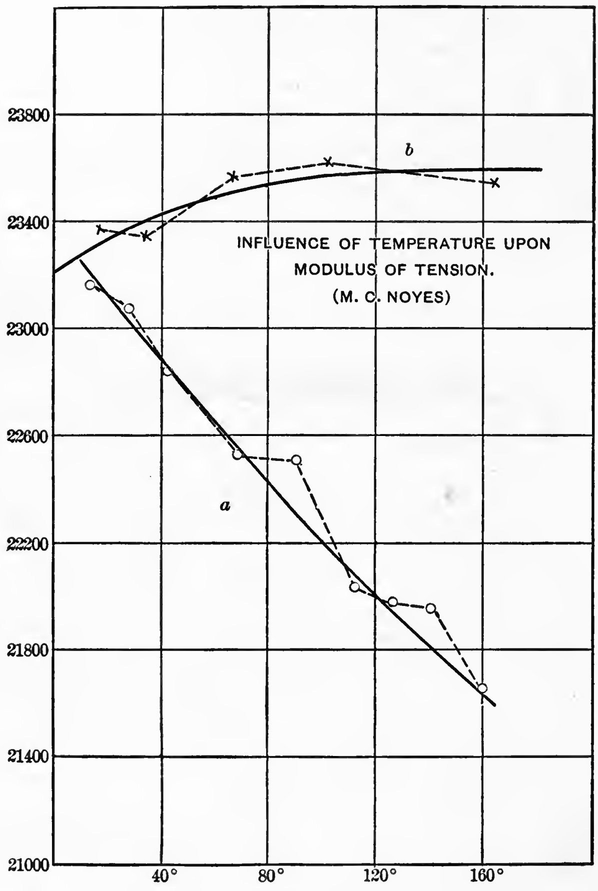


Fig. 149.

tions be taken to prevent draughts of air, by filling the ends of the glass tube with cotton waste, or, better, with shredded asbestos, almost perfect uniformity of temperature may be secured throughout the entire length of the tube.

By adding weights to the stretched wire and noting the elongations in the usual manner, with the wire at various temperatures, a curve may be obtained expressing the variation of the modulus with temperature.

Some determinations upon pianoforte wire, made by the above-described method by Miss Mary C. Noyes* (Physical Laboratory of Cornell University, 1893–94), afforded data for the curve *a* (Fig. 149).

A repetition of the measurements, in the course of which the wire under investigation was heated directly by sending a current through it, gave, however, a result entirely different from that obtained when the heating was due to the action of the helix. Under these conditions the modulus showed increase, with rise in temperature, as shown in curve *b* (Fig. 149). That the difference in these curves was not due to longitudinal magnetization under the influence of the helix was ascertained by supplying the latter from an alternating current dynamo and repeating the measurements of elongation. The resulting curve conformed in all respects to curve *a*.

Although much has been done upon the relations of magnetization to elasticity, none of the researches as yet published afford an explanation of the marked difference in the behavior of steel wires under these two conditions. An investigation of the phenomenon, extending the measurements to other materials than iron, would be of great interest.

The value of the modulus obtained in Miss Noyes' experiments was so much larger than the values usually found that it seemed desirable to repeat the observations at a single temperature, using a vertical wire. The results thus obtained were smaller by about two per cent than those used in plotting curve *a*.

Determinations of the modulus with the horizontal wire should be regarded as relative, therefore, and strictly com-

* Physical Review, Vol. 2.

parable only with one another. An absolute value should be determined at room temperature, using a vertical wire.

III.

Influence of temperature upon the thermal conductivity of a copper bar.

This investigation includes three distinct experiments: the determination of the specific heat of the bar; the determination of its curve of cooling; the study of the distribution of temperatures along the bar after the latter has been subjected to a very high or a very low temperature for a considerable time.

To determine the specific heat of a mass of copper 100 cm. long, 5.0 cm. in width, and 2.5 cm. thick, special apparatus is necessary. In the case of a bar of these dimensions, experimented* upon under the writer's direction in 1891–94, the following methods were used:

A. *Specific heat by the method of mixtures.*

In carrying out this determination there are several difficulties to be overcome.

(1) The bar must be heated uniformly throughout its entire length.

(2) The temperature of the bar must be determined at the moment when it is introduced into the calorimeter.

(3) The water equivalent of the calorimeter, which consists of a copper box, long enough to admit the bar in a horizontal position, and of the requisite length and breadth, must be ascertained.

The best way to heat the bar is by means of a spiral coil of heavy German silver or iron wire somewhat longer than the bar itself.† The diameter of the coil must be such that the bar

---

* The bar in question was one of a set made for Professor W. A. Rogers and loaned by him for the purpose of this investigation.

† See Physical Review, Vol. 1, p. 146.

can be introduced from the end and be hung within the coil in a horizontal position without touching the wires. By means of a steady current of electricity sent through the spiral coil, the latter may be maintained at any desired constant temperature.

If the coil is carefully constructed, so that the distance between the neighboring turns is very nearly equal, the region which it encloses will be at a uniform temperature throughout. After a lapse of about three hours the copper bar will itself have acquired this temperature.

The circuit may now be broken, and the copper bar removed and placed in the calorimeter. During the time necessary to carry out this operation, however, a certain amount of cooling will have occurred. In order to make the proper correction, two or three determinations of the temperature of the bar should be made in the interval between its removal from the coil and its introduction into the calorimeter, the precise time at which these temperature measurements are taken being noted. From these, by extrapolation, making use of the curve of cooling of the bar, the method of obtaining which will be described presently, the precise temperature of the bar at the moment it is inserted in the water of the calorimeter may be accurately ascertained.

Method of determining the temperature of the copper bar.

In the classical researches on thermo-conductivity by Forbes, Tait, Wiedemann and Franz, and others, temperature measurements have been made either by means of the mercury thermometer or with the thermopile. The following method, which has been successfully used in the experiments upon which this description is based, is believed to possess many advantages over those previously in vogue. The method consists in the use of a collar, made up of a single layer of fine insulated copper wire wound around the bar so as to fit the surface of the latter closely, yet free to slip along the bar to any desired

position. This collar or coil (*C*, Fig. 150), forms one arm of a Wheatstone's bridge the other arms of which consist of coils of wire compensated or else maintained at constant temperature. The measurements of resistance are made by the well-known method of the slide wire bridge. With a galvanometer of suitable sensitiveness the changes in the resistance of this collar afford a very delicate and convenient method for the measurement of temperatures.

The collar is calibrated in the following manner:

(1) A short piece of copper, the cross-section of which is the same as that of the bar, has a hole 1 cm. in diameter bored into it near its center.

(2) This hole is filled with mercury, and a thermometer is placed within it, the bulb being entirely submerged.

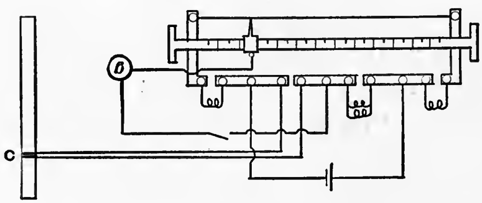


Fig. 150.

(3) The collar is now slipped over the end of this short bar and moved along until it is nearly in contact with the stem of the thermometer. The copper is then heated, with the thermometer and collar in place, by means of a coil of wire carrying current until a temperature is reached which is higher than the highest temperature to be measured in the subsequent experiments.

(4) Circuit is then broken and the copper allowed to cool, time readings of temperature and of the resistance of the collar being made simultaneously by two observers.

This calibration having been carefully performed, the resistance of the collar may be taken in all subsequent determinations as an accurate indication of the temperature of that portion of the copper bar which it surrounds. In heating the bar previous to its insertion in the calorimeter this collar should be

in place. Measurements of its resistance will then serve to show whether the bar has reached its final temperature.

After the removal of the bar from the heating-coil, the time measurements of temperature previous to its being placed in the calorimeter are easily made by the same method. When these are completed, the collar is slipped from the bar, and the bar is inserted in the water of the calorimeter. The temperature of the calorimetric bath should, of course, have been so adjusted that the entire rise of temperature due to the cooling of the bar will take place as nearly as possible, equally above and below the temperature of the room by which means, as in all calorimetric experiments, the influence of loss and gain of heat by radiation is very nearly eliminated.

In one of the sets of experiments upon which this outline is based, Mr. S. J. Saunders made use of the following method as a substitute for the usual determination of the water equivalent of the calorimeter. From the known mass of the bar and the approximate value of its specific heat, the water value of the bar was readily computed. A suitable mass of water was then selected, and this was heated to such a temperature that when added to the water of the calorimeter, the latter being identical in amount and at the same temperature as in the experiment with the bar itself, it would produce as nearly as possible the same rise of temperature within the calorimeter. Slightly smaller and larger quantities than these were then taken, the mass, initial temperature, and final temperature, after mixing with the water of the calorimeter, being determined in each case.

From the results of this series of experiments, a curve was plotted, showing the relation between the amount of water introduced and the change of temperature. The rise of temperature produced by the introduction of the heated bar into the calorimeter would fall somewhere upon this curve, and the abscissa corresponding to this point gave the true water equivalent of the bar.

B. *Specific heat by the method of the ice-block calorimeter.*

As a check upon this series of determinations the following method was also employed:

During the winter season a block of ice was obtained, the length of which was somewhat more than 1 m., and the cross-section about 20 cm. In the end of this block a hole was bored of such size that the copper bar could be introduced, the length of the hole being sufficient to admit the entire bar within the ice-block. A cap consisting of a slab of ice was provided for closing the mouth of the hole. The ice-block was inclined sufficiently from the horizontal plane so that the water resulting from the melting of the ice would not escape. After a sufficient time had elapsed to insure the fall of the bar to zero, the liquid thus obtained was drawn off and weighed. The results obtained by the method of the ice-block calorimeter were in good agreement with those of the method of mixtures, but the apparatus was a more difficult one to make and the operation altogether less satisfactory.*

C. *Determination of the final distribution of temperatures along the bar.*

In this part of the work, as in the determinations just described, temperatures are measured by means of the resistance of the collar of copper wire. Since thermo-conductivity is a function of the temperature, it is desirable to extend the range as widely as possible.

One of the principal difficulties in such work has always been to find a high temperature bath which could be maintained without fluctuation for a sufficient length of time. As a substitute for such a bath a coil of German silver wire wound around one end of the bar, the turns being separated from the copper and from each other by strips of thin asbestos paper, is very satisfactory. It is only necessary to maintain the current flow-

---

* See A. W. Shepard, Thesis, Cornell University Library, 1892.

ing through this coil at a constant intensity in order to secure a degree of constancy as regards temperature which can be obtained otherwise only at the boiling-point of a liquid. The bar itself during this operation should be supported in a horizontal position upon two V-shaped blocks of wood.

When the final distribution of temperatures has been reached, the distribution of temperatures along the bar from end to end are to be measured by means of the resistance of the collar, which is placed successively at various selected distances from the free end. The results should be expressed graphically by means of a curve, the ordinates of which are temperatures, the abscissas distances from the source of heat.

In order to extend the range of temperature for which the thermal conductivity is to be determined, one end of the bar may subsequently be subjected to a low temperature bath. The first experiments of this kind to be performed, so far as the writer is aware, are those recently completed in the Physical Laboratory of Cornell University.* An outline of the method pursued by the observers, Messrs. Quick and Lanphear, may serve as a guide to future workers. In this experiment the bar is entirely surrounded by a box of glass and varnished wood, with the exception of one end, which passes through the end of the box into the cooling chamber. The latter consists of a small metal box neatly soldered to the end of the bar. Into this is poured ether containing solid carbon dioxide in solution.

This mixture, which has been known since the time of Faraday, maintains itself indefinitely at the extremely low temperature of -79°. In order to reduce the rate at which this cooling mixture is dissipated, the bath is surrounded first by asbestos, then by a thick layer of ice. Under these conditions a few hundred cubic centimeters will suffice to cool the copper bar to its final condition and to maintain it thus for several hours.

* See Child, Quick, and Lanphear, Physical Review, Vol. 2.

A serious difficulty arises from the fact that even a trace of moisture in the atmosphere soon forms a coating of hoar frost upon the colder portions of the surface of the copper. The

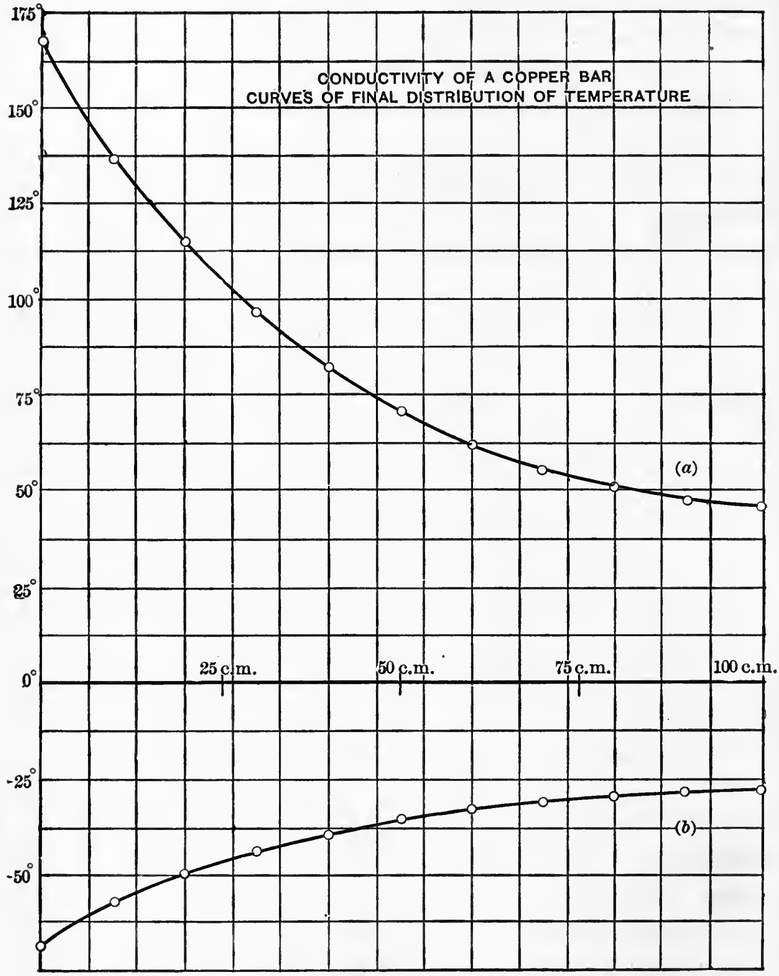


Fig. 151.

presence of such a layer would of course seriously modify the radiating power of the copper and change altogether the distribution of temperatures throughout the bar. This difficulty can

be avoided only by scrupulously closing all openings in the surrounding box and supplying within an abundance of desiccating material.

Figure 151 (*a* and *b*) gives the curves of distribution of temperatures obtained by the above-mentioned observers, when one end of the bar was heated by means of the coil of wire and when the same end was cooled in the freezing mixture.

The question as to the reliability of the readings of the copper collar for the lowest temperatures reached in this experiment is not unimportant. Recent experiments by Messrs. Dewar and Fleming, upon electrical conductivity of metals at temperatures very much lower than those with which we have to deal in this experiment, seem to show that the resistance coefficient of pure copper is fairly constant. A calibration of the copper coil will be found in the paper of Messrs. Child, Quick, and Lanphear, to which reference has just been made. Their measurements seem to confirm the statement of Dewar and Fleming, at least so far as temperatures to $-60°$ are concerned.

D. *Determination of the curve of cooling.*

The essential features of the method of obtaining the curve of cooling of such a bar as we have under consideration have been indicated in describing the method of getting the specific heat.

The bar should be heated by means of a spiral coil, such as has been described in that paragraph. The advantage of using such a coil lies not only in the fact that by means of it any desired temperature may be obtained and maintained, but also in the fact that the surface of the bar, the radiating power of which will necessarily be modified by contact with any liquid bath, is not vitiated by the heating coil. After the bar has reached its temperature within the coil, the latter may be removed, and temperature determinations by means of the collar may be begun at once. Figure 152 gives the curve of cooling respectively for the copper bar already referred to as deter-

mined by this method by Messrs. Child and Quick in the spring of 1893, and by Messrs. Quick and Lanphear in the following year. In the latter case it was found to be impracticable to cool the entire bar to $-70°$. A short piece of the same cross-section was, however, cooled, and its curve of heat

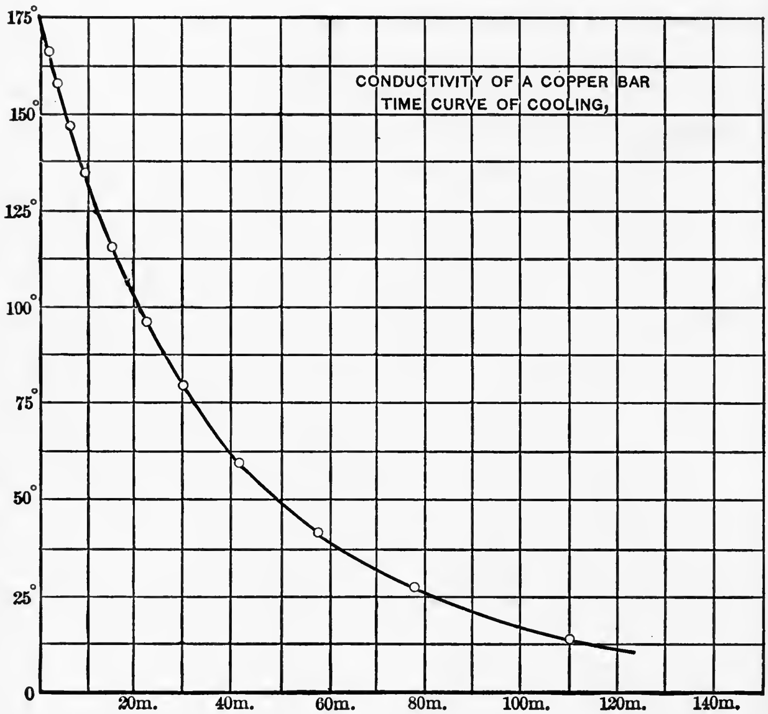


Fig. 152.

absorption was determined. The same piece was then heated, and its curve of cooling plotted. These two curves were found to be exact counterparts, and it was assumed that the corresponding curves for the whole bar would bear the same simple relationship to one another. In computing the conductivity, therefore, the curve of cooling (Fig. 152) was used with ordinates reversed.

The result of the investigation was to show conductivity a function of the temperature and rising in value throughout the entire range from $-60°$ to $+160°$. Fig. 153 shows the conductivity between $-55°$ and $-13°$ C.

Computation of the results of the measurements outlined here involves no principles other than those which have been given

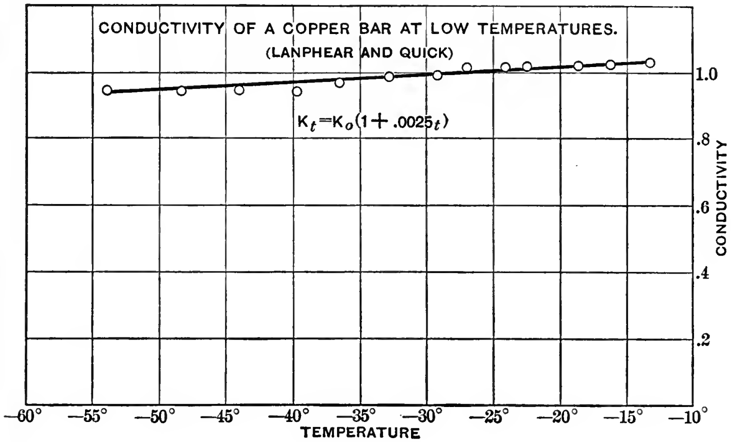


Fig. 153.

by various writers. The following are some of the most important memoirs upon this subject.

FOURIER: Theorie de la Chaleur.

POISSON: Theorie Mathematique de la Chaleur.

ÅNGSTRÖM: Phil. Mag. (4). Vol. 25, p. 130.

FORBES: Trans. Royal Soc. of Edinburgh. Vol. 23, p. 133; Vol. 24, p. 73.

WIEDEMANN AND FRANZ: Poggendorff's Annalen. Vol. 142, p. 1; Vol. 165, p. 497; Vol. 171, p. 497; Vol. 184, p. 393.

TAIT: Trans. Royal Soc. of Edinburgh. Vol. 28, p. 717.

MITCHELL: Trans. Royal Soc. of Edinburgh. Vol. 33, p. 535.

LODGE: Phil. Mag. (5). Vol. 7, pp. 198 and 251; Vol. 8, p. 510.

CALVERT AND JOHNSON: Phil. Mag. (5). Vol. 8, p. 551; Vol. 17, p. 214; Vol. 28, p. 429. Philos. Trans. Vol. 148, p. 349.

(For further references, see Thesis of Quick and Lanphear, Cornell University Library, 1894.)

IV.

The influence of temperature on the volume of liquid mixtures.

It is a well-known fact that when certain liquids are mixed together, as, for example, water and alcohol, the volume of the mixture is less than the sum of the volumes of the two components. By mixing the two liquids in various proportions, and noting the diminution in volume, a curve can be plotted giving

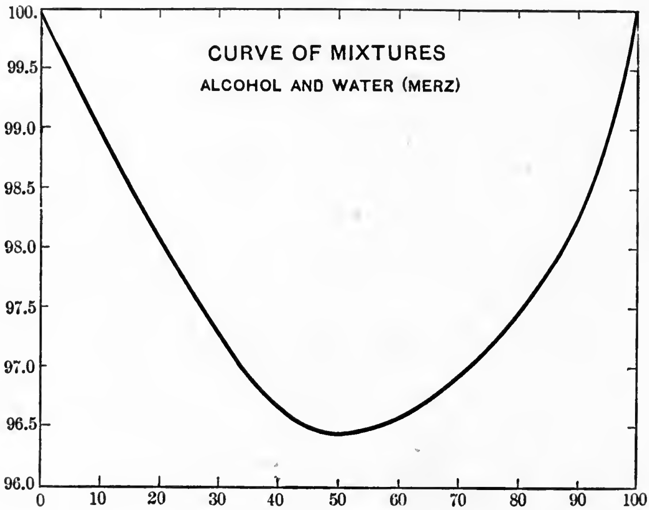


Fig. 154.

volume of the mixture in terms of the proportions in which the two liquids are combined. Figure 154, which is from measurements made by Mr. August Merz, under the writer's direction, shows the character of the curve. This curve itself is, moreover, a function of the temperature at which the mixture takes place, and the problem under consideration consists in tracing the temperature changes which it undergoes. Figure 155 shows the variations in volume of a mixture of equal parts of alcohol and water at temperatures from 0° to 30°, according to Kreitling.*

* Wiedemann's Beiblätter, 1894, p. 58.

Measurements by the same writer seem to show that the shrinkage of the 20 per cent mixture is the same at all temperatures.

The method to be pursued is that of the determination of densities by weighing in a specific gravity bottle. Since the number of such weighings will of necessity be large, it is advantageous to determine with great care the curve of sensitiveness of the balance, and to make all subsequent measurements by

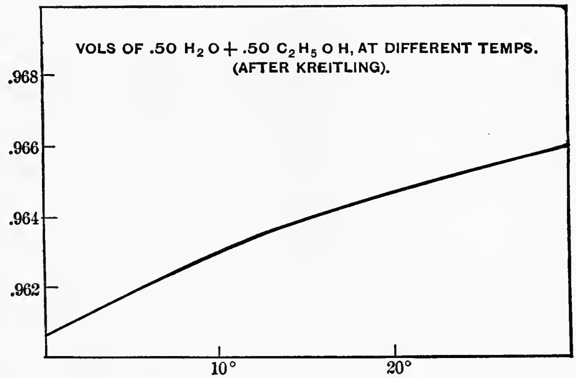


Fig. 155.

the method of vibrations, using this curve to compute the weights.

The curve of sensitiveness is to be determined by placing upon the scale-pans equal loads, noting the position of the pointer by the method of vibrations, adding one, two, or five milligrams, according to the delicacy of the balance, to one side, and determining the change in the zero point produced by this addition. This process is to be repeated with loads ranging from zero grams to a load considerably in excess of that with which one will have to do in subsequent experiments.

It will be found in the case of many sensitive balances that the curve rises to a distinct maximum at some point corresponding to some small load and then falls off slowly until the balance is loaded to its full capacity. Figure 156, which is the curve of sensitiveness of a Sartorius balance, may be regarded as typical.

The determinations of density in this experiment should be made with great accuracy. It is therefore necessary to select a specific gravity bottle the stopper of which is well fitted, and to determine the coefficient of expansion of the glass by filling the flask with mercury at three or four different temperatures and weighing with great care. A small-sized bottle is to be pre-

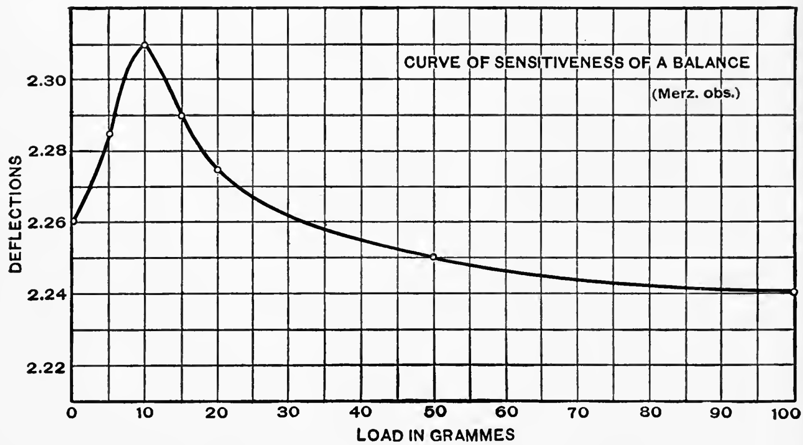


Fig. 156.

ferred in all respects. In the first place, the amount of mercury which it contains will not overload sensitive balances; in the second place, temperature control in a small flask is a much simpler matter than in bottles of large capacity, and equilibrium of temperature is much sooner reached in flasks of small size.

V.

Influence of temperature upon the volume of substances in the neighborhood of the melting-point.

The coefficient of expansion of both solids and liquids usually undergoes marked change, just before the melting point of the substance is reached. In those cases in which the difference in volume produced by melting is marked, this phenomenon is specially noticeable. The work proposed

under this head consists in tracing the change of volume in such cases and in plotting curves, showing the behavior of the material in this respect both above and below the melting-point. Kopp has made determinations of this kind in the case of such substances as beeswax, phosphorus, and sulphur. For materials which in the solid state are not soluble in water, and which, when liquefied, do not mix with water, and the melting-point of which, moreover, lies

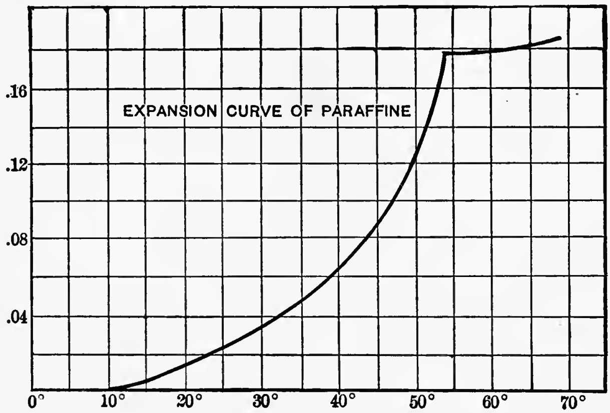


Fig. 157.

between 0° and 100°, this determination may be made by partly filling a dilatometer bulb with the substance to be tested and the remainder with water. In some cases mercury should be used instead of that liquid. By carrying the dilatometer through a range of temperature which includes the melting-point, and by making determinations of the amount of liquid expelled as the temperature rises, stepwise, the changes in volume may be plotted.

In order to compute these volume changes, it is necessary to know with some degree of accuracy the coefficient of expansion of the glass, a quantity which should have been previously determined by one of the usual methods.

Figure 157 shows the volume curve for paraffine, obtained under the writer's direction by Mr. Bliss, in 1894. The differ-

ence in volume between molten and solid paraffine is very marked, and it will be noticed by the inspection of this curve that the change which takes place occurs by continuous increase in the coefficient of expansion in the solid as the temperature rises, and not by an abrupt change from a curve to a vertical straight line as in the case of the vaporization of liquids at their boiling-point.

In the transition state many substances present interesting and important phenomena; and their behavior, under varying conditions when subject to temperature changes very near the melting-point, is worthy of careful study. An important addition to our present knowledge might be made by determining the curve of some substance which changes volume greatly upon melting, repeating the observations through a range which would include the change from the solid to the liquid state for different pressures. Thus the relation of phenomena due to changes of pressure to the changes which occur when temperature is the variable might be definitely determined.

VI.

The influence of temperature upon the color of pigments.

Change of color with temperature is a phenomenon which has long been known. Houston and Thomson, in 1871, observed the change of hue of a great number of pigments when heated. Ackroyd, in 1876, using the spectroscope, ascertained that the change of color was due to the change of absorption of light with rise of temperature, and that the absorption increased at a greater rate in one part of the spectrum than in another.

In 1891 the writer, in collaboration with B. W. Snow, made measurements of the spectra of a considerable number of pigments at temperatures ranging from 25° to the red heat. The results which were obtained were in the main corroborative of Ackroyd's statements, but the phenomenon was found to be more complicated than his observations would make it appear.

Figures 158, 159, 160, and 161 show typical results obtained by the method pursued in our investigation.

The measurements were made by means of the apparatus shown in Fig. 162, which consists of a one-prism spectroscope before the slit of which is placed a set of four rectangular reflection prisms (Fig. 163), by means of which light can be intro-

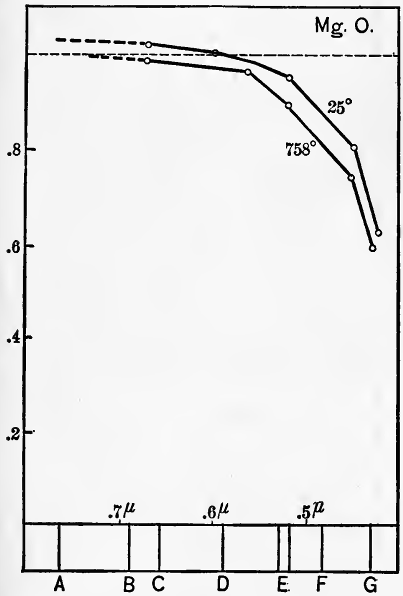


Fig. 158.

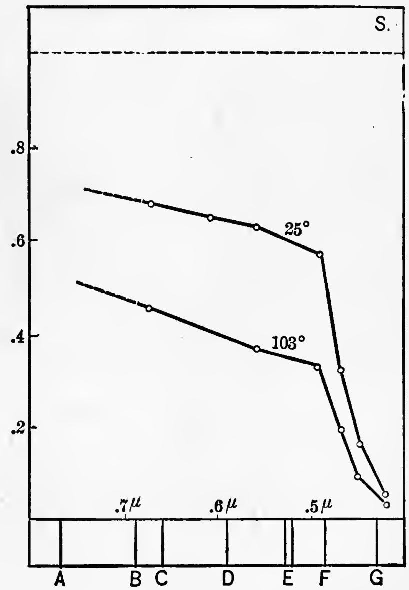


Fig. 159.

duced to the upper and lower half of the slit, respectively, from the right and left hand. At the right hand of the observer is placed an incandescent lamp, *L*, which serves as a comparison standard. The rays from this are rendered parallel by means of a lens, *A*, then passing through a pair of Nicol's prisms, *N* and *N′*, the former of which is capable of rotation upon an axis parallel to the path of the ray. The rays of the lamp, *L*, which are thus capable of being reduced in brightness by the adjustment of the angle between the polarizing planes of the Nicol's

prisms, enter the lower half of the slit and form a spectrum which fills half the field of the observing telescope.

To the left of the collimator tube was mounted a strip of platinum foil, TT', through which an electric current from a storage battery could be sent. The face of the foil on the side towards the spectroscope was coated with the pigment to be

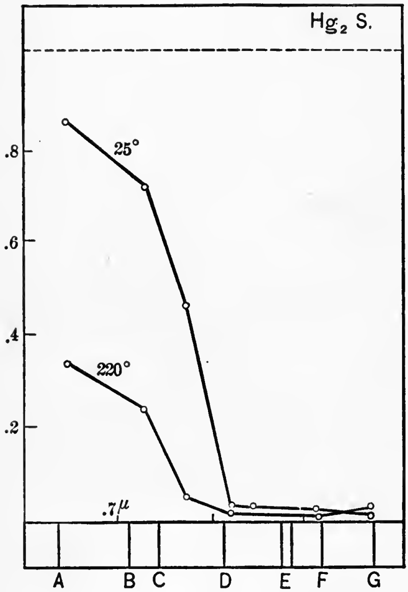


Fig. 160.

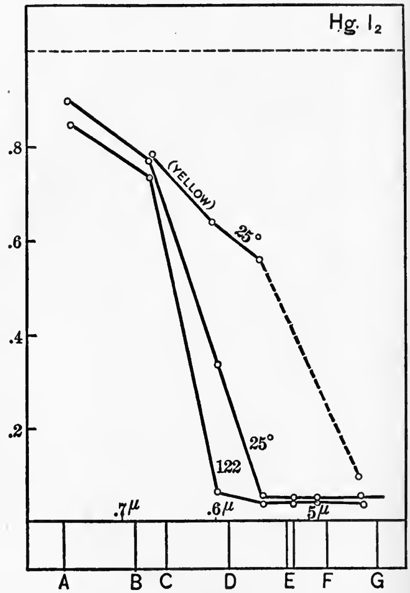


Fig. 161.

studied, and this coating was illuminated by means of an incandescent lamp, L'. Upon the back of the strip of foil two lines, *mm*, were ruled with a diamond, and two reading microscopes were mounted in such a position that the images of these lines appeared in the fields of view. By means of the movement of these images in the eye-pieces of the two microscopes, which are provided with micrometers, the temperature of the platinum foil could be determined.

The experiment consisted in comparing the spectrum of the light reflected by the pigment coating with the spectrum from

the comparison lamp L, for which purpose the two were observed, wave-length by wave-length, from red to violet. The spectrum of the comparison standard L was reduced at each observation to the same brightness as that of the pigment.

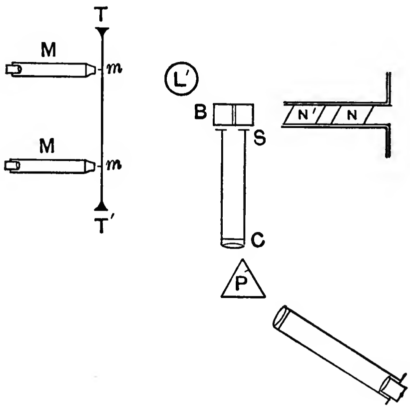


Fig. 162.

One of the forms of spectrophotometer described in Chapter IV. would serve fully as well for making measurements of the kind under consideration here as this instrument. When a photometer like that indicated in Fig. 162 is made use of, or indeed in the case of any spectrophotometer in which complete symmetry of the parts is not secured, it becomes necessary to correct the measurements for the absorption of light in the lens A and in the pair of Nicol's prisms. This correction is required, since the rays reflected from the surface of the pigment do not pass through these media.

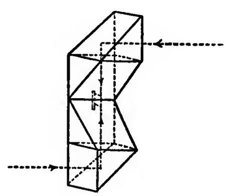


Fig. 163.

To obtain the value of the correction factor, two white surfaces, identical as to character, are obtained. A simple plan consists in cutting in two lengthwise a slab of compressed magnesium carbonate and using the pieces thus prepared as comparison surfaces. One of these is mounted in the place of

the lamp L, and the other in place of the platinum strip. They are illuminated respectively by the lamps L and L', the same being placed at distances such as to give a proper relative brightness to the two spectra. Were the rays from these two surfaces of magnesium carbonate to suffer precisely the same absorption in their paths to the eye, the spectra obtained would be identical in character. Calcite, however, and also optical glass, absorb the rays of the visible spectroscope selectively (see Chapter IV.).

By comparing the brightness of these two spectra, wave-length by wave-length, correction factors may be obtained by the use of which it is possible to eliminate the influence of selective absorption upon the measurement of the spectrum of the pigment, when the latter is placed in comparison with the direct rays of the lamp L. The curves shown in Figs. 158, 159, 160, and 161 were obtained in the manner above described. It will be seen that in all cases rise of temperature is accompanied by diminution of brightness, and that since the falling off in reflecting power does not occur uniformly throughout the spectrum, a change of color is the result.

A wide field of research offers itself to the student of this class of phenomena. The studies of pigments already made are very incomplete and fragmentary. Many substances have not been subjected to investigation at all, and some of these are of peculiar interest. The oxides of the metals, for example, such as zinc oxide and other substances which become fluorescent by heat, afford an admirable theme for the student of spectro-photometry. Both radiation and the reflection from such surfaces remain to be investigated as a function, both of time and temperature. Another important field of investigation consists in the study of the color of pigments at low temperatures. It has been observed that there is a general tendency of colors towards the white as the temperature falls. No quantitative results have as yet been obtained, but it would be an entirely practicable thing to extend measurements at least to temperatures as low as that of solid carbon dioxide.

Some references to memoirs on the spectra of pigments:

SCHÖNBEIN: Poggendorff's Annalen. Vol. 45, p. 263.
HOUSTON AND THOMSON: Journal of the Franklin Institute (3). Vol. 62, p. 115.
ACKROYD: Chemical News. Vol. 34, p. 76.
NICHOLS: American Journal of Science. Vol. 28, p. 343.
ABNEY AND FESTING: Philos. Transactions. Vol. 179, p. 549.
ABNEY: Colour Photometry. Chapter VIII.
NICHOLS AND SNOW: Philos. Mag. (5). Vol. 32, p. 401.

VII.

The influence of temperature upon the transparency of solutions.

That the solutions of many salts change color upon heating is a familiar matter to chemists, but comparatively few quantitative determinations have been described previous to those made under the writer's direction by Miss Spencer,

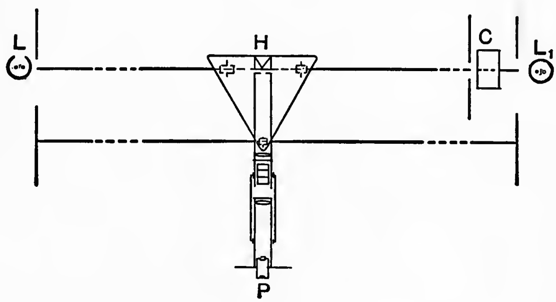


Fig. 164.

1893–94.* A suitable instrument for such measurements is the horizontal-slit photometer, for a full description of which see Chapter IV. The arrangement of the apparatus is shown in Fig. 164, in which L and L_1 are glow lamps, as nearly identical in quality as possible, placed at the ends of a photometer bar which is divided into 1000 parts. HP is the spectrophotometer, which is mounted upon a carriage and is capable of move-

* Physical Review, Vol. 2.

ment along the bar. Light from the sources is introduced into the halves of the horizontal slit by means of a pair of exactly similar rectangular total reflection prisms. The two vertical spectra which appear side by side in the eye-piece of the instrument are brought to equal brightness, wave-length by wave-length, by moving the spectrophotometer along the bar. The solution to be studied is placed in a cylindrical cell the ends of which are faced with plate glass. Between these there is room for a layer of liquid 5 cm. in thickness. The same is introduced through the hole in the side of the cylinder, which latter consists of a brass tube three inches in diameter. The hole is of such size as to admit the bulb of a thermometer by means of which the temperature of the solution can be determined. Figure 165 gives a sketch of this cell.

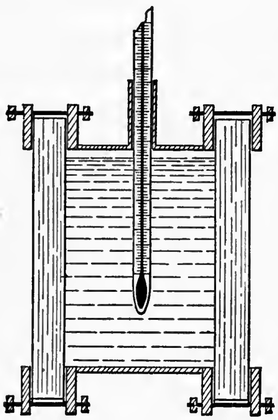


Fig. 165.

The heating of the solution is effected by means of an electric current which traverses a coil of German silver wire wound in a single layer around the cell and separated from the brass walls of the same by a layer of asbestos paper. This coil is not shown in the diagram. By sending a steady current through the coil the solution can be raised to any temperature below its boiling-point and can be maintained very nearly constant so long as the current continues to flow.

The method of comparison used in Miss Spencer's investigation, to which reference has just been made, consisted in measuring the light of each wave-length of the visible spectrum transmitted through the cell when the latter was filled with distilled water, and of comparing with the same the transmitting power, wave-length by wave-length, of the solution which it was desired to study. For this purpose, the telescope of the

spectrophotometer was set so as to bring a given region into the field of view. The alternate readings were made with the cell interposed between the slit and the glow lamp and with the cell removed. During this set of readings the temperature of the solution was that of the room; subsequently the solution was heated by means of a current to various temperatures lying between 80° and room temperature. In this way several regions

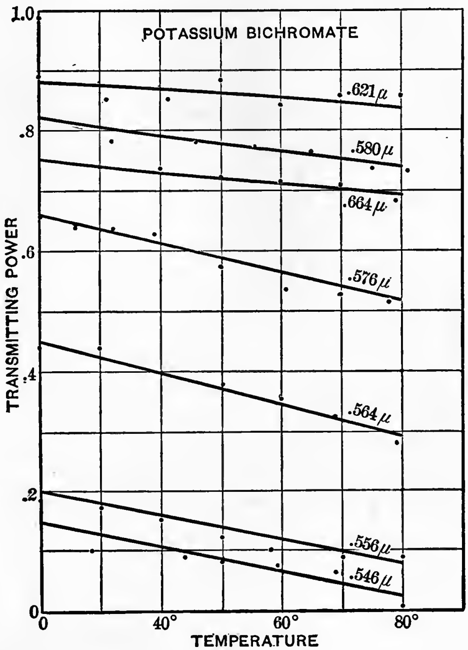


Fig. 166.

distributed at nearly equal distance throughout the spectrum were explored. The results were expressed by means of a set of curves in which temperatures are abscissas, and transmitting powers compared with that of distilled water are ordinates. Figure 166 shows such a curve for a solution of bichromate of potassium. The same results can be expressed graphically in another manner by plotting curves in which wave-lengths are abscissas and transmitting powers are ordinates. Each of these

curves gives the relative distribution of energy in the spectrum for a given temperature (see Fig. 167). It was found as the result of these measurements :

1. That in general the transparency of salt solutions falls off with rise of temperature.

2. That the change in transparency is selective, affecting certain wave-lengths more than others.

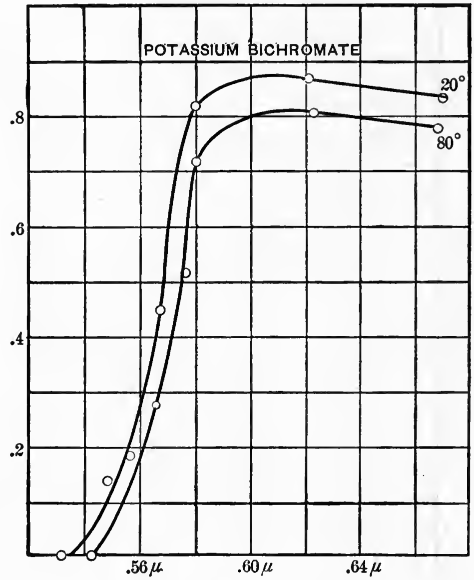


Fig. 167.

3. That certain solutions, such, for example, as nickel sulphate, suffer no change of transmitting power with change of temperature.

4. That water is equally transparent at 20° and at 80°.

5. That many of the most striking changes of color upon heating, are due to permanent changes of chemical composition, the solution not returning to its original condition when cooled again.

6. That in the cases of absorption-spectra, containing opaque bands, the edge of these bands shows movement when the

solution is heated such as to indicate a widening of the band, or perhaps, in other words, increased absorption in those regions which were most nearly opaque when the solution was cold.

There is a broad field for investigation in the study of the influence of substances, both solid and liquid, upon their power of transmitting radiant energy. Concerning the behavior of solids, such as glass, quartz, calcite, etc., but little is known with reference to the influence of temperature, while in the case of solutions, aside from the casual observations made by chemists and a few other incomplete investigations, nothing excepting the experiments just described has been undertaken.

Some references to memoirs upon absorption-spectra :

GLADSTONE: Philos. Mag. (4). Vol. 14, p. 423.
BARTLEY: Proc. Royal Society. Vol. 22, p. 241.
CONROY: Philos. Mag. (5). Vol. 31, p. 317.
BEER: Poggendorff's Annalen. Vol. 86, p. 78.
BUNSEN AND ROSCOE: Poggendorff's Annalen. Vol. 101, p. 242.
ZÖLLNER: Poggendorff's Annalen. Vol. 109, p. 254.
MELDE: Poggendorff's Annalen. Vol. 126, p. 284.
VIERORDT: Die quantitative Spectral Analyse.
GLAN: Wiedemann's Annalen. Vol. 3, p. 54.
SETTEGAST: Wiedemann's Annalen. Vol. 7, p. 242.
HESSE: Wiedemann's Annalen. Vol. 11, p. 871.
WALTER, B.: Wiedemann's Annalen. Vol. 36, p. 518.
PITCHER, F. B.: American Journal of Science. Vol. 36, p. 332.
KRÜSS: Kolorimetrie.
KNOBLAUCH: Wiedemann's Annalen. Vol. 43, p. 738.
EWAN: Philos. Mag. (5). Vol. 33, p. 317.
NICHOLS AND SPENCER: Physical Review. Vol. 2.

CHAPTER II.

STUDIES OF THE EFFICIENCY OF ARTIFICIAL LIGHT-SOURCES.

The term "efficiency," as used in this connection, is a quantity which has been very appropriately called the "radiant efficiency"* of the light-source. It is the ratio, $\frac{\text{light-giving radiation.}}{\text{total radiation}}$ Efficiency thus defined is, of course, a different quantity from that which is sometimes called the "total efficiency" of the source, which is the ratio of the energy of the luminiferous rays to the total energy expended.

The apparatus necessary to perform the experiments to be described consists of a sensitive galvanometer, a thermopile, and certain cells or screens, which may be interposed between the source under investigation and the thermopile.

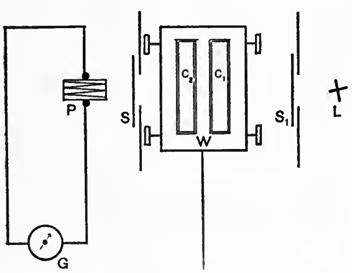


Fig. 168.

The method of operation, which is the same in its essential features, whatever the source of light to be studied, is as follows: A galvanometer (*G*, Fig. 168) having been brought into a suitable state of sensitiveness, is placed in closed circuit with the thermopile (*P*), one face of which is carefully protected from temperature changes, while the other is exposed to the source (*L*), the radiation of which is to be measured, at a distance varying, according to the intensity of the latter, from 50 cm. to 300 or 400 cm. A double screen (SS_1)

* See Rogers, American Journal of Science, Vol. 43, p. 302.

of some non-conducting material is placed between the light-source and the face of the thermopile. This contains apertures which may be readily opened and closed by the observer from his position at the reading telescope. Two cells with plane glass sides (C_1C_2) are also interposed between the thermopile and the source of light. These should be mounted upon a car (W) which travels upon a track, so that they also can be readily withdrawn and replaced. Each of them should contain a layer of distilled water, the thickness of which is

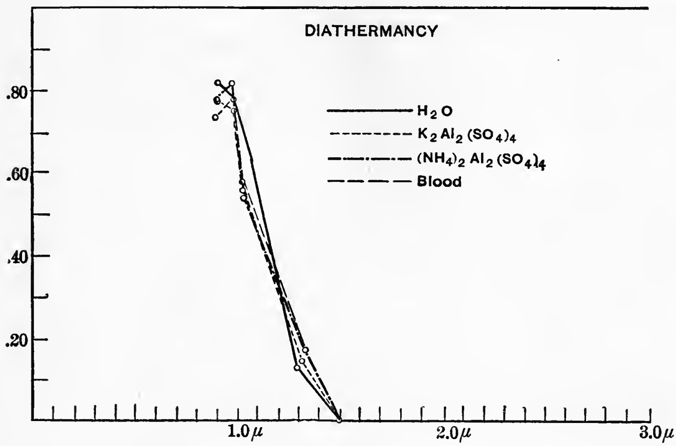


Fig. 169.

about 2 cm. It has been the practice, until recently, in such an experiment to make use of a strong solution of alum in water, but pure water is just as effective as an alum solution for the purpose of cutting off non-luminous radiation. Conclusive evidence on this point is afforded by the curves in Fig. 169, which are from measurements by Ernest F. Nichols.* Ordinates in this diagram are transmitting powers of a glass cell containing, respectively, water and solutions of potassium alum, ammonium alum, and blood. Abscissas are wave-lengths.

* E. F. Nichols, Transmission Spectra of Certain Substances, etc., Physical Review, I., p. 16, 1893.

It will be seen that there is nothing in these measurements to lead to the supposition that the diathermancy of water is modified by the presence of any of these substances in solution.

The apparatus above described being in place, and the galvanometer circuit closed once for all, the mirror is brought to rest at a suitable point upon the scale by means of a controlling magnet in the hands of the observer.

The shutter is now withdrawn during the length of time required for a single swing of the galvanometer, after which the opening is immediately closed again, and the farthest reading of the throw is noted. The point to which the needle returns is also noted. The mean of the original and the return position is taken as the zero point of the instrument, and the deflection is counted from this average.

It has been shown by Merritt* that when this method is followed, the indication of the instrument is always proportional to the final deflection which would be reached were the exposure continued until the thermopile had reached its maximum temperature. The advantages of the method are obvious, since disturbances due to changes in the condition of the light-source are almost entirely eliminated. After these readings have been made, the water-cells are withdrawn, and similar readings are obtained for the total radiation.

A series of such observations are taken alternately as above, a sufficient number of each kind being obtained to give the necessary data for computing the ratio. Since no material exists which is perfectly transparent to all light-giving rays, and totally opaque to non-luminous radiation, it becomes necessary to determine the amount of light cut off by the medium interposed, — viz. by the water-cells with their glass walls, — and to determine also the amount of non-luminous energy which these cells are capable of transmitting.

The first correction is to be obtained by placing the cells in

---

* Merritt, American Journal of Science, Vol. 41, p. 417.

question in front of a similar source of light at one end of the photometer bar, and making photometric comparisons between this source with the cells interposed and another source of light. The latter should be as nearly as possible of the same quality as regards color. The cells are then removed, and a second comparison is made between the sources. From a considerable number of such readings, alternately with and without the interposed cells, the proportion of light cut off by them can be determined with sufficient accuracy.

The following data, culled from various researches in which this method has been used, may be of service:

Authority.	Absorbing Medium.	Transparency.
E. Merritt, 1886–88	Water of calorimeter *	0.70 to 0.75
" "	One alum cell	0.70 to 0.75
H. Nakano, 1889	" " "	0.74
Ida Hill, 1890	" " "	0.74
F. J. Rogers, 1892	Two alum cells	0.641

The other correction is a more difficult one. The best method of making it is probably by means of a cell containing a solution of iodine in carbon bisulphide, this solution being of such density that when one looks at the disk of the sun through the cell, the outline of the sun can scarcely be followed. The transparency of such an iodine solution to wave-lengths greater than 0.8 μ is unfortunately not accurately known.

Some of the results which have been obtained by this method will be given in the discussion of the applications of the method of various light sources. The method has already been applied to the incandescent lamp, the arc-lamp, the magnesium lamp, to the various forms of the Drummond light, using lime, zircon, and magnesia, to the Auer incandescent burner, to various gas flames and petroleum flames, and to the English and German standard candles. The study of each of

* See the following section upon the calorimetric method for radiant efficiencies.

these light sources will be taken up in turn; and the modifications of the methods just outlined, necessary in the application of it to them individually, will be considered.

I.

Efficiency of the incandescent lamp.

The incandescent lamp lends itself especially to studies of radiant efficiency from the fact that its radiation is under control, being capable of maintenance at nearly constant temperature. It is at the same time capable of a wide range as regards the degree and character of incandescence, and can be carried through that range at will by variations of the voltage.

The best form to give the experiment is to determine the ratio of light-giving radiation to the total radiation by the method already outlined, repeating the measurements for a series of different temperatures. In order to express the results obtained in a definite manner, readings should be made of the current flowing through the filament and of the difference of potential between the terminals of the lamp.

The best source of energy is a storage battery, the electromotive force of which should be sufficient to give a difference of potential in excess of that required by the lamp at the highest degree of incandescence to which it is safe to bring the latter. Suitable resistance placed in the circuit will then enable the experimenter to bring the lamp to any required state and to maintain it there throughout an entire series of observations with the thermopile and galvanometer. By varying the temperature of the lamp step-wise, and by determining for each step the radiant efficiency, one can then plot curves expressing the change of efficiency with voltage, and what is more interesting, the relationship between radiant efficiency and the efficiency expressed in the arbitrary terms of watts per candle.

The only fundamental difficulty connected with this work lies in the depreciation of the incandescent lamp with time

(see Chapter III.). Such depreciation, when the lamp is held at abnormally high voltages, is unfortunately very rapid; and since there is no method of restoring an aged lamp to its original condition, special care must be taken to reduce this source of error to the minimum. To this end all preliminary adjustments of the apparatus should be made, using a lamp which is not to be subjected to the ultimate tests. After the experimental details are entirely within the observer's control, the lamp to be tested may be substituted for that used in the preliminary experiments. Measurements should begin with the lower voltages, and as the temperature of the lamp is increased, great care

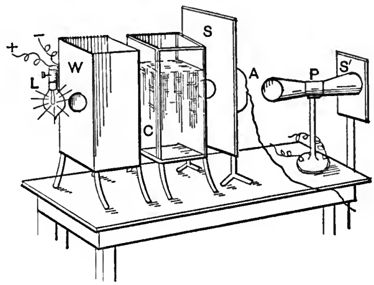


Fig. 170.

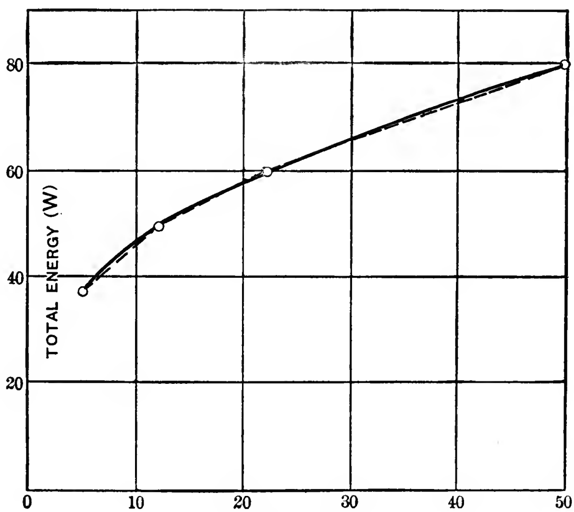


Fig. 171.

should be taken to reduce the time during which it is burning, as much as possible. In this way excellent results may be

obtained; but it will be found, if the determination be carried to extreme values, that the condition of the lamp has begun to show depreciation. Measurements of the radiant efficiency of incandescent lamps were made by Ernest Merritt, following the method already described, in 1888.* Figure 170 shows his apparatus.

In 1892 similar measurements were made, under the direction of the writer, by J. C. Shedd.† The results obtained by that observer in the case of a 16 candle-power lamp of 50

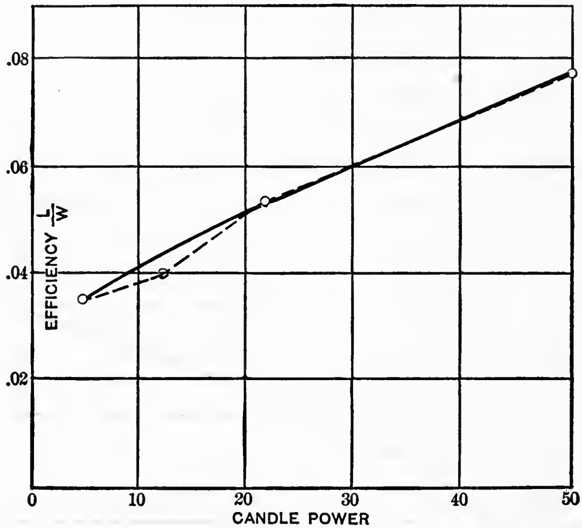


Fig. 172.

volts, the filament of which had been treated in hydrocarbon vapor in the process of manufacture, may be regarded as typical.

They are represented graphically in Figs. 171, 172, and 173. The first of these (Fig. 171) gives the relation between total radiated energy (W) and candle-power; the second (Fig. 172),

* Ernest Merritt, American Journal of Science, Vol. 37, p. 167.

† Shedd, A Comparative Study of Treated and Untreated Carbon Filaments, etc., etc. Thesis in the library of Cornell University, 1892.

the relation of radiant efficiency $\left(\frac{L}{W}=\frac{\text{light rays}}{\text{total rays}}\right)$ to candle power. The third (Fig. 173.) has total energy of radiation (W) as abscissas, and the efficiency $\left(\frac{L}{W}\right)$ as ordinates.

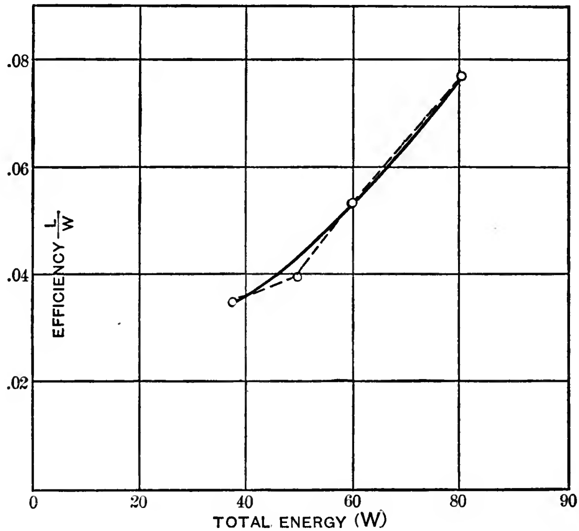


Fig. 173.

The calorimetric method for radiant efficiencies.

Owing to the fact that the filament of the incandescent lamp is entirely enclosed in glass and needs no supply of oxygen, it is possible to bring it to incandescence under water. Thus it is easy to determine the radiant efficiency by another very interesting method which was described by Merritt in the paper just referred to, and which was carried out by him in collaboration with S. Ryder as early as 1886.

This, which may properly be termed the calorimetric method, is as follows: A glass calorimeter was provided, within which the lamp was placed. The calorimeter was supplied with water from a large carboy *D* (Fig. 174) by means of a siphon tube, the lower end of which terminated in an opening near the base of the calorimeter. A second opening, or overflow, at the top of

the calorimeter discharged liquid into a beaker. The influx and efflux tubes were enlarged just outside the calorimeter, so as to admit the insertion of thermometers at E and F. The calorimeter in question was cylindrical in form. It contained 10,000 c.c. of water.

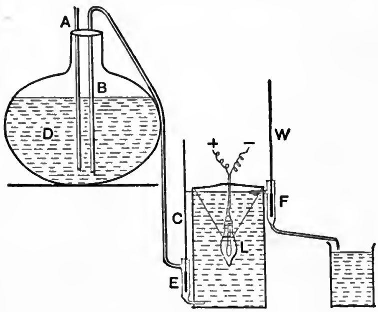


Fig. 174.

The lamp having been brought to the desired temperature by means of current, distilled water was made to flow from the carboy through the calorimeter, and was collected by means of the beaker indicated in the figure. Of the light-giving radiation from the lamp, a large proportion was transmitted by the water, and by the glass walls of the calorimeter, and escaped. The calorimeter and its contents were, however, nearly opaque to the longer wave-lengths, and these were converted into heat. The result was a rise of temperature which showed itself by means of the difference in the readings of the two thermometers.

The success of the experiment depended upon the maintenance of a constant rate of flow throughout the experiment. Under these conditions, assuming the lamp to be supplied from a battery of constant potential, the difference in temperature between the water entering and that leaving the calorimeter affords a measure of the heat delivered by the lamp to the water. The results were computed as follows (see Merritt's article, p. 169):

Let t be the time occupied by the experiment;

T_1, the temperature of the water entering;

T_2, the temperature of the water leaving;

F, the rate of flow of the water;

R, the loss of heat by radiation.

H, the heat absorbed by the water, is then expressed by the following equation:

$$H=[T_2-T_1]Ft+R.$$

Of these two methods, the one of the thermopile and galvanometer is unquestionably to be preferred. It is more rapid than the calorimetric method, and under proper conditions it is more accurate.

Although a number of studies of the incandescent lamp have been made, there remains still much of interest to be done. A comparison of gray and black filaments, with reference to radiant efficiency, might lead to important results. The question of the influence of the vacuum upon the efficiency also remains to be definitely determined, and a comparison of lamps of high vacuum with those in which an inert gas surrounds the filament would be of considerable interest.

II.

Efficiency of the arc-lamp.

In studying the radiant efficiency of the arc-lamp, a factor enters with which we do not have to deal in the investigation of other artificial light sources. The distribution of light from the continuous current arc-lamp is a peculiar one, and the radiant efficiency obtained by measurements with the thermopile and alum cells will be found to vary considerably according to the angle which the line joining the arc and the face of the pile makes with the horizontal plane. No satisfactory investigation can be completed, therefore, without some arrangement for taking measurements in a variety of positions. For this purpose the apparatus described by Nakano,* by whom the first measurements of this kind were carried out, may be used. This consists of a wooden platform about 1 m. long (see Fig. 175) and 20 cm. in breadth. The same is pivoted at *c* upon a horizontal axis. At one end the lamp is suspended in a wooden frame *bb*, which, by means of a simple, jointed parallel motion device maintains a vertical position whatever be the angle at which the platform is placed. At convenient positions at the other end of

* Nakano, American Institute of Electrical Engineers, Vol. 6, p. 308 (1889).

this platform are mounted the screens, the alum cells, and the thermopile.

The use of the galvanometer and the thermopile are the same as in the other experiments already described; but the difficulties of obtaining satisfactory readings are considerably increased by the fluctuating nature of the arc-light, which runs through a very wide range of intensities. On this account a large number of readings must be taken in order to secure a satisfactory result. Much depends upon the quality of carbons used.

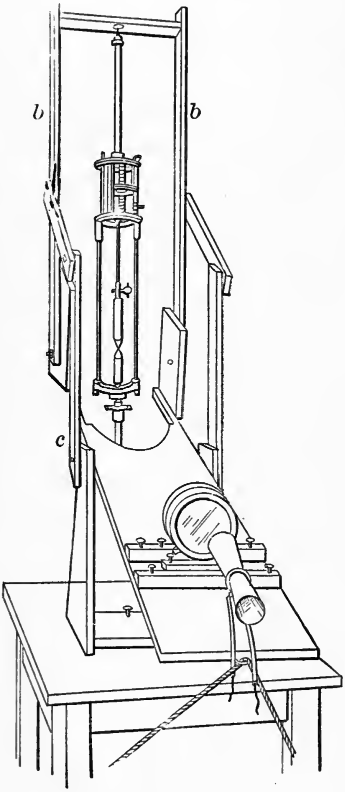


Fig. 175.

A large number of measurements has been made with this apparatus by Nakano and by Marks, under a variety of conditions. Figures 176 and 177 show the graphical method of indicating results in a typical case taken from Nakano's paper. The former gives the total radiation W, at different angles below the horizontal plane and the radiation through the alum cell L (used by Nakano as an absorbent of the dark heat instead of water cells), plotted to the same scale. The ratios of these areas is a quantity which has been called the *mean spherical efficiency*. This ratio, in the case of the lamp to which these curves apply, a continuous current arc with plated carbons 11.5 mm. in diameter (9 amperes and 45 volts), is 0.133.

The radiant efficiency, however, computed from the total and

light-giving radiation in a given direction, will be found to be a function of the angle with the horizon, and the variation may be expressed in a diagram with polar co-ordinates (Fig. 177). Attempts have been made to extend this method to the study of the radiant efficiency of alternate current arc-lamps, but thus

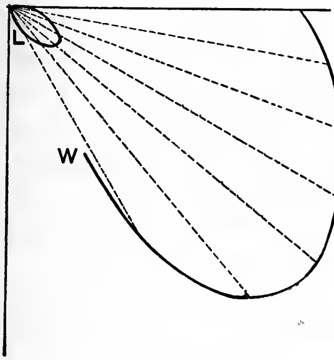


Fig. 176.

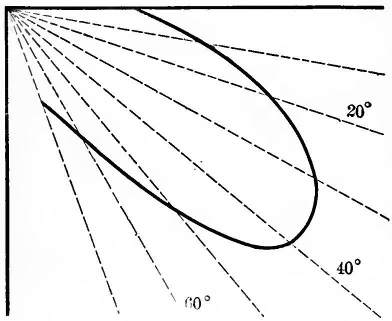


Fig 177.

far without success. The distribution of light from such lamps varies so rapidly as to make the application of the method very difficult.

By using two galvanometers and two bolometers of small mass, one of which is naked while the other is screened by water cells, interesting results could doubtless be obtained with the alternating arc. The best method would probably be, to use the galvanometers ballistically on closed circuit, exposing them simultaneously during a single swing by the removal and replacing of opaque shutters.*

* See Nichols, The Galvanometer, Lecture 9; also Chapters III. and IV. of this part of the Manual.

III.

Efficiency of the magnesium light.

Light of burning magnesium affords a source, the study of the efficiency of which is of peculiar interest on account of the unusual character of its radiation.

Magnesium ribbon when ignited attains to the degree of incandescence corresponding to an extremely high temperature. Pickering,* estimating this temperature from the nature of the spectrum of the magnesium flame, placed it at 4900° C. Measurements of the actual temperature of the incandescent material, however, show that the temperature of this source lies about midway between that of the Bunsen burner and that of the ordinary air blast lamp; viz. about 1350° C.

Since the radiant efficiency of a source of light rises as its incandescence becomes more intense, one would expect from the character of the magnesium light, which excels in whiteness all artificial sources (see Chapter IV.), a high efficiency. The subject has been investigated by Mr. F. J. Rogers,† whose value of the efficiency from eighteen sets of observations is 0.135.

The difficulties involved in the study of this source are chiefly due to the fact that no device has as yet been found by means of which to secure a uniform and steady combustion of the magnesium ribbon. Under present conditions its fluctuations in brightness are very great; greater, probably, even than those of an arc-lamp. The use of a diaphragm, of such size that its opening as viewed from the position of the face of the thermopile will always present a field entirely filled with radiant surface, affords the best means of counteracting the fluctuations of the flame, but the errors due to these fluctuations can be overcome only by increasing the number of individual observations and taking the average of a long series of readings.

---

* Proceedings of the American Academy of Arts and Science, 1879–1880, p. 236.
† American Journal of Science, Vol. 43, p. 301.

It is probable that the plan suggested for the study of the alternating arc-lamp (Section II. of this chapter)—viz. of using two galvanometers and two bolometers simultaneously, one for the measurement of the radiation transmitted by the water cells and the other for direct radiation—would entirely obviate the difficulties which arise from the vagaries of this source of light.

IV.

Efficiency of the Drummond light.

The method to be pursued with this source is similar to those already indicated. The matter is complicated, however, by the fact that the incandescent cylinder is never brought to a uniform temperature by the action of the oxyhydrogen flame. As in the case of the arc-lamp, therefore, there is a great range of incandescence. The hottest point lies directly in the axis of the flame, from which region outwards there is gradation of temperatures to the coldest regions at the base of the cylinder.

The determination of the radiant efficiency of the Drummond light, therefore, will give slightly different values, according to the manner in which the experiments are performed. By the use of a diaphragm, on the one hand, it is possible to isolate the radiation from the parts of highest temperature; while without a diaphragm, radiation is necessarily received from surfaces varying from the brilliant whiteness of this central region to temperatures far below the red heat.

It is characteristic of this source of light, moreover, that its brightness falls off steadily from the moment of first ignition; consequently, experiments made with freshly ignited lime or magnesia cylinders will not apply to those which have been subjected to the action of the flame for a longer time. The changes in question are not confined altogether to total brightness; they affect also in some degree the quality of the light.

Careful determinations of the radiant efficiency of the Drum-

mond light have been made under the writer's direction, by Miss Mary Crehore,* some data from whose work is incorporated below.

Miss Crehore's results, so far as they relate to the efficiency of the lime light, are indicated by Fig. 178, in which the falling

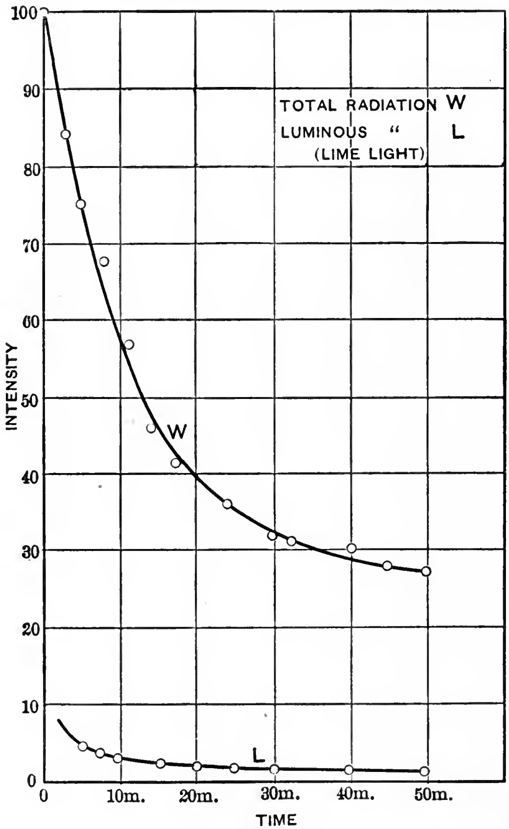


Fig. 178.

off of total radiation W and of light-giving radiation L from the moment of ignition are shown graphically. The ratios of these values, $\frac{L}{W}$, is plotted in Fig. 179 as a function of the time.

---

* See Physical Review, Vol. 2.

This ratio which is corrected for diathermancy and absorption gives the radiant efficiency of the source.

The method by means of which these curves were obtained was that of the thermopile with intervening water cells. A diaphragm placed before the glowing lime cylinder screened all

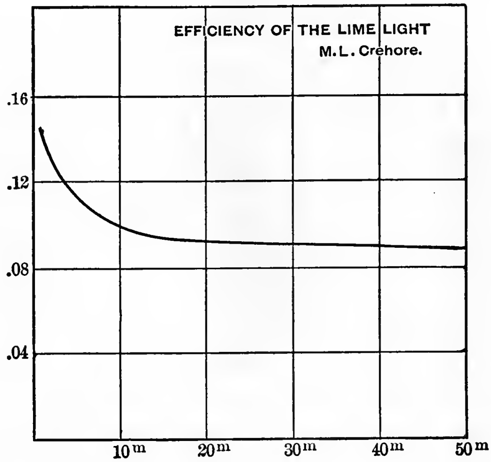


Fig. 179.

but the hottest portions of the latter, and it is to the radiant efficiency of these highly incandescent regions, taken by themselves, that the curves apply.

It will be seen that after thirty minutes the lime reaches a nearly stable condition of incandescence, in which its total radiating power is less than one-third of the initial value. The ratio of total radiation to radiation which passes the water cells (corrected for diathermancy and losses in transmission) is, after thirty minutes, 0.086, having fallen from an initial value of about 0.14.

An inspection of the time curves of visible radiation, for which see Chapter IV. (Fig. 210), shows that in the region of the *D* line radiation falls off in thirty minutes to one-sixth of the initial value.

Efficiency studies of the Drummond light with a magnesia cylinder, also by Miss Crehore, gave results similar to the above, excepting that decadence with time was less marked (see further Chapters III. and IV.).

V.

Efficiency of various other sources of light.

A study of the radiant efficiency of naked flames of gas and oil and of the candle may be readily carried on, using the method already described in the discussion of the incandescent lamp. The very earliest of such researches upon the energy of light sources were made by Julius Thomsen.* Measurements have been made in the case of candles and gas, by more recent observers also, among others by Rogers in the paper just cited. Langley † determined the radiant efficiency of the argand burner by comparison of the areas of the energy-curve of those portions of the spectrum which are luminous and obscure. The values obtained by these observers ranged between 0.012 and 0.024. Further study of these sources is much needed.

Thomsen found the efficiency of a petroleum flame to be 0.02; Rogers found for the candle, 0.0153; for a "bat's wing" burner, 0.0128. Measurements of an argand burner, by the latter observer, gave as the radiant efficiency 0.0161, while Langley, estimating the efficiency by the integration of areas enclosed by the intensity curves of the luminous and non-luminous portions of the spectrum, respectively, found for the last-named flame 0.024. Measurements of the radiant energy of chimneyed flames, however, are of little significance unless the energy of the long waves from the heated glass is taken into account.

---

* Poggendorff's Annalen, Vol. 125, p. 348.

† Science, Vol. 1, p. 483.

It is possible to eliminate the influence of the chimney by using a bolometer of small mass. The method is as follows:

After the determination of the total and luminous radiation by the method of the water cell, the lamp under investigation is extinguished at a carefully observed time. A time curve of the cooling of the chimney is then made by means of the bolometer and galvanometer, and this curve is extended by extrapolation to the time of the extinction of the flame. Such a curve was made by Messrs. Sharp and Turnbull in 1894, in the course of some experiments upon the Carcel lamp (see Fig. 180), but the method has not been applied to other efficiency determinations.

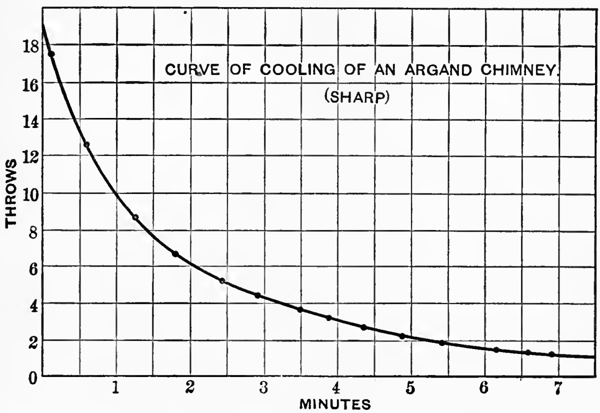


Fig. 180.

A very interesting study of efficiencies might be made, taking the incandescent mantle gas-burner of Auer as the subject.

Preliminary measurements of this source were made, indeed, at the writer's suggestion, by Miss Ida M. Hill.* Her investigations showed that the efficiency is a function of the gas pressure, and also of the age of the mantle. Although her measurements are open to the objection that the radiation from the chimney was not taken into account, they are of interest as showing that mantles giving a white light are distinctly superior

* Thesis in the library of Cornell University, 1890.

in efficiency to those in which yellow rays predominate; that new mantles are more efficient than those which have been in use for a considerable time, and that the efficiency of a *yellow light* mantle rises steadily with increasing supply of gas from 0.019 at 3 feet to 0.028 at 5½ feet per hour.

The results obtained by Miss Hill concerning the last-named relation are shown in Fig. 181. In this diagram abscissas are gas supplies, and ordinates are efficiencies.

In addition to the researches of which some account has been given in this chapter, and to the early investigations of

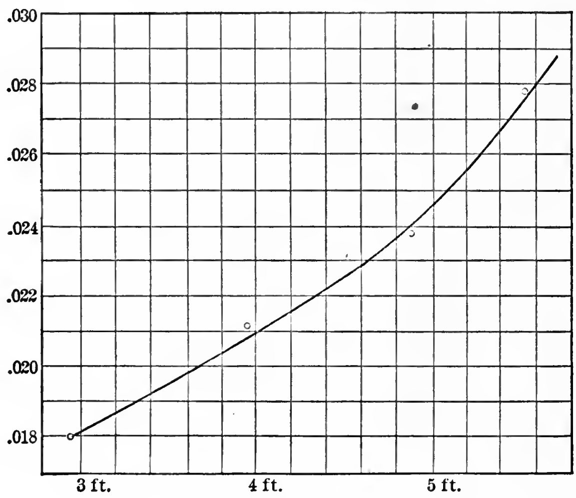


Fig. 181.

Melloni and of Tyndall, measurements of radiant efficiency have been made in the case of the electric discharge *in vacuo* by Blattner and by Ångström, and in the case of the light of the Cuban fire-fly (Pyrophorus noctilucus) by Langley. The latter writer, indeed, has studied many sources, including the light of the sun, and he has made use of a more complete, though more difficult, method than that which has been outlined here. I refer to the method of plotting energy curves for the entire spectrum.

References to the literature relating to radiant efficiency:

MELLONI: Thermochrose. 1850.

THOMSEN: Poggendorff's Annalen. Vol. 125, p. 348.

TYNDALL: Heat a Mode of Motion. 1868. (See particularly the appendix to Lecture XII.)

LANGLEY: Professional Papers of the Signal Service, No. 15. (Memoirs of National Acad. Sciences, Vol. IV; Science, Vol. 1, p. 483; American Journal of Science, Vol. 40, p. 97. 1890.

MERRITT: American Journal of Science. Vol. 37, p. 168.

BLATTNER: Der Optische Nutzeffect der Glühlampen: Frauenfeld, 1886.

NICHOLS: American Institute E. E. Vol. 6, p. 158. The Artificial Light of the Future. (Electric Club Pamphlets, No. 24. New York, 1890.)

NAKANO: American Institute E. E. Vol. 6, p. 308.

MARKS: American Institute E. E. Vol. 7, p. 175.

HILL, IDA M.: Thesis, Cornell University. 1890.

ROGERS: American Journal of Science. Vol. 43, p. 301.

ÅNGSTRÖM: Königliche Gesellschaft der Wissenschaften zu Upsala. April, 1892. Also Wiedemann's Annalen. Vol. 48, p. 493.

SHEDD: Thesis, Cornell University.

NICHOLS AND CREHORE: Physical Review. Vol. 2. 1894.

CHAPTER III.

DETERMINATIONS OF THE LIFE CURVES OF ARTIFICIAL LIGHT-SOURCES.

The life curve of a source of light is a curve the co-ordinates of which are time and intensities. Curves may be drawn showing the total radiation as a function of the time, or we may plot the light-giving radiation, or the radiation of selected wave-lengths, each with reference to the time, or analogous curves showing changes in the efficiency of the source with the time.

I.

Incandescent oxides.

A class of illuminants, the life curves of which are especially interesting and significant, are those in which stored energy is utilized, or in which the radiation of the incandescent body or bodies to which the light is due is modified by the peculiar initial molecular condition of the source. Sources of light to which this applies are the lime light, the light of incandescent magnesia and zircon, and such lights as the Auer's incandescent mantle.

In all of these the oxide which is brought to incandescence by the application of a flame of high temperature is undoubtedly capable of luminescence. In other words, its radiation is at first different, both in quantity and quality, from that which the solid is capable of emitting after having been exposed for a considerable time to a high temperature. In the formation of such oxides a certain amount of such energy is probably stored, which energy is released again upon heating, and manifests itself by increasing and modifying the radiation. As this source

of supply is exhausted, the radiating surface comes into a condition of more stable equilibrium, in which its radiation differs less and less from that of such inert substances as carbon, platinum, iron, etc. The life curve of the total luminous radiation of such a source has for its purpose to show the decadence in the brilliancy of the light. A series of analogous curves, each of which applies to a given wave-length, will afford information with reference to the relative decadence of different parts of the spectrum, from which data the change in the quality of light with the time can be deduced.

In the case of some of the sources already mentioned — viz. the Auer mantle burners — the decadence is slow, and the life curve runs over a period of many hours. Such curves have been plotted by Miss Ida M. Hill* (to whose work

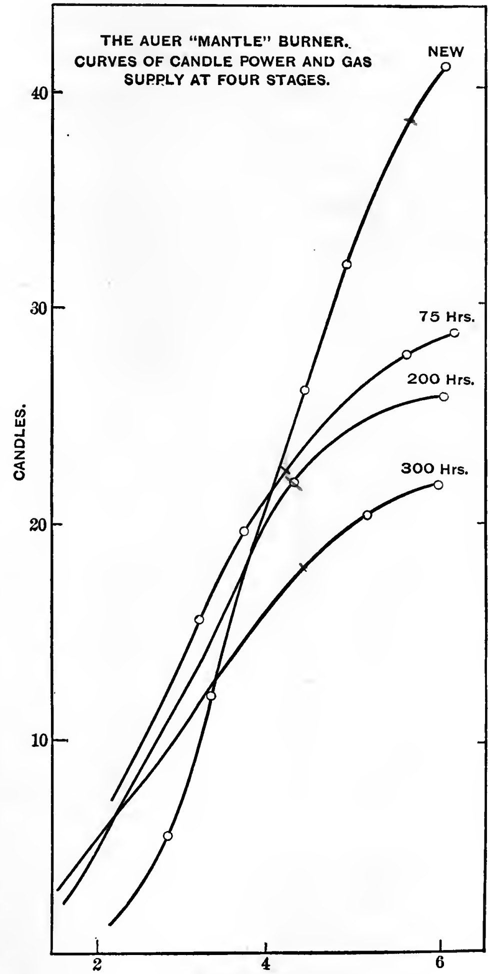


Fig. 182.

* Ida M. Hill, Thesis in the Library of Cornell University, 1890.

reference has already been made in Chapter II.), in connection with the study of the spectrum of these lamps. Some typical results obtained by her are given in graphical form below. Two methods were pursued in the life study of this source. In the first place, measurements of candle-power at different gas pressures of light were made in the case of a burner when new, and after it had been in continuous operation for 75 hours, 200 hours, and 300 hours, respectively. The results are shown in Figs. 182 and 183.

The first of these shows the relations of candle-power to supply of gas at the times indicated above. The second

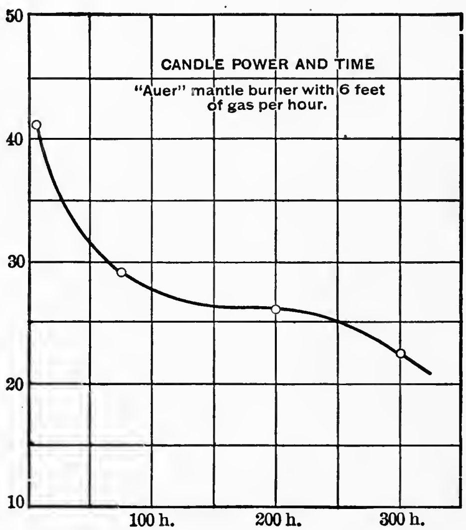


Fig. 183.

(Fig. 183) shows the decadence of candle-power with time in the case of a burner fed with six feet of gas per hour.

Subsequently the horizontal-slit photometer was used to determine the changes which the spectrum underwent with age. The results are given graphically in Fig. 184, which shows the spectrum curves of a "white" mantle burner when new and after 50 hours. The abscissas of these curves are wave-lengths,

and the ordinates are the ratio $\frac{A_\lambda}{G_\lambda}$, or the intensity of the Auer light, wave-length by wave-length (A_λ), in terms of that of the light (G_λ) of an argand burner for the same wave-lengths. The

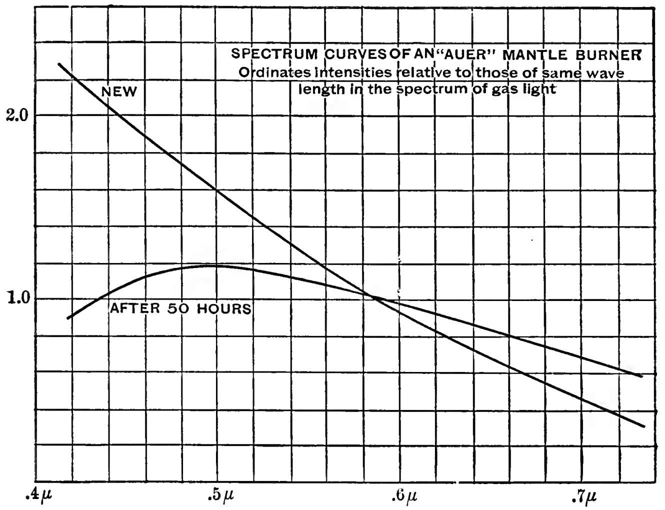


Fig. 184.

study of the decadence of this source of light with the thermopile and galvanometer, using the method to be described in the next paragraph, would doubtless yield results of interest (see further, Chapter IV.).

In the case of the light emanating from incandescent lime, magnesia, and zircon, these time changes occur with much greater rapidity. Some attempts to trace them have already been described in Chapter II.

Measurements of the spectrum of the freshly ignited oxide, and of the same at various times, until it has reached a final condition, were also made in connection with these life curves. These spectrophotometric results will be described in Chapter IV. In that chapter will be found, likewise, a brief account of some life studies of the visible radiation from zinc oxide at 1013°, made by the writer in collaboration with B. W. Snow.

II.

The glow lamp.

Life studies of the incandescent lamp have led to results of great practical importance, and there are at the same time many features of such work that are of scientific interest. The variations in this case are not due to a loss of stored energy in the carbon, but rather to the following causes:

(1) To a diminution in radiating surface, due to the slow disintegration of the filament.

(2) To changes in the amount of energy imparted to the lamp when maintained at constant voltage (on account of the change of resistance of the filament, due to this same disintegration).

(3) To changes in the amount of heat lost by convection, owing to continued depreciation of the vacuum of the lamp.

(4) To loss of light, due to the formation of a carbonaceous deposit upon the interior of the lamp bulb.

These causes of variation all work in the same direction, tending to diminish the candle-power of the lamp. It is possible, by making candle-power measurements from time to time upon a lamp maintained at constant voltage, and accompanying these by measurements of the current flowing through the lamp, as well, and by studying the growth of the age-coating upon the interior of the bulb, to obtain life curves and the data for the analysis of the same. In this way one may determine the proper proportion of the total decadence to be ascribed to each of the various causes already indicated. Studies of the change of candle-power of incandescent lamps, with the time, have been made by Pierce,* by Moore and Ling,† by B. F. Thomas,‡ and by various European observers.

---

* Pierce, Transactions American Institute E. E., Vol. 6, p. 293.

† See Nichols, American Journal of Science. Vol. 44, p. 277; also Moore and Ling, Thesis in Library of Cornell University, 1890.

‡ Thomas, Transactions American Institute E. E., Vol. 9.

The results of all these experimenters are in agreement, and the extent and rapidity with which this change in incandescent lamps takes place is sufficiently shown from the accompanying curves, from measurements by Moore and Ling (Fig. 185), which

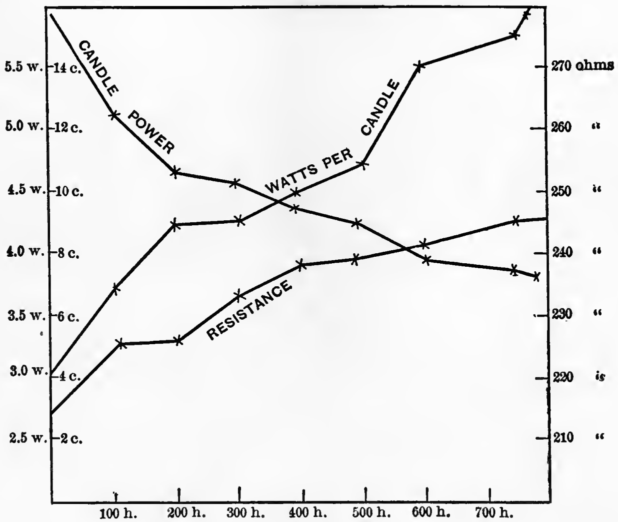


Fig. 185.—Life Curves of a Glow Lamp.

indicate the candle-power of a lamp maintained at constant normal voltage throughout its life, also the resistance of the filament and efficiency in watts per candle. All of these are given as functions of the time. That this is not an extreme case may be shown by comparing these curves, which apply to an individual lamp, with the average of a large number of measurements made by Pierce and described in the paper just referred to.

It will be seen that in this typical case, which is that of any ordinary glow lamp, maintained at normal voltage throughout its entire life of nearly 800 hours, the candle-power fell off to less than half its initial value, while the resistance rose from 214 ohms to 246 ohms. The watts per candle rose meantime from 3.015 to 5.750. In the figure, abscissas are hours, and ordinates are respectively candles, ohms, and watts per candle.

Moore and Ling accompanied their studies of the life history of incandescent lamps with careful spectrophotometric measurements of the age-coating. The results of their investigation, which have been fully described in the paper just cited, are shown graphically in Figs. 186, 187, and 188. The first of these (Fig. 186) shows the transparency of the coated lamp bulb after 100, 200, 400, and 800 hours in terms of the transmitting power of the new bulb. These measurements were made in several regions of the spectrum. The nearly horizontal lines, which were obtained when the results were plotted, indicate that the coating possesses a nearly neutral tint, its absorbing power being almost entirely non-selective.

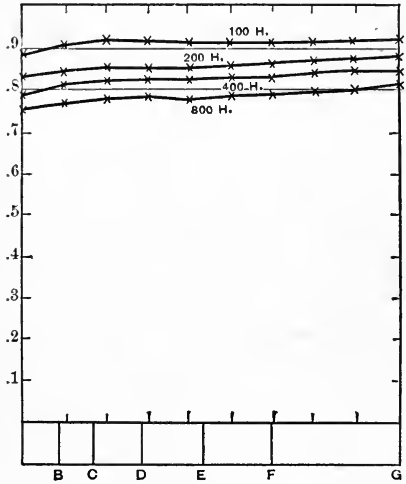


Fig. 186.

Figure 187 shows the resultant transparency of the lamp bulb as a function of the time and in its relation to the diminishing brightness of the lamp. It makes it evident that while the coating is an important factor in the falling off of candle-power, it is not the only element of decadence.

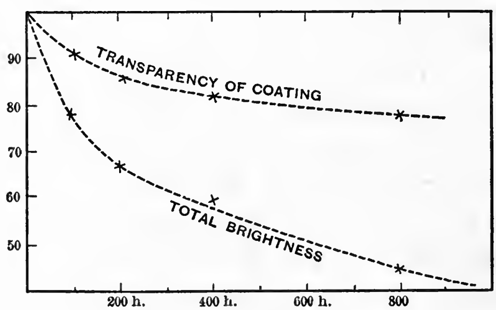


Fig. 187.

Figure 188, finally, shows the results of measurements for horizontal distribution of light at frequent intervals during the

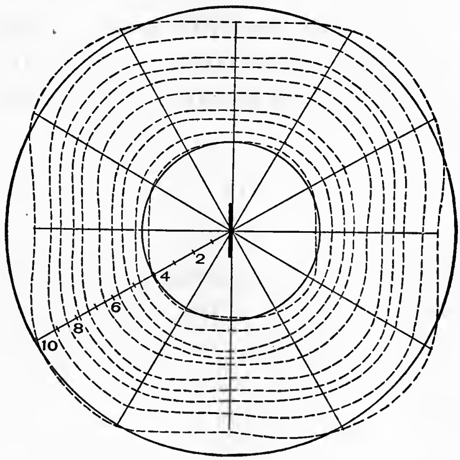


Fig. 188.

life of the lamp. The slight irregularities which the curves (the dotted lines) show are the result of observational errors of a kind almost unavoidable in photometry. The essentially parallel character of the successive curves as we pass inward from the first curve, of largest mean radius, to the final measurements upon a lamp the intensity of which had fallen below half its initial brightness, show the distribution of the age-coating to be nearly uniform.

These results have been introduced here to illustrate the range and scope of life studies of light-sources. They leave many interesting questions still to be determined. It is important to extend such investigations, for example, to lamps still upon the pumps, maintaining the vacuum at a constant and carefully measured value. For this purpose a MacLeod gauge or similar device should be employed. Experiments are needed also upon lamps exhausted under conditions which exclude mercury vapor. This might be accomplished by the interposition of a tube filled with gold foil between pump and lamp.

Investigations similar to those just described should be extended also to lamps in which the vacuum is supplanted by the presence of a heavy inert gas, such as bromine. Such experiments, to be of the greatest value, should be applied to lamps specially constructed, precautions being taken to secure an atmosphere the constituents of which are known and the pressure of which is within control.

III.

Candles, oil, and gas.

In the case of the old illuminants, such as candles, oil, and gas, where new products of combustion are continually supplied, the life curve does not possess the same significance as does that of the sources already considered. Nevertheless, these flames are subject to fluctuations in brightness which interfere with their usefulness as photometric standards. It becomes of interest, therefore, to study these fluctuations and to obtain a life curve of the candle or standard lamp from the moment of ignition. In this way we may determine the changes which the conditions of combustion undergo in consequence of greater heating of the fuel contained within the lamp, and of the parts which constitute the burner (wicks, draught, orifices, chimneys, etc.).

Students of photometry are already well aware of the lack of steadiness of flames used as standards of comparison. The physiological elements of the methods usually employed, however, leave it always in doubt whether variations in the reading of the photometer are due to errors of judgment on the part of the observer, to physiological fluctuations in the character of the eye, or to actual changes in the brightness of the lights to be compared.

For the purpose of eliminating the first two of these sources of error, a bolometer may be used to good advantage. If the bolometer strip be very light and thin, it will respond instantly to changes of temperature. If such a strip be exposed

to the radiation of a source of light, the constancy of which is to be tested, and if it form one arm of a properly constructed Wheatstone bridge, the adjacent arm of which consists of a similar strip of metal which is protected from radiation, readings may be obtained which will show every change in the brightness of the source of light.

The conditions of success are a galvanometer of the highest sensitiveness, of constant zero-point, with freedom from magnetic drift, and a bridge which is readily adjustable, and which at the same time is properly protected from sudden temperature changes. Under these conditions, the intensity of a flame from the moment it is lighted may be traced by means of the readings of the galvanometer. Special precautions must be taken to separate the fluctuations due to actual changes in the source of light from those which arise from magnetic disturbances or from thermal effects other than those which are due to the lamp itself.

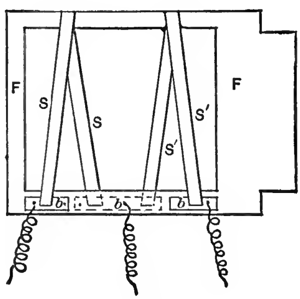


Fig. 189.

A number of the ordinary standards of light have been tested in the manner just indicated by Messrs. Sharp and Turnbull.*

The bolometer used in their investigation was a strip of iron, reduced to extreme thinness by dissolving in sulphuric acid containing potassium bichromate. This strip was mounted as shown in Fig. 189, and formed two adjacent arms of a Wheatstone bridge, half of it being sheltered from radiation by means of a screen, while the rest was exposed. The other two arms of the bridge consisted of an adjustable resistance of German silver wire.

The bolometer and the compensating resistance were placed in boxes *B* and *B′* (Fig. 190), within the door, *DD*, of an inner

* Sharp and Turnbull, Physical Review, Vol. 2.

room of constant temperature. From this room wires, *gg*, led to the bridge galvanometer, and other wires, *ss*, to a calibrating shunt.

The source of light, *C*, was placed outside the door, its rays reaching the bolometer through an aperture in the latter and through holes in various intervening screens, which were so placed as to exclude stray light.

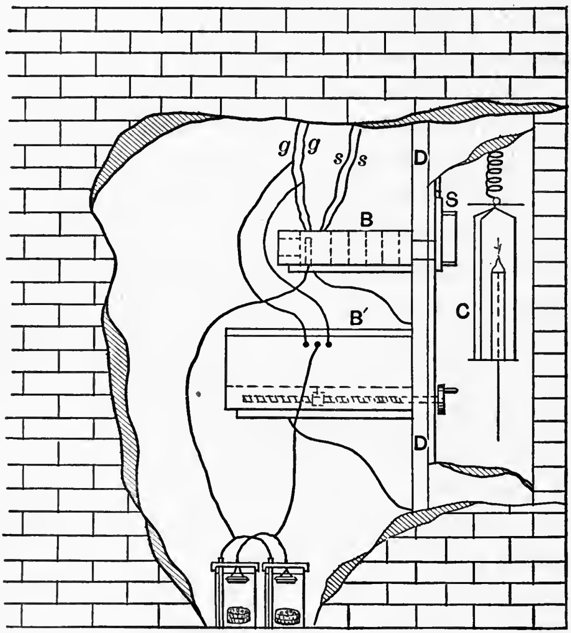


Fig. 190.

The bridge galvanometer, constructed by W. S. Franklin, possessed a figure of merit of about 68×10^{-11} amperes, with a 5-second period, and was so sensitive that it could be used in an artificially strengthened field.

Measurements of the radiation from British and German candles, the Methven slit, the Carcel lamp, the Hefner lamp, the Blondel arc-light standard, and other sources, gave a complete record of the fluctuations of the flame from the moment of light-

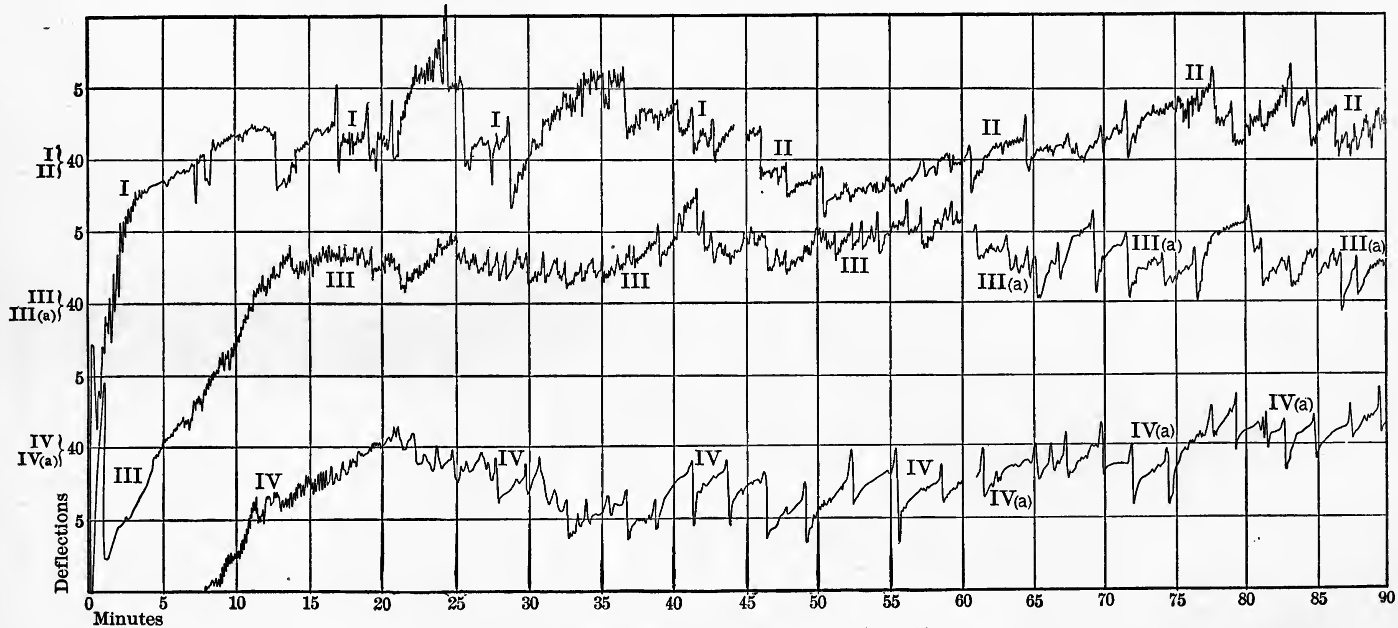


Fig. 191.—Life Curves of a Standard Candle (British).

ing until the end of the run. Some of the life curves obtained in the case of the British standard candle are shown, plotted to a greatly reduced scale, in Fig. 46.

The analysis of these and of other similar curves published in connection with the articles of Messrs. Sharp and Turnbull, already cited, affords very definite information concerning the nature and causes of the fluctuations to which the various flames in use as light standards are subject. By means of such results, it is possible to determine the accuracy of agreement of such standards, their reproducibility, steadiness, and relative freedom from disturbance. The method is one the use of which should be extended to other sources.

CHAPTER IV.

SPECTROPHOTOMETRY.

I.

Preliminary remarks concerning spectrophotometers.

The spectrophotometer, in its essential parts, consists of a spectroscope which is so arranged as to admit the rays from two light sources to the collimator, and to permit the comparison of the spectra of the rays, as to brightness, wave-length by wave-length. This instrument is given almost as many forms as the spectroscope itself, but there are certain requirements which must be met in its construction whatever form be adopted. To render the comparison of two spectra easy and accurate, they must be brought into the eye-piece side by side, corresponding regions in the same transverse line; the boundary between them must be sharp, and should consist of a narrow black band, the edges of which are as clearly marked as possible. These conditions can be fulfilled only by proper methods of introducing the rays of light into the collimator tube. The usual method of introducing the comparison ray into chemical spectroscopes (Fig. 192) is faulty in this respect, although it answers very well where wave-lengths instead of intensities are under consideration. A clearly defined boundary between spectra can be obtained only by bringing the edge of the reflecting prisms or mirrors, by means of which the rays are given direction, in focus at the middle of the slit.

The prism should be turned until its oblique edge is perpendicular to the slit (Fig. 193), and cuts the latter in a line which will bring the boundary between the spectra into the middle of the field of view. Symmetry demands, moreover, that the two rays entering the field should be treated alike, so that, excepting in cases in which other considerations overrule the requirement of symmetry, there will always be a pair of reflecting prisms at the slit, each covering half of the latter.

A serious practical difficulty arises in securing perfectly smooth edges where these prisms come together. If ground

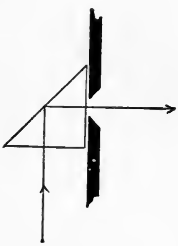


Fig. 192.

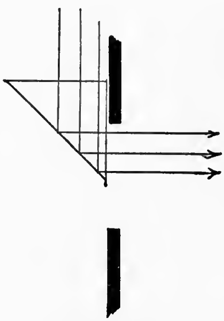


Fig. 193.

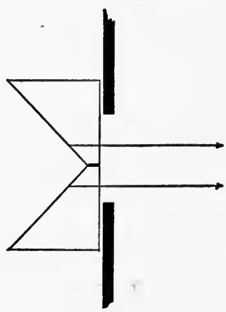


Fig. 194.

to a true edge, both prisms are soon broken away irregularly. In the magnified image within the eye-piece these broken edges give a ragged boundary line which is distinctly detrimental. The best procedure in the construction of this part of the instrument is as follows:

The prisms are cut away, leaving a ground face about 1 mm. wide, normal to the face which is to be parallel to the jaws of the slit. This narrow face of each prism is blackened, and the blackened faces are placed in contact when the prisms are mounted as in Fig. 194. This device gives a somewhat broader band between the spectra to be compared than though the prisms were not truncated, but it affords a uniform boundary of sufficient sharpness.

The arrangement shown in the last figure serves only to introduce rays from above and below into the halves of a vertical slit. In many cases it is more convenient, and sometimes it is necessary, to bring horizontal rays from the right hand and the left into the collimator tube, for which purpose four prisms arranged as in Fig. 163 (Chapter I.) may be used. A better plan, when practicable, is to use a horizontal slit, following the construction to be described later.

The question as to the method of adjusting two spectra to equal brightness, which is the operation upon which all spectrophotometry depends, is the most serious one connected with the design of the spectrophotometer.

The earliest plan, in point of time, was that of Vierordt, who depended upon increasing the brightness of one or the other spectrum by opening the slit. For this purpose he devised a double slit, each half of which was capable of movement separately by means of a micrometer screw and springs. In order to maintain the symmetry, both jaws of the slit moved, so that however wide either half might be, the line which bisected it longitudinally would bisect the other half also as in Fig. 195.

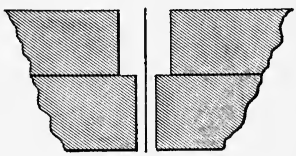


Fig. 195.

The Vierordt slit can be used only through a very narrow range of intensities. The opening of either half to the point where impurity of the spectrum begins to show itself limits its usefulness in one direction; a narrowing of either half to the width at which the field loses uniformity of brightness brings the experimenter to the other limit.

Where the spectra to be compared are of nearly equal brightness throughout, the Vierordt slit affords the simplest means of bringing them to equality. Where it is in itself inadequate, and especially where very great ranges of intensity exist, it is a valuable accessory to other means of adjustment.

Nearly all subsequent forms of the spectrophotometer — as,

for example, those of Hüfner, Glan, Crova, A. König, Vogel, etc. — have been constructed with a view to the reduction to equal brightness of the two spectra by the polarization of one or both rays. W. H. Pickering, however, at an early day in the history of spectrophotometry (1878), substituted the method of moving the comparison standard back and forth along a track in line with the slit of the spectroscope. In 1890 the writer mounted his "horizontal-slit" photometer upon the carriage of the usual photometer bar, while in 1892 Lummer and Brodhun described a modification of their well-known photometer which adapts that instrument to the comparison of similar wave-lengths. The ingenious devices applied by E. C. Pickering to stellar photometry cannot be considered here.

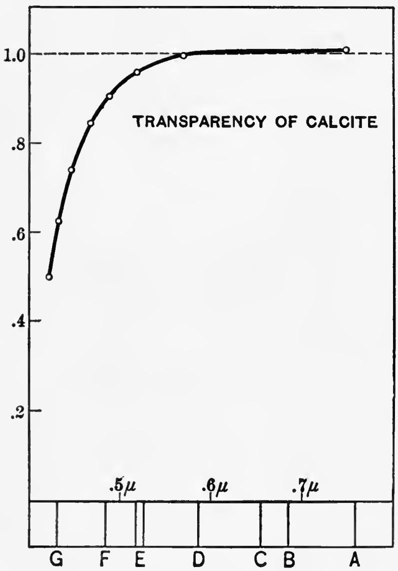


Fig. 196.

The polarizing spectrophotometer, although it is the best type for many kinds of work, is open to serious objections. Of these the most important are:

(1) *Selective absorption in calcite.* This substance appears to the eye to be nearly colorless, but the measurement of the light transmitted by it shows increasing absorption as the wave-length diminishes.

A pair of Nicol's prisms, the transmitting power of which was studied by the writer in collaboration with B. W. Snow (1891), show the transparency to be only half as great for the region 0.43 μ as for the region 0.60 μ. Figure 196 shows the variation in relative brightness of the transmitted ray with the wave-length. In the diagram, the transparency for the region of the D line is taken as unity.

This source of error may be eliminated by symmetry of construction. Otherwise a correction must be applied.

(2) *Selective losses by reflection of polarized rays at the various surfaces of the optical parts of the spectrophotometer.* This error occurs whenever in a spectrophotometer one ray is polarized and the other unpolarized, or whenever the two rays are polarized in planes perpendicular to one another. The remedy is symmetry of construction, extending the term so as to include symmetry as regards the planes of vibration of the two rays which are under investigation. Failing of symmetry, the error can frequently be reduced to small amount by selecting the most favorable arrangement of the apparatus.

(3) *The necessarily imperfect performance of the Nicol's prism and of other polarizing devices.* This objection arises whenever one has occasion to work at very low intensities; as, for example, in the measurement of regions of maximum absorption in the study of pigments or solutions, or in the investigation of visible radiation at the red heat.

It is a common experience in such cases to find that the field with crossed Nicols is brighter than that which we wish to measure, even while the latter is still readily distinguishable from the blackness which we get by the exclusion of all light. The remedy is to use a Vierordt slit, with as great difference between the widths of the two slit-halves as can exist without the introduction of the troublesome errors due to color-mixing or to striation of the field.

(4) The narrowing of the field, and the considerable losses of light which are unavoidable in polarizing spectrophotometers, are also serious objections to the adoption of this type for very many kinds of work.

Symmetry of construction in a spectrophotometer, to which reference has been made several times, is attained when the form of the instrument is such that the two rays to be compared will suffer precisely similar treatment along their paths from the respective sources to the eye. By this is meant that

the rays must traverse the same thickness of glass, and of whatever other media may form their paths; also, that they must suffer loss by reflection at the same surfaces and at the same angle from each surface.

In so far as it is impossible to fulfil these conditions, cognizance must be taken of the errors thus introduced. That the error due to selective absorption in calcite is a large one has already been indicated. The loss in glass is of the same general nature, and it is scarcely less important. The almost complete opacity of the heaviest varieties of flint glass to the extreme violet is well known, and the same tendency to increasing absorption with diminishing wave-length shows itself in the lighter optical glasses. Figure 52 gives graphically the results of studying the relative transparencies for various wave-lengths of a condensing lens made of glass of medium density.

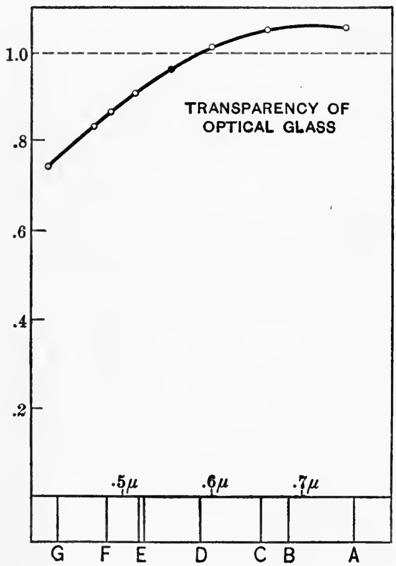


Fig. 197.

It was with a view to meeting the requirements of symmetry, and of avoiding some of the errors which are indicated in a previous paragraph, that the writer devised two forms of horizontal slit photometer. The original examples of these pieces of apparatus were constructed in the instrument-maker's shop of the department of physics of Cornell University, by Fred. C. Fowler.

The first of these instruments (1890) is mounted upon a photometer bar, the length of which can be varied at will up to 5 m., according to the intensity of the light-sources to be

compared. One advantage possessed by this form of apparatus consists in the avoidance of all polarizing devices. The two

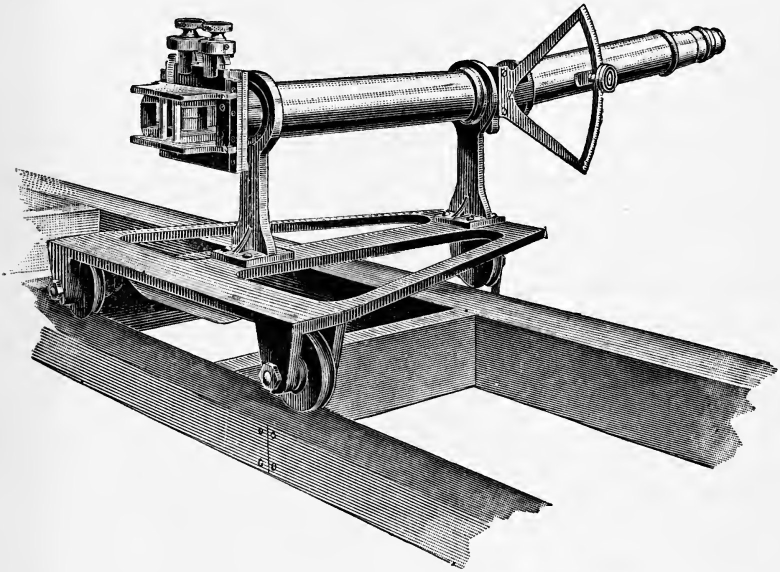


Fig. 198.

spectra to be compared are brought to equality by moving the spectrophotometer along the bar until the proper position has been reached. The arrangement of the instrument is perfectly

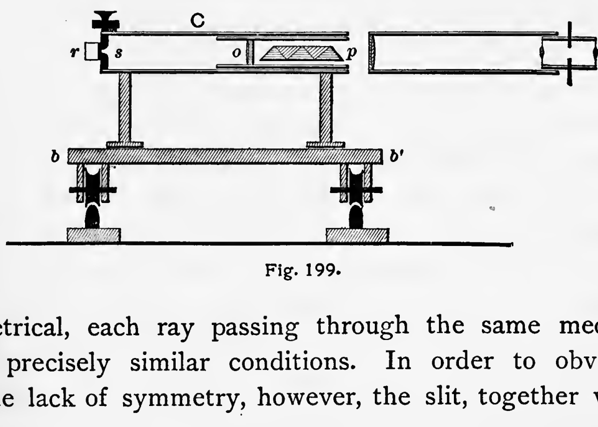


Fig. 199.

symmetrical, each ray passing through the same media and under precisely similar conditions. In order to obviate all possible lack of symmetry, however, the slit, together with its

reflecting prisms, is capable of being turned through an angle of 180°; and further than this, the entire instrument may be turned around, the observer passing from one side of the photometer bar to the other. To gain the last-named advantage,

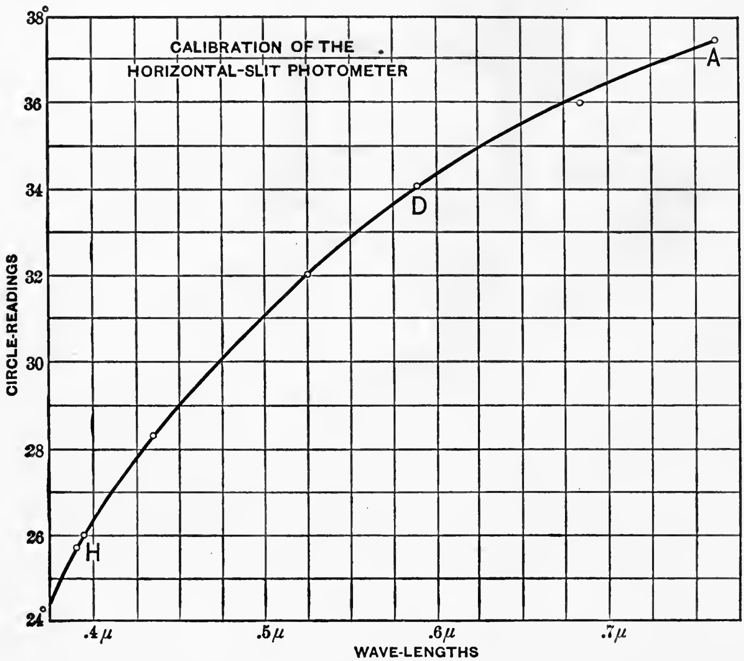


Fig. 200.

the photometer bar must be situated near the middle of the dark room instead of being fastened, as is too often the case, to one wall.

Figures 198 and 199 show the essential features of this instrument. In the latter diagram bb' is a triangular base of cast iron, which is carried upon three wheels. These are fitted to the photometer track. C is the collimator, with its Vierordt slit s, its objective o, and its set of direct vision prisms p.

The sources of light to be compared are placed at the ends of the photometer bar, and their rays are introduced through the

halves of the horizontal slit by means of the two prisms rr', the arrangement of which has been already described. The observing telescope is pivoted to the collimator tube with freedom of rotation in a vertical plane. It can be clamped at will to a vertical metallic arc, and its position can be determined by reference to a graduated scale upon the rim of that arc. The eye-piece is provided with the usual diaphragm in the focal plane. By the adjustment of this diaphragm corresponding regions of the two spectra, which appear in the field side by side with a sharply defined dividing band of black, may be isolated. Wave-lengths are determined in terms of the circle readings, once for all, by the location of the principal Fraunhofer lines. The result of such a calibration is shown graphically in Fig. 200. In this diagram wave-lengths are abscissas,

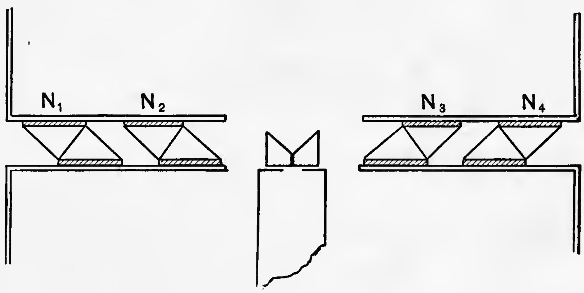


Fig. 201.

and scale readings are ordinates. By means of such a curve the wave-lengths of the various parts of the spectrum under examination can readily be ascertained with an accuracy sufficient for the purposes of the student of spectrophotometry.

In the use of this form of spectrophotometer the instrument is moved along the bar until the two regions of the spectrum which are in the field are equally bright, and the position of the carriage is then noted. This reading is repeated for each wave-length which is to be measured. It is convenient to use a bar divided into 1000 equal parts, in which case computations of

the light ratio may be simplified by means of curves of light ratios (see Part III. of this volume).

When the spectra to be compared are of nearly equal brightness, the right and left halves of the slit are set at the same width. Where great differences of intensity exist, the slit should be adjusted so as to keep the photometer near the

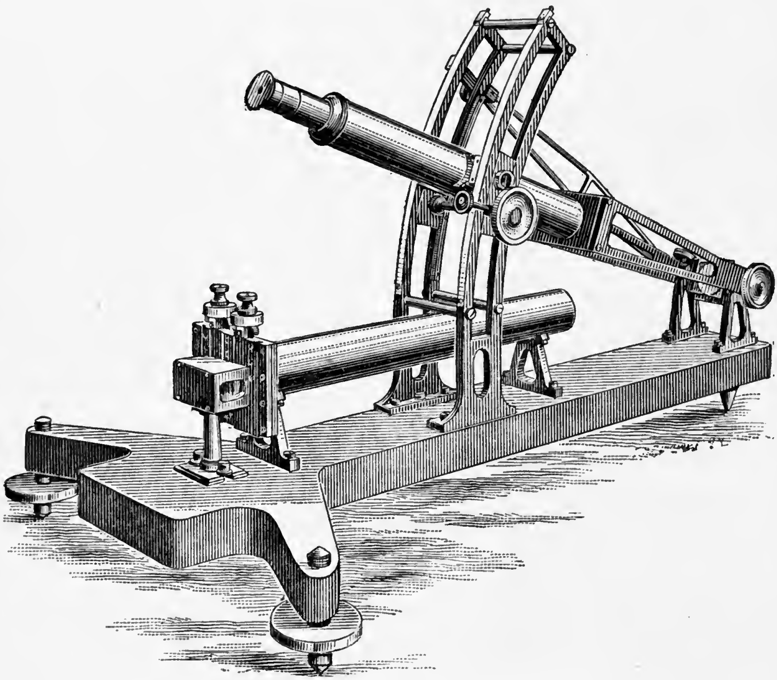


Fig. 202.

middle of the bar. By this combination of the principles of the Vierordt slit and of the Bunsen photometer the range may be greatly extended without introducing the errors which limit the usefulness of each when applied singly.

This form of the instrument is to be preferred in all cases where the light-sources to be compared are of sufficient intensity. When spectrophotometric work is to be done under conditions of illumination such as to make the method of the

photometer impracticable, the horizontal slit photometer is still a desirable form. In such cases it is mounted permanently, and polarizing apparatus (preferably two precisely similar pairs of Nicol's prisms) are placed with their common axis at right angles to the axis of the collimator tube, and in line with the reflection prisms in front of the slit (see Fig. 201). With this arrangement two light-sources, S and S', may be compared by the method of the polarizing spectrophotometer, and if the planes of the inner Nicols N_2 and N_3 be parallel and fixed, and the outer ones only be given freedom of rotation, the measurement will be made under conditions of almost complete symmetry.

The prismatic spectrum, in spite of its brightness, is open to objection in spectrophotometry because of the crowding together of the less refrangible rays. In the orange and yellow, excepting with a very narrow field of view, uniformity of tint cannot be secured without the aid of a telescope of long focus and high power. With prisms of high dispersion, moreover, the violet is a region of great dimness. For some purposes, therefore, the diffraction spectrum is to be preferred.

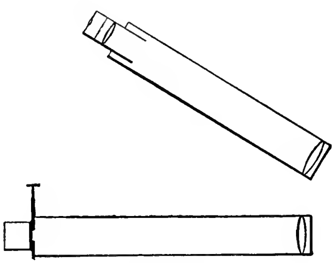


Fig. 203.

A modification of the horizontal-slit photometer was designed by the writer (1893) to admit of the convenient use of the plane grating in spectroscopy, and at the same time to bring the slit, the light-sources, and the intervening apparatus for varying the brightness of the spectra immediately under the hand of the observer.

The collimator of this instrument is mounted upon a substantial base (Fig. 202), to which it is rigidly attached. It carries a Vierordt slit, placed horizontally, with reflecting prisms arranged

in the manner described in a previous paragraph. Fig. 203 is a diagram of the essential parts of this instrument.

About a horizontal axis which is normal to the optical axis of the collimator in a point about 20 cm. beyond the objective of the latter, swings the observing telescope, which travels smoothly and without lost motion between two vertical guiding sectors of cast brass. To these it may be clamped at any desired angle. The final adjustment is made by means of a tangent screw.

The grating is mounted in a holder which swings upon bearings in the line of the axis of rotation of the observing telescope. Its position is controlled by means of a light arm which plays along the outer face of one of the vertical sectors. A set screw serves to fasten this arm in any given position. The rims of the two guiding sectors carry scales, the graduation being in equal parts of convenient size. For the measurement of wave-lengths, the instrument is to be calibrated by reference to the Fraunhofer lines. The telescope and also the arm of the grating-holder carry verniers adapted to these scales.

The apparatus for adjusting the spectra to equal brightness in the use of this instrument is to be placed between the sources of light and the reflecting prisms, to the right and left hand of the observer. Two similar pairs of Nicol's prisms may be used, as shown in Fig. 201, or any other device suitable to the work in hand. A simple and very satisfactory method which is adapted to many investigations consists of variable slits or diaphragms placed directly in front of the light sources. These taken in combination with the Vierordt slit in the collimator afford abundant range. They are admissible, however, only in cases in which the sources of light offer surfaces of uniform brightness which fill the entire field of the diaphragm, or when a glow-lamp is used, the filament of which is straight and of uniform cross-section. In the latter case the filament should be placed across the diaphragm, which serves to increase or diminish the exposed portion in proportion to its own width.

The grating used in this instrument may be replaced by a 30° prism silvered at the back (after Abbe) when a bright spectrum of small dispersion is needed.

II.

Comparison standards and the method of plotting results.

One of the chief difficulties in comparing the spectrum of artificial sources of light consists in finding a suitable standard in terms of which the various spectra may be defined. Daylight and the direct light of the sun have been used for this purpose by some of the earlier observers, but no source of light could be worse than these, on account of the very great fluctuations in quality to which sunlight, both direct and diffused, is continually subject. Gaslight, and the light of petroleum lamps also, vary in quality and brightness with the fuel used, and with the varying conditions of combustion. The incandescent lamp is a steadier source, but the character of its spectrum varies with the temperature of the filament, and it is not possible as yet to define with perfect certainty the degree of incandescence of the lamp used as a comparison standard in such a manner as to enable other observers to reproduce the same. With our present knowledge, the most satisfactory plan is probably to adopt the naked gas flame, viewing the brightest portions of the same through a rectangular aperture, as a standard of quality. The best actual comparison standard, however, is a glow-lamp, which should be of the type with heavy filament used in constant current circuits. This lamp should be placed at one end of a photometer bar, and the naked gas flame at the other. The former should be in circuit with an accurate ammeter and voltmeter, and with an adjustable rheostat. The resistance should be varied until a photometer of some type sensitive to color shows no distinguishable difference in the quality of the two. The Ritchie photometer is well adapted to this adjustment.* An incandescent lamp maintained at a definite

* For a description of a modification of the Ritchie photometer, adapted for use upon the photometer bar, see Physical Review, Vol. 1, p. 339.

efficiency, say 4 watts or 5 watts per candle, according to the quality of the light desired, would be preferable to the standard just described; but the fact that a statement of the efficiency of a lamp, in watts per candle, does not define the quality of its light with even approximate accuracy prevents the adoption of this method of standardization.

Extended experiments upon the character of the spectrum of incandescent lamps, made by Mr. John C. Shedd in 1892,* afford abundant evidence of the wide range in quality shown by different lamps. The difficulty could, doubtless, be overcome were it possible to secure incandescent lamps manufactured with a view to constancy of performance. Precautions similar to those which are taken in the manufacture of standard candles would give us standard lamps, the dimensions of the filaments of which would be of fair uniformity as regards the character of the radiating surface and uniformity of vacuum. It is probable that lamps thus prepared would give always the same character of spectrum when they were maintained at a normal efficiency, but in the present state of the art of manufacturing incandescent lamps no such result is to be expected.

The reference standard having been selected, the arrangement of the apparatus for a comparison of its spectrum with that of any source of light which is to be investigated, must be determined. Almost any type of spectrophotometer may be used, but the horizontal-slit photometer is especially adapted to such work. The procedure consists simply in setting successively to regions of the spectrum lying respectively in the red, yellow, green, blue, and violet. In each region determinations are made by moving the photometer back and forth along the track, precisely as in the case of the Bunsen photometer, until equality of the fields of view is obtained.

The sensitiveness of the horizontal-slit photometer varies with the brightness of the spectra, and also with the wave-

* "A Comparative Study of Treated and Untreated Filaments for Incandescent Lamps," Thesis in the Library of Cornell University.

length of the light under inspection. It is somewhat less sensitive than the Bunsen photometer when the latter is used for the measurement of lights nearly identical in character. The horizontal-slit photometer, however, soon overtakes the Bunsen photometer, as regards sensitiveness, when we pass from sources which are of similar composition to those varying more and more in quality.

In expressing the results of their measurements there has been a difference of practice among spectroscopists. It is usual, however, and a matter of convenience, to take the brightness of the standard as unity for each wave-length of the visible spectrum separately and to express the brightness of corresponding wave-lengths of the spectrum under investigation in terms of these values. Where the only question of interest is that of the quality of the light, as distinguished from its total brightness, it is best to reduce all values so as to represent the spectrum under such conditions that the lamp under investigation is just as bright in the region of the D line as the comparison standard. The result of a series of measurements, then, will always be represented by means of a curve, the ordinate of which is unity at the wave-length, $0.589\ \mu$. The other ordinates will represent respectively the brightness of each wave-length of the light to be studied divided by the brightness of the spectrum of a gas-flame at that wave-length. The abscissas of such a curve are wave-lengths, and the ordinates are everywhere the ratios just defined. In the following sections are given some of the results already obtained by the application of this method in the study of artificial light sources. They are introduced here for the benefit of those who desire to pursue this line of investigation further. The measurements have not all been made by means of the horizontal-slit photometer, but they are all strictly comparable with those which might have been obtained with that instrument.

III.

The incandescent lamp.

This is a most interesting and fruitful subject of spectro-photometric study. The growth of the spectrum, as the filament attains more and more brilliant incandescence, may be determined with reference to the change in voltage, in current flow, in electrical activity (expressed in total watts), in candle-power,

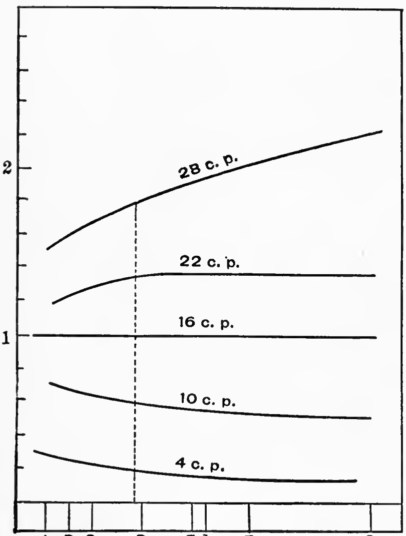


Fig. 204.

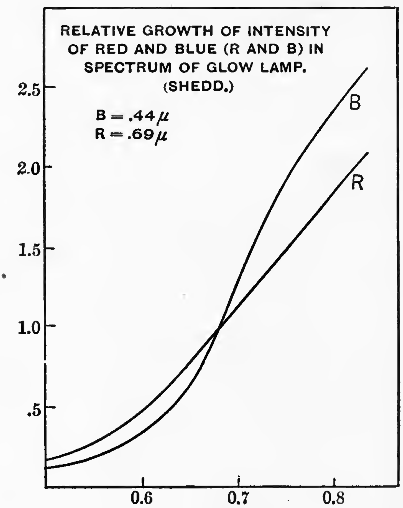


Fig. 205.

in efficiency (expressed in watts per candle), or in radiant efficiency. The ultimate basis of reference should be temperature of the filament; but that is as yet a matter very difficult of determination, unless it be by following the method described by Weber.*

Whichever of the above-mentioned relations be selected, measurements always show increase of intensity with rise of

* Bericht ueber die Verhandlungen des Electrotechnischen Congresses zu Frankfurt am Main, 1891, II., p. 49; also Physical Review, Vol. 2.

temperature which is least in the red, and which tends to become more and more rapid with diminishing wave-length. Adopting candle-power, for example, as the criterion of incandescence, the writer, in collaboration with W. S. Franklin (1888), obtained curves showing these changes in the case of an ordinary incandescent lamp of that period (bamboo filament, untreated).

The general character of these is shown in Fig. 204, in which the distribution of light in the spectrum of the lamp at sixteen candles is taken as unity throughout, and the curves for four,

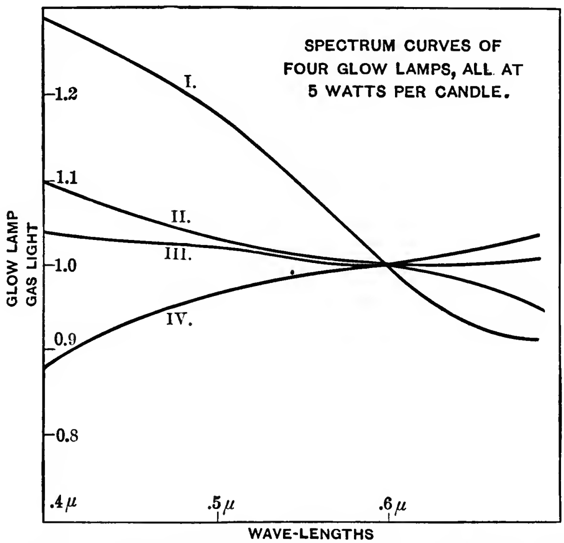


Fig. 206.—I, An Untreated Filament. II, III, and IV, Treated Filaments.

ten, twenty-two, and twenty-eight candles are plotted with reference to the same.

When we come to trace the rise in intensity of each wave-length separately, in the case of the incandescent lamp we find that the change (as the wave-length changes) in the law of increasing radiation is a slow one. J. C. Shedd,* who made many spectrophotometric measurements under the writer's

* Shedd, *l.c.*

direction in 1891–92, found for wave-lengths 0.69μ and 0.44μ, in the spectrum of a glow-lamp with untreated carbon, results which are expressed graphically in Fig. 205. In this diagram, abscissas give the current traversing the filament, while ordinates are intensities in terms of the brightness of corresponding wave-lengths in the spectrum of a standard lamp which was maintained at constant candle-power throughout the experiment.

The quality of the light of the glow-lamp is not a function of the current nor of any of the factors such as candle-power, voltage, or efficiency, which depend upon temperature, taken alone. It is related to the pressure within the bulb, the constitution of the atmosphere which surrounds the filament, and the nature of the radiating surface. How different the quality of the radiation from different glow-lamps when maintained under incandescence such that the watts expended per candle-power are the same in all, may be seen from the following typical curves obtained by Shedd (Fig. 206). All of these lamps were under activity of five watts per candle. It will be noted that the curve for an uncoated filament shows a whiter light, and, therefore, presumably a higher temperature, than any of the others. Shedd's measurements seem to show that, in general, the untreated filament is whiter for a given efficiency, but that there are other potent factors besides the character of the surface. The most important problem before the student of the spectrum of the glow-lamp is that of defining its performance in such a way that from observation of its spectrum the temperature of the filament can be determined. It is probable that this can be done only when the pressure of the atmosphere surrounding the filament is known, and certain radiation constants are given. The similar problem of defining the quality of the light by reference to the various conditions under which the lamp is maintained also demands investigation. Before these problems can be solved the influence of the various factors already indicated must be determined.

IV.

Other artificial sources.

The magnesium light. — This source approaches more nearly to daylight than any other artificial light. Its spectrum may be determined by the use of the horizontal slit photometer, but the investigation is rendered difficult by the fluctuating character of the flame. The error due to this cause can be reduced to a minimum by placing a diaphragm of small

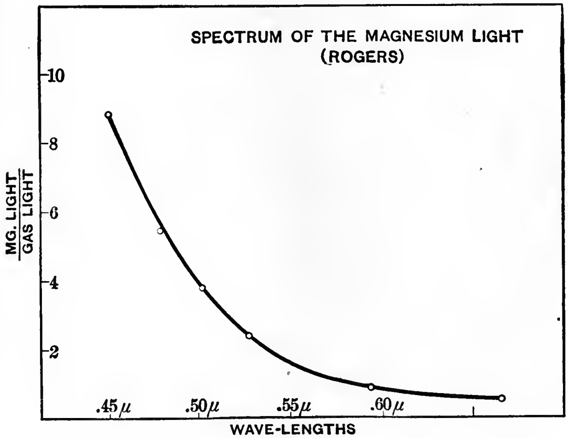


Fig. 207.

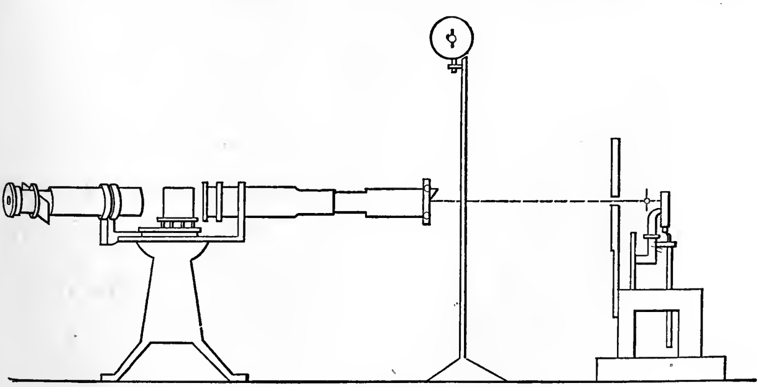


Fig. 208.

aperture in front of the burning ribbon. The area of radiation is thus nearly constant although the total amount of incandescent oxide may vary through a wide range. Measurements of this source by W. H. Pickering (1878) and by F. J. Rogers (1892)

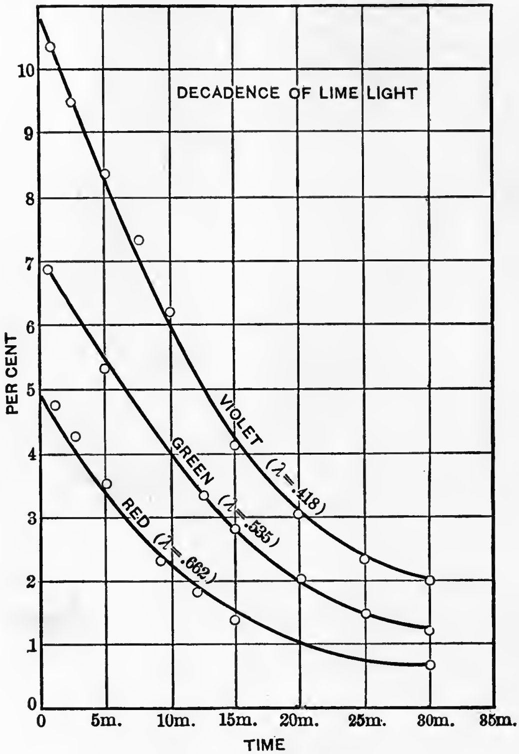


Fig. 209.

are in excellent agreement. The results obtained by the latter are shown graphically in Fig. 207. It will be seen that the ratio $\frac{\text{mg. light}}{\text{gaslight}}$ reaches 8.6 at 0.45 μ. The arc-light in the same region gives only 2.5, and the lime light 2.0. An interesting problem in connection with this source of light is that of the influence of impurities upon the quality.

The Drummond light. — This source has been shown, in

Chapters II. and III., to be subject to continuous decadence from the moment of ignition. To follow its changes, especially during the first ten minutes, one should be provided with a spectrophotometer capable of rapid adjustment. The instrument having been set to a selected region and a fresh lime having been mounted, the oxy-hydrogen burner is lighted, the time is noted, and readings are taken as rapidly as possible at known times after ignition. To get a good result, the hottest portion of the incandescent cylinder should be viewed through a diaphragm. The horizontal-slit photometer is well adapted to these measurements, or a spectroscope with the Vierordt slit may be mounted as in Fig. 208. This figure shows the arrangement of the apparatus used by Miss Crehore in her study of the lime light, to which reference has already been made. It does not fulfil the conditions of complete symmetry, but no errors are likely to occur which would be of consequence in the investigation of a variable source of light. The comparison standard is an incandescent lamp, mounted vertically above the slit, with its filament horizontal. This lamp is maintained at a voltage such as to give its light as nearly as possible the quality of gaslight (see Section I. of this chapter). Its rays are introduced into the upper half of the slit by means of a

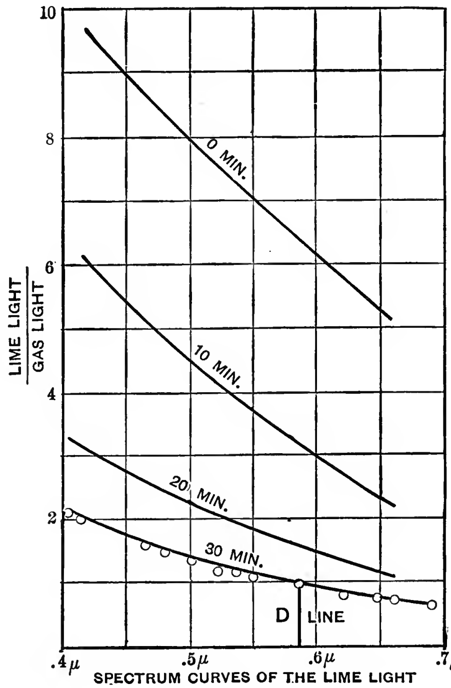


Fig. 210.

small reflection prism. Since the entire range of intensities never greatly exceeded 1 : 5, the Vierordt slit furnished a sufficient means of adjustment. Three wave-lengths were selected for time observations; these were 0.418 μ, 0.535 μ, and 0.662 μ. Their time curves of decadence are shown in Fig. 209, in which ordinates are intensities. In addition to these curves, explorations of the entire spectrum were made in the case of lime

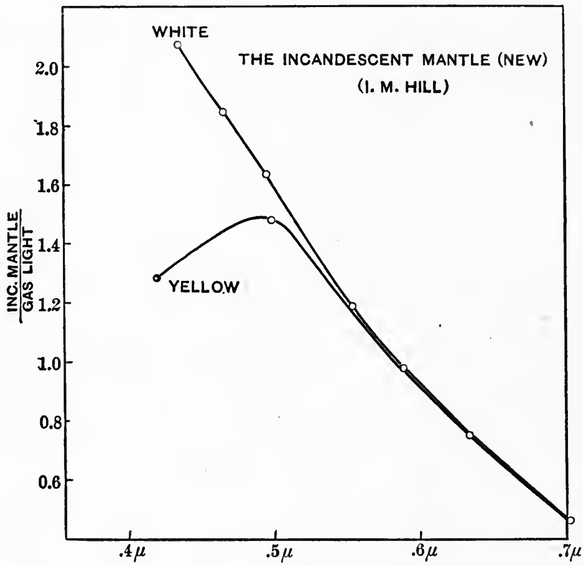


Fig. 211.

which had been incandescent long enough (30 minutes) to reach a nearly stable condition. The results are shown in the curve marked 30 min. (Fig. 210), in which the intensity of the region of the D line is taken as unity. From this as a basis, and the curves of decadence, curves of distribution can be drawn for any required intervening time. In the same diagram such curves for 0 min., 10 min., and 20 min. after ignition are given.

Time studies of the Drummond light, with magnesia and with zircon cylinders instead of lime, using the method just described, would doubtless lead to results not unlike the above.

No spectrophotometric determinations of those oxides have been made as yet (see also Chapter II. (IV.)).

The incandescent-mantle burner. — In this source of light we have as incandescent solid material the oxides of certain rare elements, and these possess the property, in common with the oxides of calcium, magnesium, zirconium, zinc, etc., of being temporarily luminescent, with gradual decadence into a condition of ordinary incandescence after ignition (see further Section V. of this chapter).

In the case of the Auer burner, this change is much slower than with lime or with zinc. Measurements by Miss Ida M. Hill* upon both "white" and "yellow" varieties of mantle, the method being that of the horizontal-slit photometer, gave results from which the curves in Fig. 211 were plotted. These show the spectrum of new mantles. They exhibit an intensity of the more refrangible rays relatively much higher than would be the case with the radiation from carbon for the same temperature. The influence of age upon the spectrum of the "white" mantle has been already discussed (see Fig. 184, Chapter III.). Very interesting results are to be expected from the study of these oxides, separately and under conditions such that the temperature is within control and measurable.

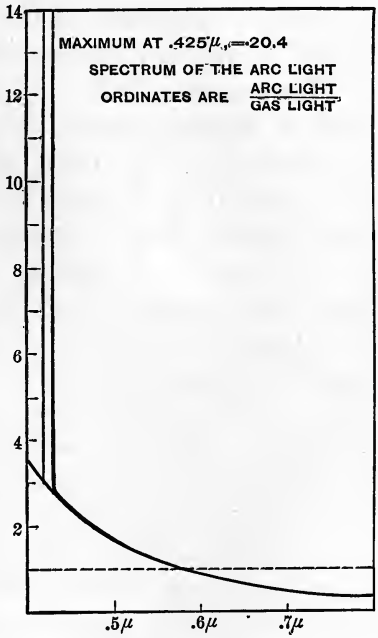


Fig. 212.

* Thesis in Library of Cornell University, 1890.

An extension of the method described in Chapter II. for the study of the influence of temperature upon the color of pigments would be applicable for this purpose. To reach the desired temperatures, it might, however, be found necessary to substitute carbon for platinum, and to work in a vacuum.

The arc-light. — This is an unsatisfactory source to deal with, on account of the fluctuations in brightness from moment to moment, and because of the great range of temperatures exhibited by different portions of the incandescent surfaces.

Measurements made upon the arc, taking light from all portions of the radiating area, made by W. S. Franklin and the writer, gave the result indicated in Fig. 212. The continuous spectrum from the carbons indicated by the unbroken curve, showed steady increase from red to violet as compared with the spectrum of gaslight. The pencils of the lamp used in these experiments were larger than those of the only lamps previously tested, which were of the Foucault regulator pattern with carbons one-quarter of an inch in diameter,* and the curve indicates a lower average temperature of incandescence. This result is in accordance with the fact established by Nakano,† that the radiant efficiency of the arc-lamp increases as the size of the pencils diminishes.

Superimposed upon this continuous spectrum is the line spectrum of the arc itself, the only stable portion of which within the regions subjected to measurement was the group of carbon lines in the neighborhood of wave-length 0.425 μ. These were not separated under the conditions of dispersion and slit aperture which existed. They appeared as a violet band of great brilliancy, the position of which is indicated in the diagram by the vertical dotted lines. The reading at this point gave 20.4, in terms of the corresponding region in the spectrum of gaslight. The bolometric explorations of B. W. Snow‡

* See Pickering, Vogel, also Crova in list of references to spectrophotometry, at the end of this chapter.

† Nakano, Transactions, American Institute Electrical Engineers, Vol. 6 (1889).

‡ Physical Review, Vol. I.

show this band and also a more intense group of somewhat shorter wave-length, which lies beyond the range of the spectrophotometer.

An important field for spectrophotometric investigation is to be found in the study of the quality of light from the crater of the arc, isolated by means of an intervening diaphragm. The researches of Blondel, Violle, and Abney seem to indicate an unlooked for constancy of condition in the crater, both as regards temperature of incandescence and intrinsic brightness.

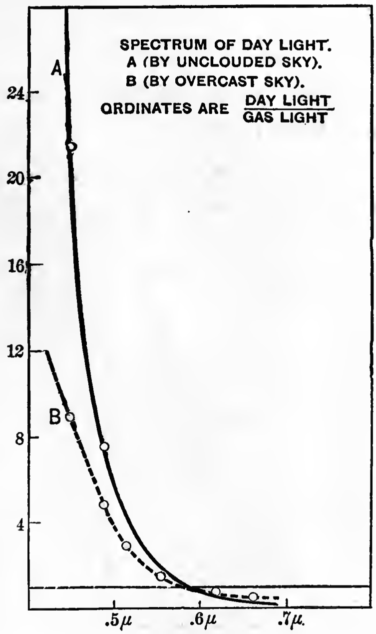


Fig. 213.

Daylight. — The light of the sun, both direct and after diffusion, as daylight and skylight, has been subjected to extended investigation by means of the spectrophotometer. These measurements indicate that the quality of daylight varies through an extraordinary range, according to the condition of the atmosphere through which the rays reach the instrument. Measurements by Vogel, for example, have shown the violet intensities to be relatively 100 times greater than the corresponding intensities of gaslight. At other times the same region may be found not more than ten times as bright as gaslight. In Fig. 213 two curves are given to represent graphically something of the range of fluctuation to which daylight is subject. The curve *A* represents the quality of daylight as compared with gaslight, wave-length for wave-length, upon an unusually clear day. The other curve is that for daylight as compared with

gaslight, measured at a time when the sky was overcast with rain clouds. Spectrophotometric studies of daylight taken in connection with the meteorological conditions might lead to results of considerable scientific and practical importance, but the problem is one of great complexity.

V.

The spectrophotometric study of radiation as a function of the temperature.

This is one of the most important fields which is open to the worker with the spectrophotometer. Some investigations

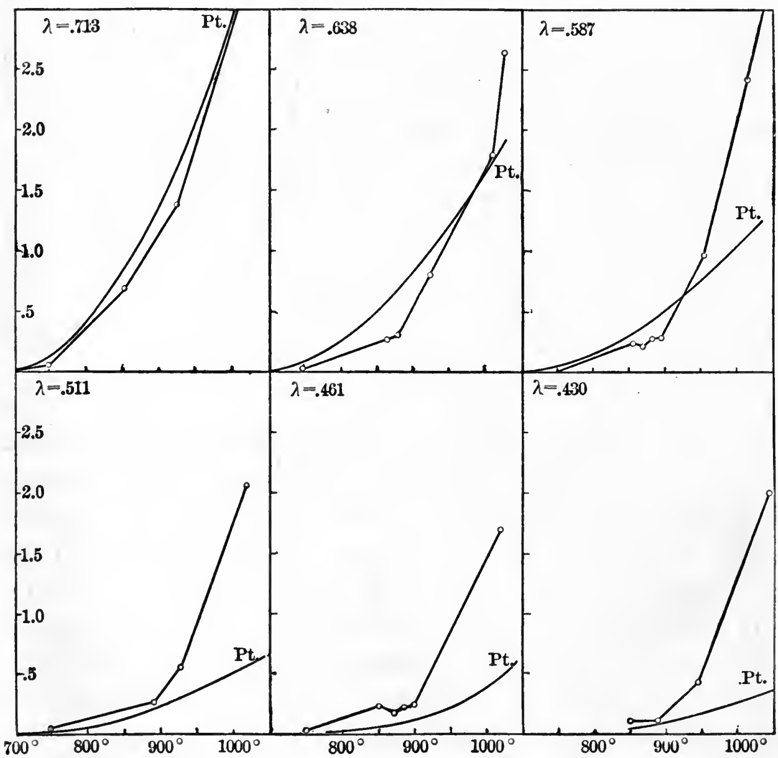


Fig. 214.

have been made of the radiating power of platinum, wave-length by wave-length, at temperatures above the red heat, but so far as the law of radiation from other surfaces is concerned, under

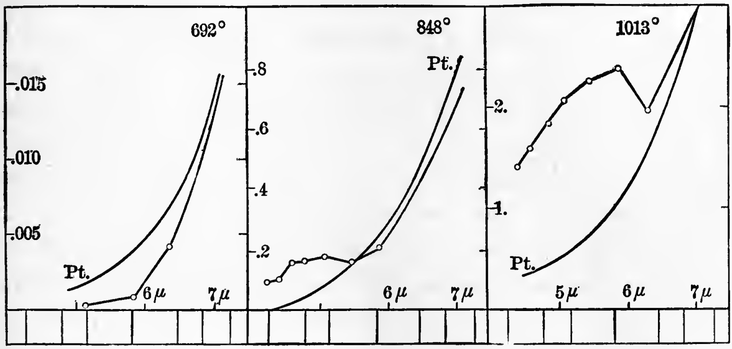


Fig. 215.

conditions which permit of temperature determinations, the beginning has scarcely been made. Some experiments by the writer, in collaboration with B. W. Snow (1891), simply serve to indicate the very interesting and important character of the results which may be expected to reward research in this domain. Figures 214 and 215 show the character of those results for the radiation of zinc oxide at various temperatures from the red heat to 1013°. For the purposes of comparison, platinum was taken as a basis, and the radiation of the oxide was compared, wave-length by wave-length, with that of platinum at the same temperature. It will be seen that the behavior of zinc oxide is very different indeed from that of platinum, and that the former shows a marked tendency to what may be termed selective radiation.

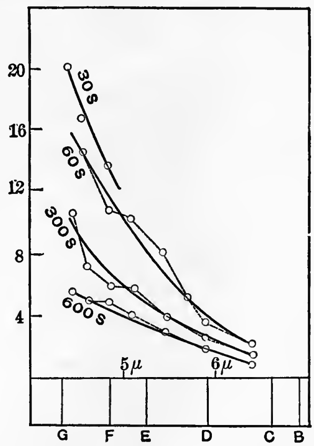


Fig. 216.

In Fig. 214 the results are plotted as isochromatic curves, each wave-length as a function of the temperature. Figure 215 shows the spectrum of platinum and zinc oxide, respectively, at 692°, 848°, and 1013°. It became evident in the course of our work that in the case of some substances radiation must be studied as a function of the time as well as of temperature. Zinc oxide, for example, is found to exhibit decadence of radiating power not unlike that which has already been described in the discussion of the lime light. Figure 216 shows a set of time curves obtained by the observation of the spectrum of this oxide at 1013°.* The method pursued in these investigations is a modification of that used in the study of the influence of temperature upon the color of pigments, to which reference has already been made. The substances the radiation of which it was desired to investigate were laid in a thin layer upon the surface of a strip of platinum, and this strip was heated to the desired degree of incandescence by means of an electric current. The measurement of temperature was obtained by means of the device described in Chapter II.

VI.

The spectrophotometry of pigments.

For the study of the quality of light reflected by various substances, two incandescent lamps, as nearly identical as possible, should be selected. These should be arranged in multiple circuit, and should be adjusted to give the same quality of light. Either the voltage of each or the current necessary to maintain them in such condition should be carefully noted. One of these lamps is taken as the reference lamp, and is mounted at the right hand of the slit of a suitable spectrometer. The brightness of its spectrum may be regulated by any of the methods already described. To the left hand of the slit a layer of the pigment under investigation should be

* Time is expressed in seconds.

placed at an angle of 45° with the horizontal plane. Its surface should be illuminated from above by means of the other lamp, and the distance of this lamp should be adjusted so as to give

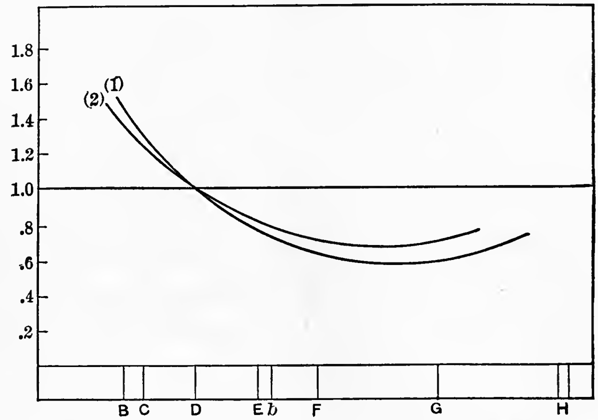


Fig. 217.—Spectrum Curves of Two Specimens of $MgCO_3$.

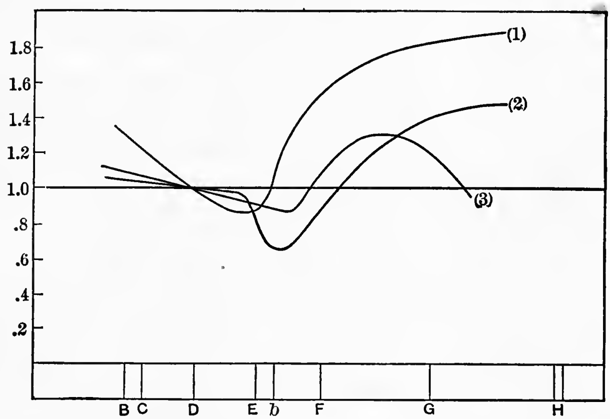


Fig. 218.—Spectrum Curves of Adulterated Whites.
(1) Plaster of Paris.
(2) Magnesium Carbonate.
(3) Paper.

a spectrum as bright as possible without exceeding in any wave-length the intensity of the spectrum of the reference lamp.

In order to avoid errors arising from lack of symmetry in

the apparatus, it is advisable to express the results of the measurements, which may be made in the usual manner, in terms of the light reflected from a surface of neutral whiteness similarly placed. Unfortunately, there exists no pigment which fulfils even approximately the conditions requisite in the reference standard of whiteness. All so-called white surfaces, so far as they have been studied, show a tendency toward yellow, or, in other words, a weakness as regards reflecting power for the shorter wave-lengths.

Many materials, in which whiteness is supposed to be an indication of purity, are adulterated by the addition of a trace of indigo or of artificial ultra-marine. The result is to correct this tendency towards yellow; but for the purposes of the spectroscopist, the addition of such coloring matter renders the surface useless. Figures 217 and 218 show the character of the light reflected by unadulterated carbonate of magnesium, and by specimens of various white substances to which a trace of blue pigment has been added. It will be seen that the addition of the coloring matter produces marked increase of intensity in the blue and violet, but that there still remains a distinct region of absorption in the middle of the spectrum.

The most satisfactory reference standard for white with which the writer is acquainted is obtained by smoking a piece of glass in the fumes of burning magnesium. The coating thus obtained, which should be thick enough to be quite opaque, is uniform in color, and it approaches a true white more nearly than that of any other more available material. That it is, however, far from being neutral in tint will be seen by the inspection of Fig. 158, in the section of Chapter I. which treats of the influence of temperature upon pigments, to which reference has already been made.* The two curves represent the distribution of intensity in the spectrum of the light reflected from such a coating at 25°, and at a high temperature, respectively. The only way of securing a reference standard of perfect neutrality is by studying the character of some actual surface

* Philosophical Magazine (5), Vol. 32, p. 401.

like that of this coating of magnesium oxide, and plotting a curve similar to that shown in this figure. The oxide may then be used as a surface of reference, and correction may be made for its deficiency in the shorter wave-lengths.

These preliminary studies having been completed, and the character of the spectrum of the oxide film having been accurately determined, all measurements of the spectra of pigments may be expressed in terms of the reflection of a hypothetical surface of neutral whiteness. It is convenient to assume the brilliancy of this surface to be such as would be given by a neutral pigment reflecting all wave-lengths in the same proportion as the oxide coating reflects light of the wave-length of the D line. Figures 159, 160 and 161 (Chapter I.) give curves for a number of substances in which this method of expressing the results has been used. While a considerable number of measurements of the spectrum of pigments have been made, especially by Abney in England, there still remains a very large field for investigation. The study of the absorption spectra of solids and of solutions likewise offers interesting topics. Some reference to the method to be pursued in such work has been made in Chapter III. With the exception of the research there described, nearly all the spectrophotometric work which has been done upon solutions has been carried out from the point of view of the chemist. The thorough and quantitative measurement of the transmitting power of solids and liquids, especially of fluorescent materials and of those which exhibit dichroic properties, offers problems of considerable complexity and of very great interest.

References to some memoirs on spectrophotometry:

VIERORDT: Poggendorff's Annalen. Vol. 140.
GLAN: Wiedermann's Annalen. Vol. 1, p. 351.
HÜFNER: Journal für praktische Chemie, N. F. Vol. 16.
CROVA: Comptes Rendus. T. 87, p. 322.
NICHOLS: American Journal of Science, Vol. 28; Kansas Academy of Sciences, Vol. 10; American Journal of Science, Vol. 36; Philosophical Magazine, (5) Vols. 32 and 33; Physical Review, Vol. 2.
ABNEY: Philosophical Transactions, Vol. 179; also Colour Photometry.
KRÜSS: Kolorimetrie.
EWAN: Philosophical Magazine. (5) Vol. 33.

CHAPTER V.

STUDIES OF THE INVISIBLE SPECTRUM.

I.

The infra-red.

The study of the invisible portions of the spectrum is naturally divided into two parts, — that relating to the infra-red spectrum, and that which has to do with the short wave-lengths of the ultra-violet. The heating effect of the infra-red, as has been abundantly shown by Langley, Ångström, Rubens, Snow, and numerous other observers, is sufficient to enable us to make use of the thermopile or bolometer for the measurement of the spectrum of artificial light-sources, and even of bodies at temperatures below the red heat.

The requisites for such work are a galvanometer of the very highest sensitiveness; a bolometer or thermopile, as nearly linear as possible; and a prism, the law of dispersion of which, for the long wave-lengths to be experimented upon, is accurately known. By means of a prism containing bisulphide of carbon, a material the dispersion of which can be expressed by use of the Cauchy formula, the very serious difficulty of determining wave-lengths in this portion of the spectrum with prisms of glass or rock salt may be obviated. The work of Rubens and Snow upon such prisms shows that no dependence can be placed upon values for wave-length obtained by extrapolation, the only reliable method being to actually calibrate the prism to be used by some such method as that proposed by Mouton, for the entire range of wave-lengths to be considered.

The principal objection to carbon bisulphide lies in its great sensitiveness to changes of temperature. Figure 219 shows this

property as exhibited by a prism studied by E. F. Nichols in 1891.* By taking proper precautions to keep the prism in a nearly constant condition, however, and by measuring accurately such changes of temperature as cannot be avoided, it is possible to render negligible the error due to this cause.

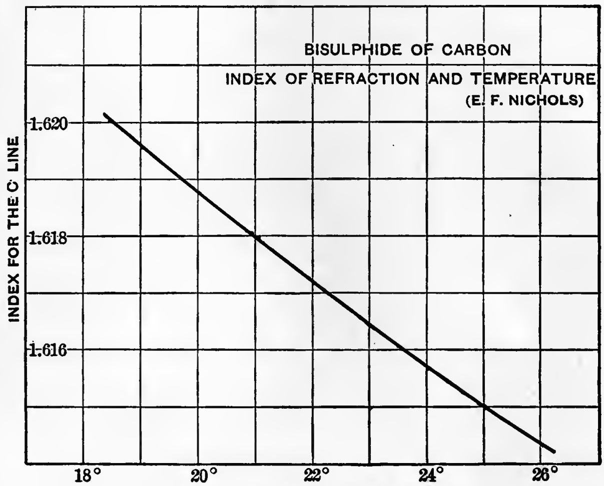


Fig. 219.

The correction for the amount of light absorbed by the material of which the lenses and prism are constructed is a more serious matter. Attempts have been made to substitute diffraction gratings for the prism; but, as has been pointed out by Rowland, and shown experimentally by Paschen, a grating, although superior to any prism where it is a question of a determination of wave-lengths, is totally unfitted for researches in which the distribution of energy in the spectrum is the chief object.

Fortunately it is not always necessary to obtain measurements of the absolute distribution of energy throughout the spectrum. Many interesting and valuable results may be

* Physical Review, Vol. 1, p. 1.

obtained, without eliminating the effects of selective absorption, by the comparison of various sources of light throughout these regions, the intensity, wave-length by wave-length, of the sources in question being expressed in terms of some arbitrarily selected standard. All of the results thus obtained can be converted to absolute measure as soon as the actual distribution of energy in the spectrum of the standard is known.

The relative merits of the thermopile and bolometer for work of this description is a matter not easily determined. Spectrobolometers of iron, nickel, tin, and platinum have given excellent results in the hands of the various observers already mentioned. While seemingly less sensitive than the other metals, platinum offers one great advantage, in that it is capable of being drawn into wire of exceedingly small diameter. The cross-hair wire drawn in silver, for the use of makers of astronomical instruments, has been found by Snow* to afford an excellent bolometer. Before removing the silver coating, the wire is rolled or hammered until it has attained several times its original width. Upon dissolving away the envelope, there remains a ribbon of platinum the breadth of which does not exceed 0.05 mm., and the thickness of which need not be more than one-tenth that quantity.

A bolometer of this description fulfills more nearly the conditions of being strictly linear than any other form as yet devised. Snow found an instrument thus constructed to be of sufficient delicacy to admit of the exploration of the bright-line spectra of the metals. Even the violet and ultra-violet bands of the spectrum of the arc-light gave large deflections with it. The experience of others has been that a linear thermopile made up of antimony and bismuth couples exceeds in delicacy even the best of bolometers, and when used directly in closed circuit with a suitable galvanometer, the thermopile is the more readily handled. It is true that the width of the face of

* Wiedemann's Annalen, Vol. 47, p. 208; also, Physical Review, Vol. 1.

such a thermopile cannot well be reduced much below a half millimeter, which precludes the detailed investigation of wave-lengths greater than 3μ; but with prisms of bisulphide of carbon, or of glass or rock salt, the lack of dispersion for the greater wave-lengths limits the range of investigation in any case to a region not extending greatly beyond that point. Fluorspar possesses properties which make it possible to deal quantitatively with still longer waves, but this is material which, in the form of lenses and prisms, is in the possession of but few physicists.

The method of observation which gives the most satisfactory results in this work is as follows: The source of light to be studied is placed in front of the slit of a spectroscope (Fig. 220)

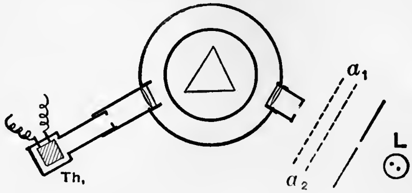


Fig. 220.

in the eye-piece of which the thermopile is mounted. Between the light-source and the slit a double shutter, a_1a_2, is so arranged that it can be withdrawn and replaced by the observer without the necessity of his taking his eye from the reading telescope. The thermopile is put in closed circuit with the galvanometer, the position of the needle of which is then adjusted by means of a controlling magnet. As soon as a condition of thermal equilibrium has been reached, which condition is indicated by the coming to rest of the galvanometer needle, a scale reading is taken. The shutter is then withdrawn, and the light is allowed to fall upon the thermopile during an interval corresponding to the first swing of the galvanometer. The reading corresponding to the extreme of this first swing is noted, the shutter is replaced, and the return swing is read. The galvanometer needle will not in general return to its zero point, but

it does reach a position not widely divergent from the same. By taking the average of the first and third readings instead of the original zero, the drift of the galvanometer during the short intervening period is annulled.

A sufficient number of readings should be taken in this manner to afford data for the determination of the value of the intensity of radiation of the wave-length reaching the thermopile. The circle reading of the spectroscope is then changed, and the operation is repeated. Thus the entire region of the infra-red may be explored. The sensitiveness to thermal disturbances is always so great that a considerable interval of time must be allowed to pass after each setting before satisfactory readings can be undertaken. It is not unusual, indeed, to be compelled to wait for half an hour or longer after each readjustment of the spectroscope before the galvanometer indicates, by the return to its old position, that the thermal balance has been restored. While working in the weaker portions of the spectrum, it is a matter of necessity to bring the galvanometer to such extreme sensitiveness as to introduce a rather rapid drifting of the needle. It frequently happens, indeed, that the drift is sufficient to carry the needle entirely off the scale during the interval between each setting and the restoration of equilibrium. It is necessary, therefore, to bring the needle back by rapid manipulation with the controlling magnet.

Such changes cannot be made without varying appreciably the constant of the instrument, and it is out of the question to maintain a sufficiently sensitive galvanometer with its figure of merit unchanged even for short intervals of time. In order to render successive readings comparable, some device must be used, therefore, to determine the constant of the instrument from moment to moment. An excellent plan of accomplishing this is by means of a subsidiary coil placed in a fixed position behind the galvanometer. By sending a known current through this coil, a current of such intensity being selected for the purpose as will produce a deflection, the size of which is of the

same order as those obtained from the thermopile in the course of the experiment, it is possible to correct for the ever-fluctuating figure of merit of the instrument.

Figure 221 shows a convenient arrangement for carrying out this method. Current from a storage battery is sent through a

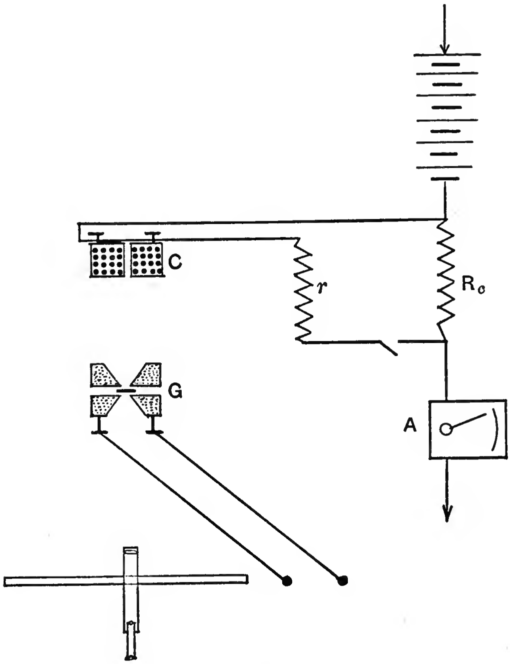


Fig. 221.

compensated resistance R_c and the ammeter A. The calibration coil C is shunted around R_c. Changes of constant brought about by outside magnetic disturbances, such as manifest themselves continually in the laboratory as well as those which are due to the rapid shifting of the controlling magnet, are thus readily taken account of and eliminated. A switch at the hand of the observer enables him to close this circuit before and after every set of readings and to make the necessary corrections for changes in conditions of the sensitiveness of the galvanometer.

Details concerning the construction of galvanometers of extreme sensitiveness will be found in the various memoirs upon the bolometric study of radiation, to which reference is made at the end of this chapter.*

The method of studying the infra-red just described, modified to suit the requirements of each problem, is of wide application. It will be illustrated here by two examples.

(1) *Study of the infra-red spectrum of the glow-lamp.*† — It was the purpose of the measurements to be described to trace the growth, wave-length by wave-length, of the radiation from the lamp-filament, with increasing expenditure of electrical energy.

Three lamps were tested, two of which had been especially constructed for the purpose. These two were attached to a common exhaustion tube, so that the atmospheric conditions within the two bulbs might be identical (see Fig. 222). The filaments also were similar, save that one possessed a surface of lampblack, while the other was given the silvery surface characteristic of carbons flashed in hydrocarbon vapor.

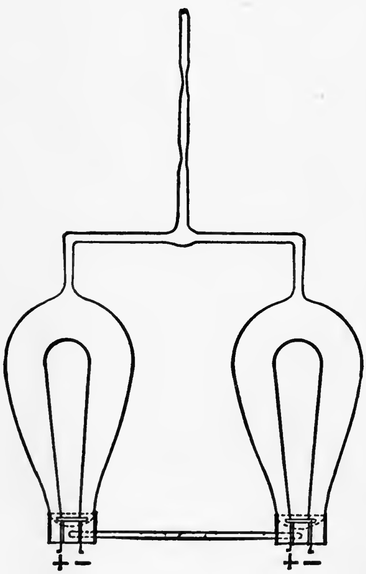


Fig. 222.

The method employed in this experiment was that outlined in a previous paragraph; and the procedure was repeated with each lamp until a series of curves showing the distribution of energy in the region lying between $0.8\,\mu$ and $3.0\,\mu$ was obtained under conditions of electrical activity varying from 35 to 85 watts in the black-coated filament, and between 15 and 75 watts

* See also Nichols, The Galvanometer, Lecture VII.

† Nichols, Physical Review, Vol. 2, 1894.

in the treated filament. The lower values corresponded in each case to a temperature just below the red heat, the larger amount of energy necessary to raise the blackened filament to that temperature being due to the presence of the layer of lamp-black. That coating added nothing to the conductivity of the filament, while it greatly increased the radiating surface.

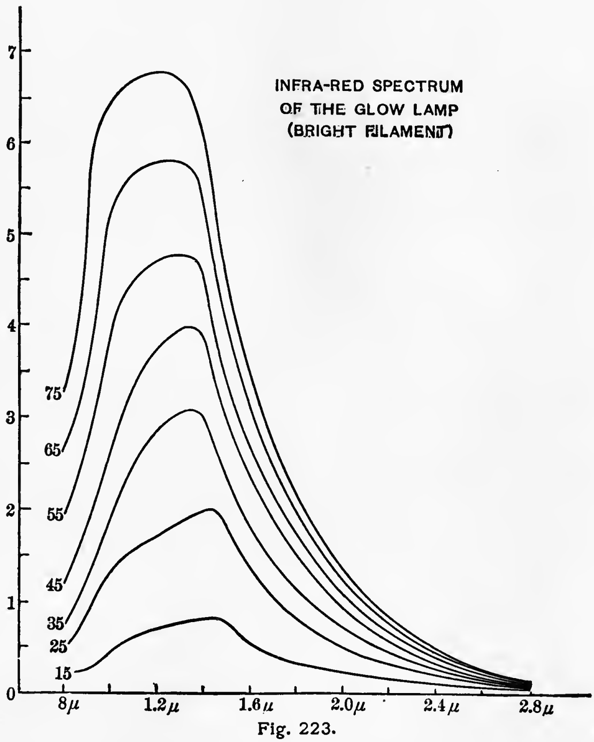


Fig. 223.

The results of these determinations, after all corrections for wave-length and intensity were made, are given in Figs. 223 and 224. It will be seen that these two lamps showed marked difference in the distribution of intensities. The high values in the curves of the treated filament, for regions bordering upon the red (Fig. 223), indicate the superiority of carbons with gray or silvery surface, over untreated filaments, for the purposes of

illumination. This is a result which had been previously reached by quite different methods.*

The most important correction in measuring radiation in the infra-red is that due to the width of the thermopile or bolometer. Whatever be the width of the exposed surface, it will cover a

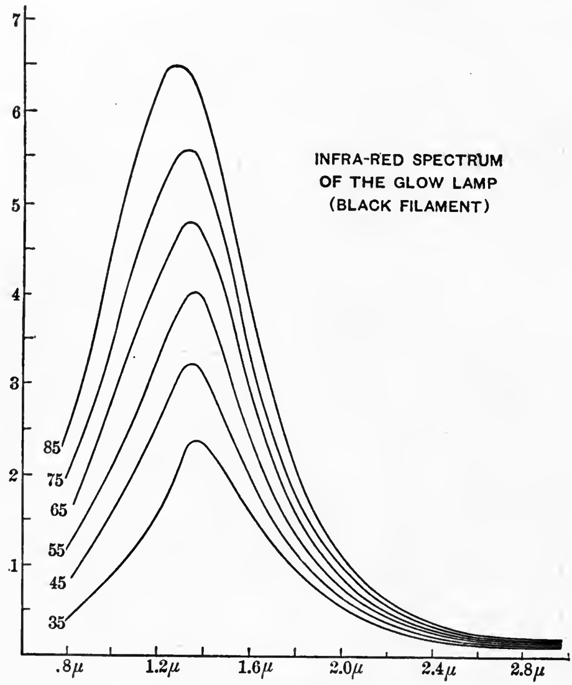


Fig. 224.

greater and greater portion of the spectrum, measured by wave-length differences, as we deal with longer waves. The normal curve would be one made with a strip the width of which contracted in proportion to the increasing contraction of the spectrum. The correction which must be applied to reduce actual readings to normal ones is very large. Indeed, shortly after passing 3 μ, if a bisulphide prism be used, it will be found that

* Evans, Proceedings of the Royal Society, 1886.

all the remaining rays are dispersed through a space but a fraction of a millimeter wide.

Figure 80 shows the relation of the corrected to the uncorrected curve in a typical case. The width of the thermopile was taken as normal, for convenience, at the D line.

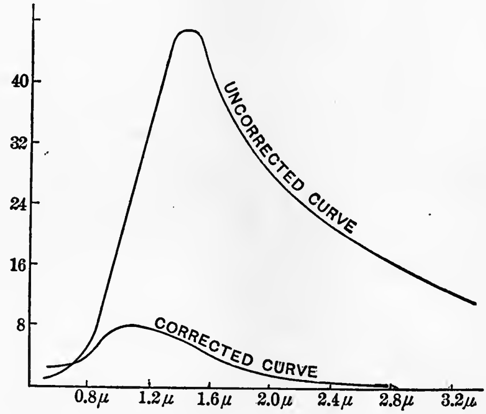


Fig. 225.

The third lamp studied by this method was a commercial lamp with untreated filament. Its spectrum, which was explored only in a single stage of incandescence, gave a curve of distribution (Fig. 226) similar in type to that obtained with the black filament. In these determinations no correction was made for the selective absorption by the glass of the lamp-bulb and of the spectroscopic lenses. Indeed, no such correction was necessary, since glass is of very nearly uniform diathermancy throughout the region embraced in these experiments (see Figures 227 and 228).

(2) *Absorption spectra in the infra-red.* — The second illustration of the method is taken from the researches of E. F. Nichols,* in which the glow-lamp, the spectrum-curve of which has been shown in Fig. 226, was used as a standard. This lamp was placed before the slit of the spectroscope (Fig. 220), and its

* Physical Review, Vol. 1, p. 1, 1893.

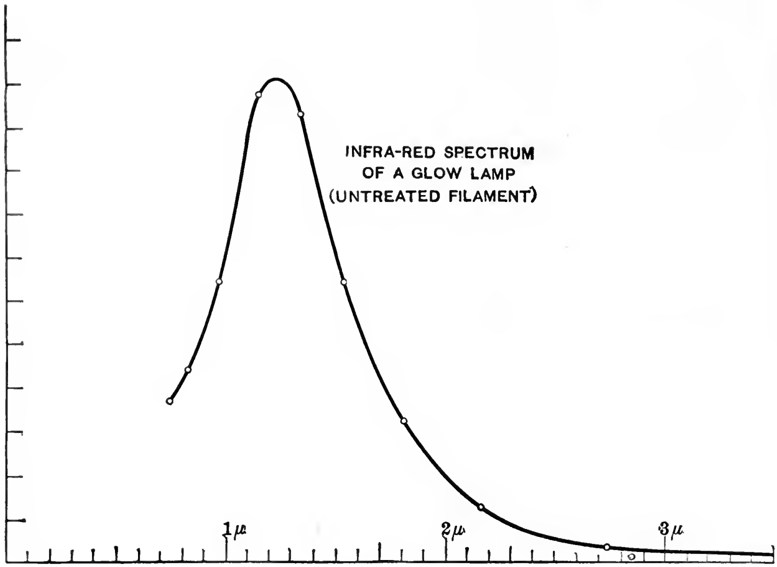


Fig. 226.

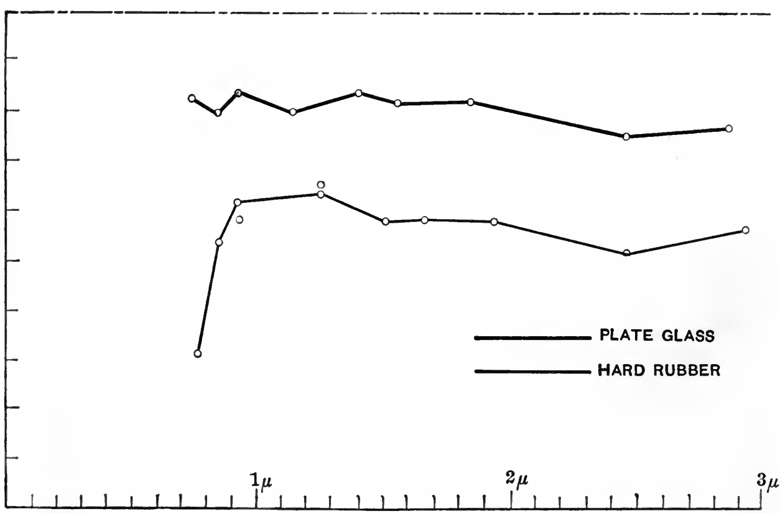


Fig. 227.

rays were made to pass through a cell containing various solutions, the diathemancy of which were to be tested; also through various solids which it was desired to subject to inquiry. Differences between the spectrum curves of the direct and of the transmitted rays indicated the transmitting powers of the substances which had been interposed. From these the relative diathermancy could be determined, and the regions in which relative absorption occurs could be located. The results, as

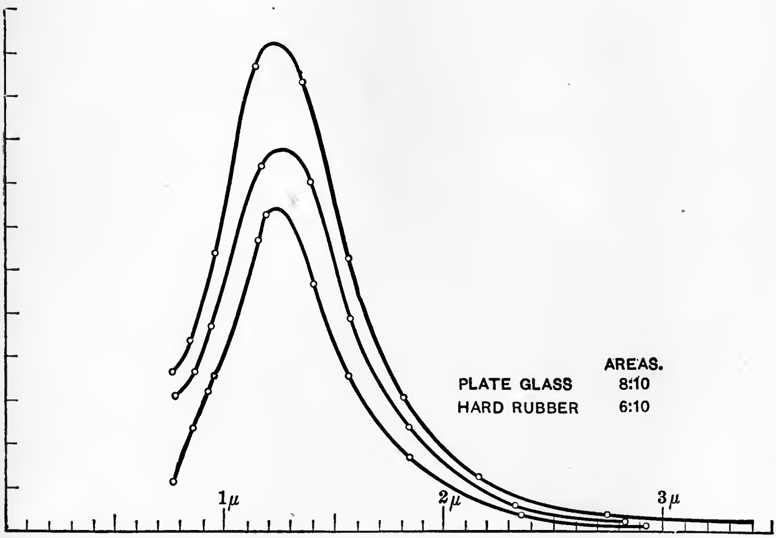


Fig. 228.

regards alum in solution and water, have been discussed in Chapter II. They are indicated graphically in Fig. 169. Other interesting cases are those of plate-glass and vulcanite, the relative diathermancy of which is shown in Fig. 227. There are two ways of exhibiting the result of such measurements. The first is that followed in Fig. 227, while the other consists in plotting upon the same sheet the curve of distribution for the direct and for the transmitted rays. This method is illustrated in Fig. 228.

References to memoirs upon the infra-red spectrum:

HERSCHEL: Proc. Royal Society. p. 209. 1840.
DRAPER: Philos. Mag. Vol. 24, p. 456. 1842.
FIZEAU AND FOUCAULT: Comptes Rendus. Vol. 25, p. 449.
LAMANSKY: Poggendorff's Annalen. Vol. 146, p. 207.
E. BECQUEREL: Comptes Rendus. Vol. 69, p. 999 (1869); Vol. 77, p. 303 (1873).
H. BECQUEREL: Comptes Rendus, Vol. 96, p. 123; Vol. 97, p. 71; Vol. 99, p. 417; Annales de Chimie et de Physique, Vol. 30, p. 33.
JACQUES: Proc. Am. Academy of Sciences. 1879.
MOUTON: Comptes Rendus. Vol. 89, p. 298.
ABNEY: Philos. Mag. (5), Vol. 7, p. 316; Philos. Trans., II., p. 653 (1880); Proc. Royal Society, Vol. 32, p. 443; Philos. Trans., p. 457. (1886); also p. 887 (1891).
DESAINS: Comptes Rendus. Vol. 95, p. 435.
LANGLEY: Philos. Mag. (5). Vol. 26, p. 511.
ÅNGSTRÖM: Wiedemann's Annalen, Vol. 26, p. 262; Vol. 36, p. 715; Vol. 39, p. 267; Öfversigt af K. Vet Akad. (Stockholm. 1889), p. 549; also 1890, p. 331, and 1893, p. 389.
JULIUS: Arch. Neerl. (22), p. 310; Verhandlungen des Vereins zur Beförderung des Gewerbbleisses (1890 and 1893).
RUBENS AND RITTER: Wiedemann's Annalen. Vol. 40, p. 62.
RUBENS: Wiedemann's Annalen. Vol. 37, p. 256; Vol. 45, p. 238.
KAYSER AND RUNGE: Wiedemann's Annalen. Vol. 41, p. 306.
SNOW: Physical Review. Vol. 1. 1893.

II.

The ultra-violet.

The energy of the ultra-violet rays of all but a few sources, perhaps of all save the electric arc, which possesses two or three groups of lines of such intensity as to give considerable deflections when a suitable bolometer is exposed to them, is so small as to preclude measurement. Photography enables us to establish the presence of the rays which make up this part of the spectrum, however, and as a means of determining wave-lengths, it has, in the skilled hands of Rowland, of Kayser and Runge, and of others, furnished results of highest accuracy. All attempts to establish a law by means of which, from the effects produced in the sensitive film, the energy of the impinging ray

may be computed, have, thus far, been futile, and the photographic study of the ultra-violet is per force confined to the discovery and location of bright and dark lines and of selective absorption.

Determination of transparency to the ultra-violet rays.—Experiments analogous to those upon diathermancy, described in a previous paragraph, have been made by the same investigator,* upon the transparency of various substances to the rays lying beyond the violet. The apparatus employed was a concave grating with a radius of six feet, a silvered mirror by means of which the sun's rays could be brought upon the slit and celluloid films of high sensitiveness.

By interposing the materials under investigation before half of the slit, two spectra are obtained—the one produced by the undiminished rays of the sun, the other by the same rays after transmission through the absorbing medium. The dark solar lines are present in both spectra, and serve to indicate wave-lengths. Regions of absorption are discovered by means of the diminished brightness of the photograph.

The materials thus far tested by this method appear to be of four classes:

(1) Substances entirely transparent throughout the ultra-violet.

(2) Substances quite opaque throughout the ultra-violet.

(3) Substances the transmitting power of which diminishes steadily with the wave-length.

(4) Substances the spectra of which have well-defined absorption bands.

Figure 229 is from photographs of the absorption spectrum; I., of a pane of common window glass compared with that of sunlight, which had suffered no diminution excepting by passing through the solar and telluric atmosphere; II., of a solution of sulphate of quinine compared with that of the empty cell.

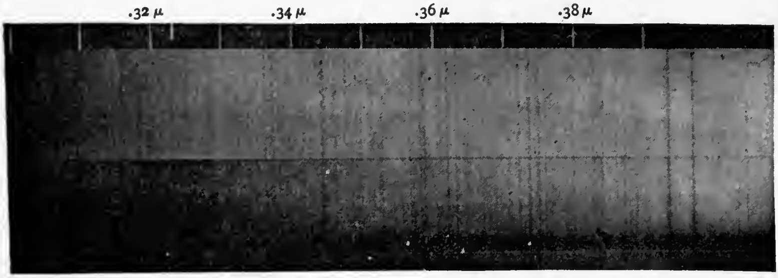
---

* E. F. Nichols, in Physical Review, Vol. 2.

The spectrum of the electric arc. — The photographic study of the ultra-violet of this source is of especial importance on account of its very high temperature. Its spectrum consists of bright lines due to the incandescent vapors of whatever elements may happen to be present or may be introduced for the purpose.

I.

.32 μ .34 μ .36 μ .38 μ



II. HH

.32 μ .43 μ .36 μ .38 μ .40 μ

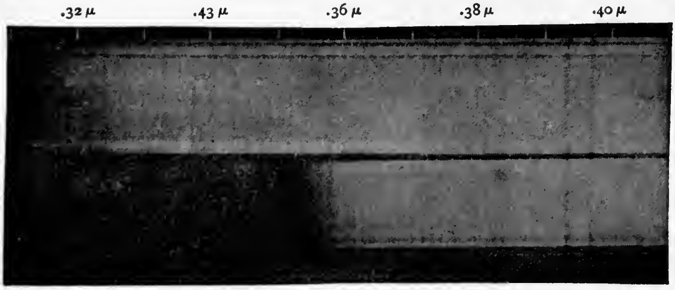


Fig. 229. Photographs of Absorption Spectra in the Ultra-violet.
I. Absorption by a pane of window glass.
II. Absorption by a solution of sulphate of quinine.
E. F. Nichols.

For nearly all work in this field, the Rowland concave grating of six feet radius is well adapted, while the mounting constructed by Brashear, a sketch of which is shown in Fig. 230, reduces the operation to its very simplest form.

In this apparatus two iron rails, RR_1 (Fig. 231), forming a right angle at the apex of which the slit S is placed, carry the grating (G) and plate-holder (P), respectively, upon wheeled

cars, one on either rail. The cars are rigidly connected by means of an iron bar, *B*, the distance between them being that of the radius of curvature of the grating. A quartz lens is

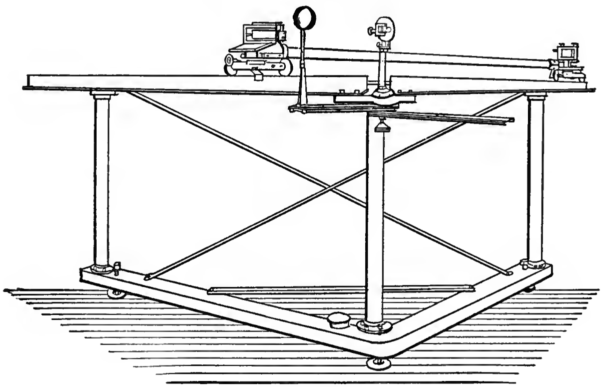


Fig. 230.

sometimes used to bring rays from the source, the spectrum of which is to be studied, to a focus upon the slit.

A valuable accessory to this apparatus is a silvered concave mirror of large surface, good figure, and of long radius. By means of such a mirror, the rays to be photographed can be brought to the slit without the intervention of a lens, and the investigation can be extended to regions which would otherwise be out of range.

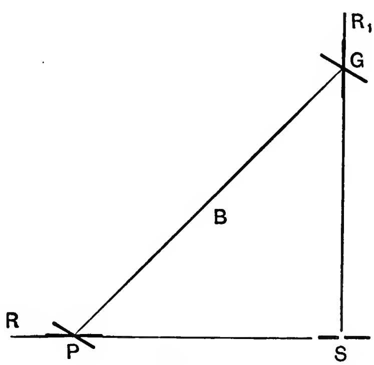


Fig. 231.

It is convenient in the study of the arc-light to place the lamp, *L*, and the mirror, *M*, in an adjoining room, as shown in Fig. 232. The lamp to be mounted is at a distance from the mirror somewhat less than the radius of the latter, which is so adjusted that the cone of reflected rays will pass through an aperture and form an image of the arc upon the slit *S*.

With a grating of the focal length described above, photographs in the first spectrum will present the general appearance

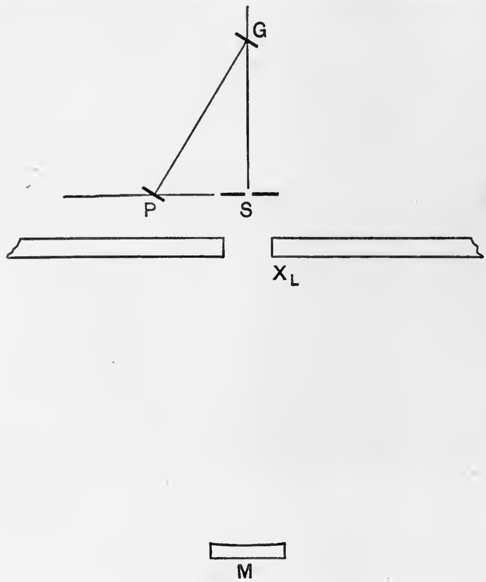


Fig. 232.

of Fig. 233, which is from a photograph of a portion of the spectrum of a flaming arc made by Miss C. W. Baldwin under the direction of the writer. This shows two distinct spectra, the one consisting of long, well-separated bright lines. These

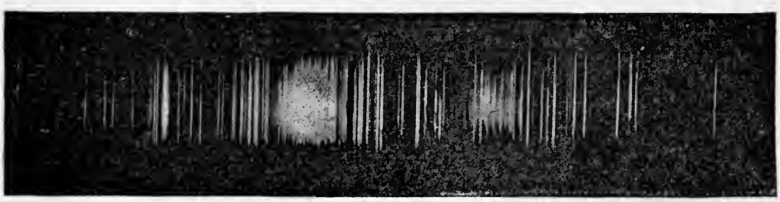


Fig. 233.

are superimposed upon bright patches of light, which in the reproduction appear to be continuous, but which in the original are seen to consist of a multitude of very fine lines closely placed and equidistant.

A portion of the brightest group, many times enlarged, is shown in Fig. 234. The determination of wave-lengths in such

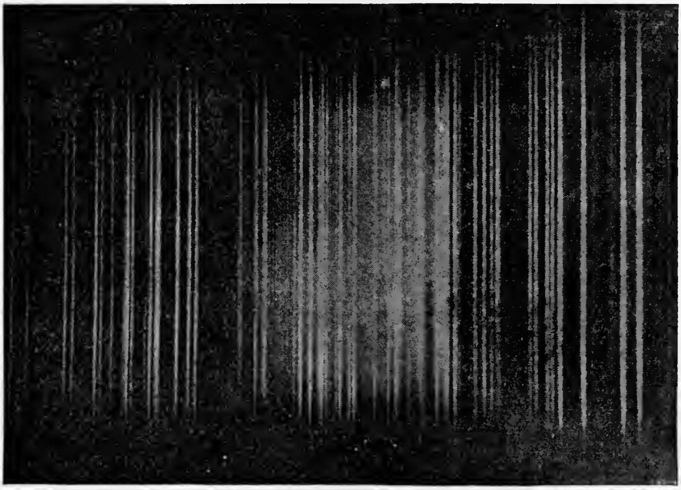


Fig. 234.

investigations is most conveniently made by photographing sunlight upon the same plate as a reference spectrum. Even in the primary spectra, with such a grating, the dispersion is too great to give the whole spectrum upon a single plate. By using films and a holder which will enable them to be exposed with the surface lying everywhere upon the circle of exact focus, the region taken at each exposure may be somewhat extended, but the only satisfactory method in any case is to take overlapping photographs throughout so much of the spectrum as is to be explored.

References to memoirs upon the spectrum of the electric arc:

FRAUNHOFER: 1817.
WHEATSTONE: B. A. A. S. Reports. 1835.
FOUCAULT: 1849.
DESPRETZ: Comptes Rendus. 1850.
VAN DER WILLIGEN: Poggendorff's Annalen. 1859.
STOKES: Philos. Trans. 1862.
LOCKYER: Philos. Trans. 1874. Proc. Royal Society. 1879.
TROWBRIDGE AND SABINE: Philos. Mag. 1888.
KAYSER AND RUNGE: Die Spektren de Elemente. Berlin. 1888–1892.
SNOW: Physical Review. Vol. 1. 1893.

CHAPTER VI.

STUDIES IN PHYSIOLOGICAL OPTICS.

I.

The measurement of duration of impressions upon the retina.

The duration of impressions upon the retina has long been known to depend upon the color of the light producing it, and E. S. Ferry,* making use of a method described by the writer in 1881,† has shown that the duration of impression is in all cases inversely as the luminosity of the ray to which it is due.

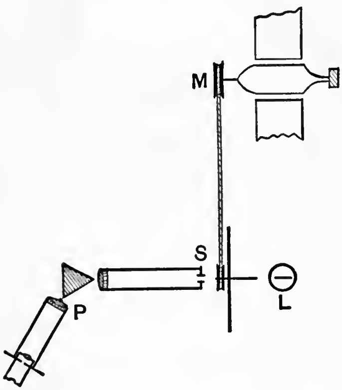


Fig. 235.

The apparatus used in making this experiment consists of a spectroscope (Fig. 235), before the slit of which revolves a disk with open sectors, the speed of which is under the control of the observer. The best means of driving this disk is by attaching it to a suitable electric motor (M), the velocity of which can be regulated by the current flowing around its field magnet.

The telescope of the instrument having been set so as to bring a given portion of the spectrum into the eye-piece, the wheel is set in motion and is driven at greater and greater speed up to the point where the flickering of the region of the

* Ferry, American Journal of Science, Vol. 44, p. 192.

† Nichols, American Journal of Science, Vol. 28, p. 243.

spectrum under inspection ceases. The rate of revolution of the disk is then recorded upon a revolving drum, following some one of the various well-known methods for that purpose.

The results obtained by exploring the visible spectrum from end to end in this way and plotting wave-lengths in their rela-

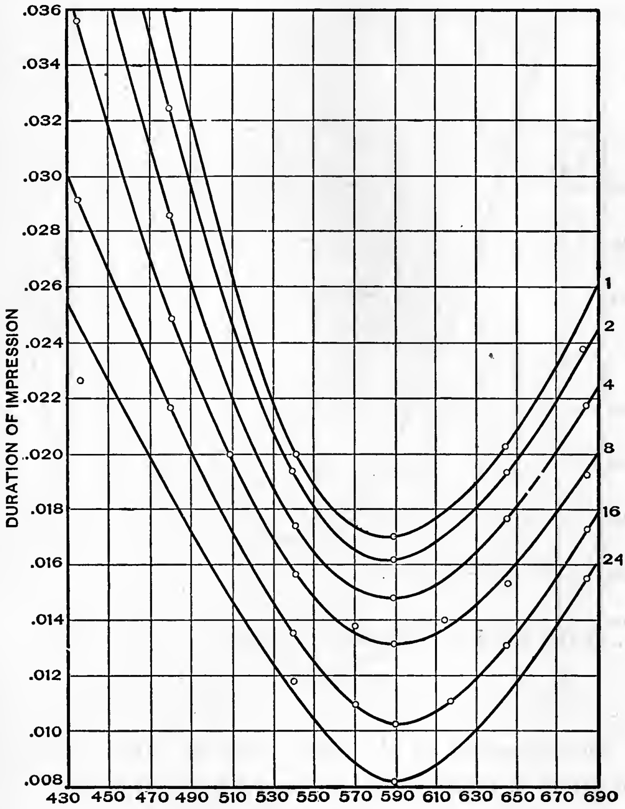


Fig. 236.

tion to duration of impression give curves similar to those in Fig. 236, which shows the duration of impression of different regions in the spectrum of the glow lamp. The various curves relate to spectra, the brightness of which varied from 1 to 24. The best source of light for this purpose is an

incandescent lamp, since it may be placed in the neighborhood of the revolving disk without being in any way disturbed by the air current set up in consequence of the motion of the latter.

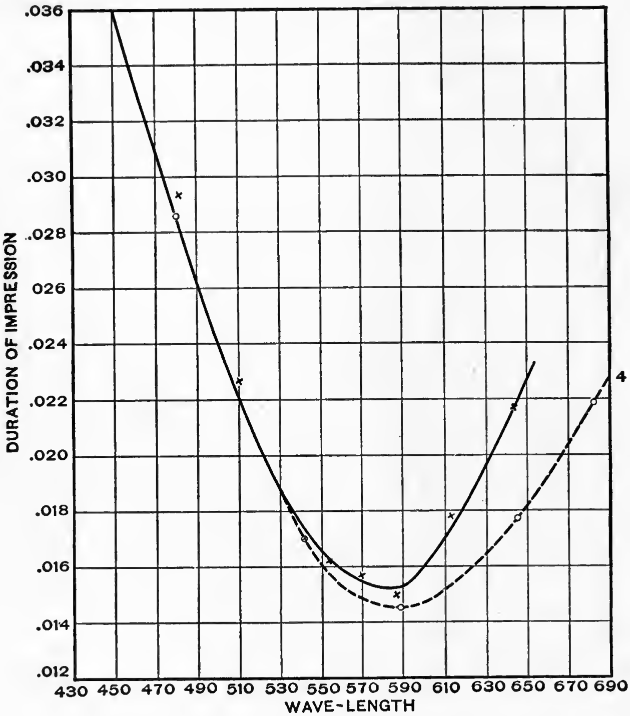


Fig. 237.—Duration of Impression for the Red Blind Eye.

The determination of the luminosity of the spectrum, for which duration of impression has been made, gives us curves of which that shown in Fig. 239 (on a subsequent page) is a typical example. The character of this curve is plainly in close relation to the curves of duration. The maximum of luminosity and minimum of duration occur at the same wave-length, and the values throughout are evidently reciprocal to each other.

A study of color-blind individuals shows that for the dichroic eye this same reciprocal relation exists between duration and

luminosity. Figures 237 and 238 show the curves obtained by Ferry for two cases of dichroism, results which can be verified by means of the luminosity curves of other individuals possessing these peculiarities of vision.* The student who repeats this investigation should endeavor to extend the observations just

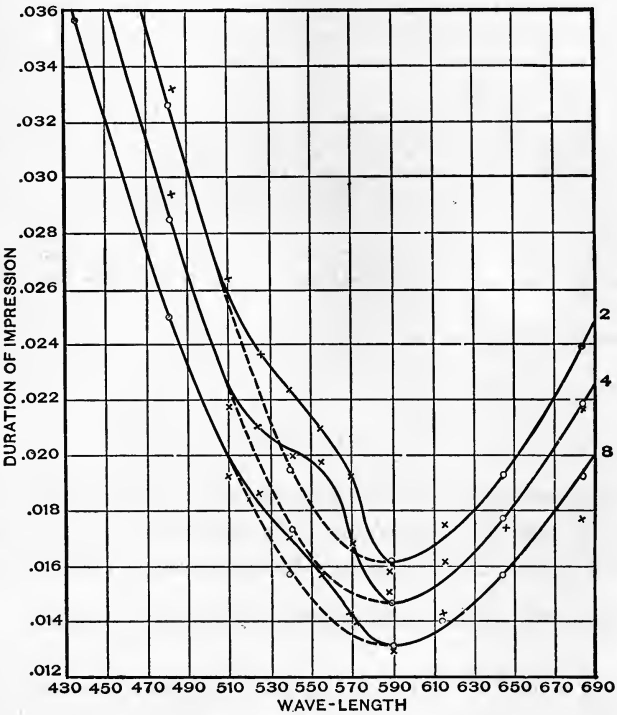


Fig. 238.

described to very weak spectra, and also to very bright spectra, for the purpose of determining through how wide a range the reciprocal relation between luminosity and duration of impression extends. He should also study the effects of temporary color-blindness produced by persistent gazing through screens of

* See Fig. 240, and the pages treating of the distribution of luminosity in the spectrum.

colored glass at some bright source, such as the positive carbon of the arc-lamp, or better still, by exposing the retina for a given time to some previously selected wave-length of the spectrum.

References to the earlier literature:

SEGUER: De raritate luminis. Göttingen. pp. 5–8. 1740.
D'ARCY: Mémoire sur la durée de la sensation de la vue. 1768.
CAVALLO: The Elements of Natural or Experimental Philosophy. London, 1803. Vol. 3, p. 135.
PARROTT: Entretiéns sur la physique. Dorpat, 1819. Vol. 3, p. 235.
PLATEAU: Dissertation sur quelques propriétés des impressions produites par la lumière sur l'organs de la vue. Liege, 1829.
EMDMANN: Ueber die Dauer des Lichteindrucks. Poggendorff's Annalen, 91, p. 611. 1854.
PLATEAU: Sur une loi de la persistance des impressions dans l'œil. Bruxelles, 1878, Bulletin de l'Academie royale de Belgique (2). T. 46. 1878.
VON HELMHOLTZ: Handbuch der Physiologischen Optik.

II.

The determination of the distribution of luminosity in the spectrum in case of normal and of various dichroic eyes, with comparison of the various methods hitherto used for this purpose.

The line of work contemplated under this heading is as follows:

Given a source of light of the greatest possible steadiness, preferably an incandescent lamp upon a storage battery circuit: to determine the luminosity of the various portions of the visible spectrum by the methods of Vierordt, Schumann, Abney and Festing, Macé and Nicati, and Ferry, extending the observation to such cases of dichroic vision as are available, and also to temporary color-blindness of the observer himself, produced by the method indicated in the last paragraph. The accompanying diagram indicates the general character of the results which should be obtained. Of these, Fig. 239 gives the luminosity of

the spectrum of the incandescent lamp to the normal eye, as determined by Ferry. Figure 240 is from measurements upon the spectrum of the positive pole of the arc-light, the observers being normal and red blind, respectively. It is from Abney's book on Color Measurements.

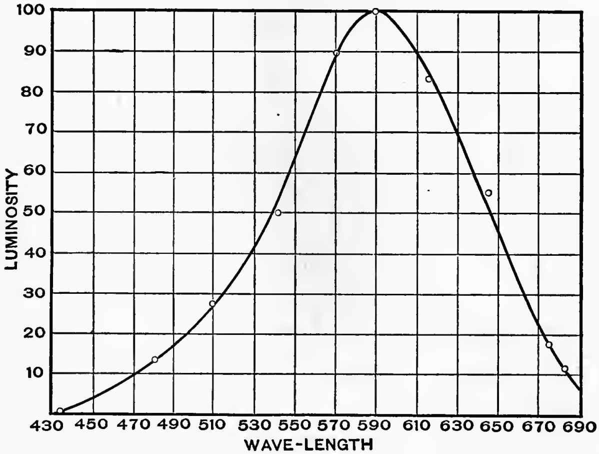


Fig. 239.

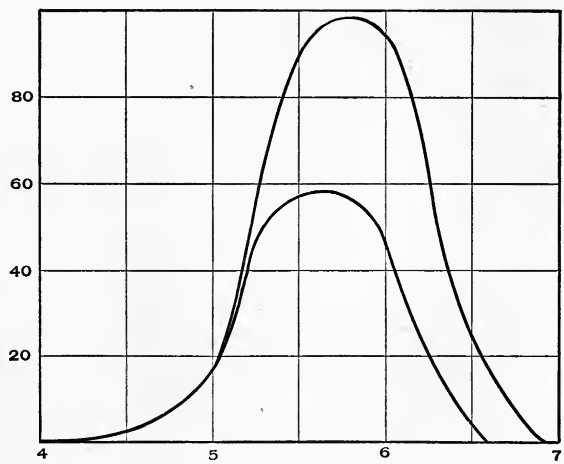


Fig. 240.

References:

VIERORDT: Annalen der Physik und Chemie. Vol. 37, p. 200.
MACÉ AND NICATI: Journal de Physique (2). Vol. 2, p. 75. 1883.
SCHUMANN: Elektrotechnische Zeitschrift. Vol. 5, p. 224.
LANGLEY: Memoirs of the National Academy of Sciences. Vol. 5, p. 1. 1888.
ABNEY AND FESTING: Trans. Royal Society of London, p. 547. 1888. See also Abney, Color Measurement and Mixture, London, 1891.
FERRY: American Journal of Science. Vol. 44, p. 192.

III.

A study of the position of the neutral zone in the spectrum for dichroic observers following the method described by Koenig, and extending his observation to the case of temporary color-blindness.

The instrument with which Koenig's method is usually carried out is the color-mixing spectroscope of von Helmholtz, but any spectrometer with movable telescopes and revolving table for the prism, can be used in place of that instrument with good results.

Figure 241 shows the arrangement of the spectrometer. The prism, which should have three polished sides, is mounted with one of its vertical edges intersecting the optical axis of the reading telescope, which axis should bisect the angle between the adjacent faces. The collimating telescope is placed so as to bring a well-defined spectrum into the field of the observing telescope, with the prism still in the position just described. The face *aa* of the prism is to be covered with the screen *S*, consisting of a strip of sheet metal, the outer surface of which has been carefully smoked in the flame of burning magnesium ribbon.

The deposit thus formed affords a surface, as has already been pointed out (Chapter IV.), the character of which approaches more nearly that of the neutral white than any other which can be obtained.* It is not a true white, but its devia-

* See Nichols and Snow, Philosophical Magazine (5), Vol. 32, p. 408, 1891.

tion from neutrality is not such as to interfere in the least with its usefulness in this experiment.

Two sources of light which should be as nearly as possible identical in quality are to be used. These may be two gas flames from similar burners or two incandescent lamps which had been previously adjusted to identity of color by comparing them on the photometer bar by means of some form of instrument which, like the Ritchie photometer, is sensitive to color differences. The modification of the Ritchie photometer, devised by the writer and described by Ferry,* is especially well adapted for such a test. Of these two light sources, one, L_1, is placed before the slit of the spectrometer, while the other one is mounted upon a sliding block or upon a car running upon a track at right angles to the axis of the observing telescope.

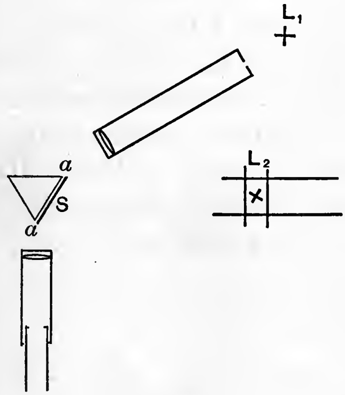


Fig. 241.

The spectrum produced by the dispersion of the rays from L_1 having been brought into focus by the adjustment of the observing telescope, the eye-piece of the latter is removed. The field of view will then be seen to consist, as Koenig has pointed out in his paper, of the two faces of the prism. Of these, the side covered by the screen of magnesium oxide will appear white, its brightness depending upon the distance of the light, L_2. The other face, that through which the dispersed rays pass, will have the color of that particular region of the spectrum the rays of which happen to be parallel with the axis of the telescope.

Under these conditions it is possible, by moving the telescope

* Ferry, Physical Review, Vol. 1, p. 339, 1894.

upon its arm, to bring into comparison with the white screen S all portions of the visible spectrum in succession. In the case of the dichroic eye there will be always found a region in the green-blue which can be matched with the white of the screen. The two halves of the field of view in this region can be brought to equal brightness by moving the source of light L_2 along the track already described, or by opening and closing the slit of the spectrometer.

The wave-length of this neutral region is well defined in the case of each individual. Its existence is one of the best tests of dichroism. Koenig finds the wave-length of this zone to lie between 0.490 μ and 0.505 μ.

IV.

The detailed study of the personal errors in photometry, with an attempt to trace these to their physiological sources.

It has been noticed that observers with the Bunsen photometer tend to set the instrument either to the right or left hand of its true position by an amount constant and quite well defined for each individual case.* The tendency is for the great majority of observers to set to the left hand of the true position, while a few make settings persistently to the other side. This personal error does not disappear as the result of long-continued practice, and it seems to be directly traceable to a lack of symmetry on the part of observers who use the two eyes simultaneously. The simplest method of checking these facts consists in the comparison of readings made by means of the Lummer-Brodhum photometer, in which only one eye is used, with those obtained by means of the usual form of the Bunsen disk. In the photometry of sources which differ somewhat in color a new difficulty arises in the refusal of the eye to recognize equality between two fields of view which differ in tint. A comparison

* Transactions of American Institute E. E., Vol. 6, p. 335.

of various types of photometer in which these color differences are more or less easily recognizable seems to show a definite relation between the errors of observation and the sensitiveness of the instrument to color differences. Some observations made by Mr. C. H. Sharp in 1893 will serve to indicate the character of the results which may be expected from the study of this phenomenon.

His results are as follows: A considerable number of observations were taken with different types of photometer to determine their relative color sensitiveness. This was expressed in terms of the difference in voltage between two similar incandescent lamps necessary to produce a noticeable effect. The mean of 16 observations with the Lummer-Brodhum photometer, 20 with the Bunsen, and 21 with the Nichols-Ritchie, taken on six different days, gave for voltage difference corresponding to a barely perceptible color difference:

For Lummer-Brodhum,	8.6 volts.
For Bunsen,	5.78 volts.
For Nichols-Ritchie,	5.37 volts.

From 40 settings with Lummer-Brodhum, and 50 with each of the others, for two similar incandescent lamps at equal voltages, the mean error of a single setting was found; viz.:

Lummer-Brodhum	$= \pm$ 0.884 divisions.
Bunsen	$= \pm$ 1.459 divisions.
Nichols-Ritchie	$= \pm$ 1.448 divisions.

The product of mean error and color difference voltage gave

For Lummer-Brodhum,	7.60,
For Bunsen,	8.43,
For Nichols-Ritchie,	7.78,

from which it appears color-sensitiveness is very nearly in inverse proportion to photometric sensitiveness.

The student should extend these observations with a view

to determining in how far photometric observations which are based upon the attempt to balance fields of view, which are illuminated by sources of light not identical in color, is warrantable. The fundamental principles of photometry laid down by von Helmholtz in the first edition of his handbook* involves identity of quality in the lights to be compared as a necessary basis for all photometry. There have been, however, of recent years a number of methods which depend upon the quantitative comparison of color with neutral fields. The establishment of the validity, on the one hand, or the rejection on the other, of such methods in photometry is a matter greatly to be desired.

V.

Studies of the physiological effects of the light passing through sectored disks in rapid revolution.

One of the best known methods of cutting down the light of sources which are too bright for ordinary photometric comparisons, is by the interposition of a rapidly driven disk containing alternate open and closed sectors. By the use of two such disks arranged upon a common axis, and adjustable with reference to one another (Fig. 242), any desired ratio of light and darkness can be secured. Photometric work with the revolving disks has brought out the fact that the effect of the light passing through them is not in direct proportion to the width of the open sectors. The most recent and complete study of this phenomenon is due to E. S. Ferry,† who finds that as the an-

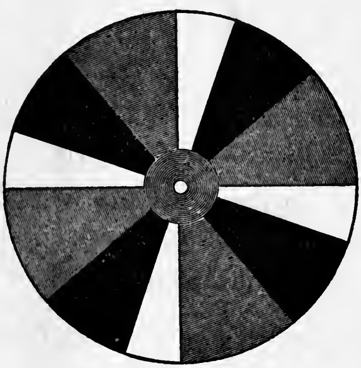


Fig. 242.

* Von Helmholtz, Handbuch der Physiologischen Optik.
† Physical Review, Vol. 1, No. 5, p. 338.

gular width of the open sector is diminished, the effect of the light which reaches the photometer falls off more rapidly than it should do. For small angular openings, this error amounts to several per cent. The rise of this correction from negligible values to very large ones is shown in Fig. 243, in which ab-

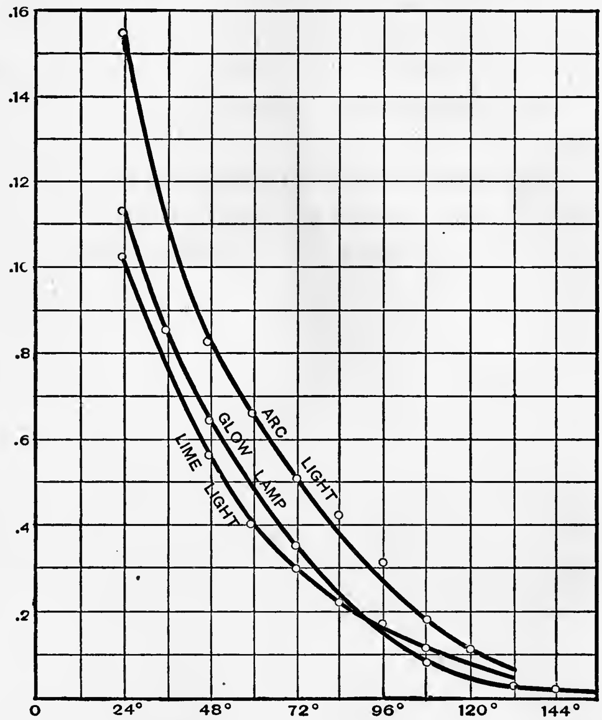


Fig. 243.—Total Angular Opening.

scissas are total angular openings for the entire circumference, and ordinates are errors.

The following conclusions, reached by Mr. Ferry as the result of his investigations, afford a basis for further study of this interesting subject:

"(1) While the ratio of the energy of the light transmitted by a rapidly rotating sectored disk to that of the total incident illumination is equal to the ratio of the total aperture of the

disk to the entire disk, yet the effect of this light upon the retina will not always be proportional to the ratio of the total aperture of the disk to the entire disk.

"(2) With mixed light containing elements of different luminosity shining upon the retina, a rotating sectored disk will appear to not cut off all the elements in equal proportion, but will intercept most strongly the elements of low luminosity.

"(3) With any given light, the error introduced by the use of the rotating sectored disk increases as the aperture of the disk diminishes.

"(4) With ordinary illuminants, the error is negligible when the total aperture of the disk is more than one-half the entire disk, but rapidly increases as this aperture is diminished."

CHAPTER VII.

EXPLORATIONS OF THE EARTH'S MAGNETIC FIELD.

The object of this work is primarily to obtain an accurate knowledge of the values of the horizontal component of the earth's magnetism throughout a limited region, such as a portion of a laboratory building or the region comprised in and surrounding a magnetic observatory. Wherever iron is present in any considerable mass, the earth's magnetic field is subject to local disturbance; and the value of H will be found to vary appreciably, sometimes, indeed, through a wide range. Even materials generally regarded as non-magnetic are found to produce marked variations in the earth's field, as has been shown in the magnetic explorations of the Jefferson Laboratory of Harvard University.*

In any locality where absolute measurements of current are to be made, involving the use of instruments the constant of which contains H, it is most important to have a knowledge of the character of the field. This subject of investigation has been introduced here for another reason also; viz., on account of the excellent training which a precise determination of the absolute value of the earth's horizontal force affords. The investigation may be carried on by either of the following methods:

I.

Method of the Kew magnetometer.

The magnetometer is set up in some position selected as a reference point, and the absolute value of H is determined

* Wilson, American Journal of Science, Vol. 39, p. 87, 1890.

with the utmost care, following the general directions for this instrument.*

In the determination of the rate of vibration, either the eye and ear method may be followed, or records may be made upon a chronograph sheet. The use of the chronograph adds slightly to the accuracy of the observation.

The various corrections to be applied in the use of the Kew magnetometer are given in the official blank furnished with this instrument, the essential parts of which are reproduced in the appendix to this chapter.

A satisfactory determination of H having been obtained, it is then only necessary to set up the instrument anew at various selected points in the neighborhood, and to repeat the observations for the rate of vibration. If the locality is of such a character that the Kew magnetometer cannot be brought into all the positions for which a knowledge of H is desired, it will be necessary to substitute a modified form of support for the needle which will permit of the necessary exploration. In some cases the best plan is to use another magnetometer needle, swinging the same first in the precise position where the magnetometer had stood, — viz. at the point of reference, — and then getting its rate of vibration in other regions of the field to be explored. The results are to be represented graphically in the form of a magnetic map.

---

* For brief discussions of this method, see

STEWART AND GEE: Practical Physics. Vol. 2.

NICHOLS: The Galvanometer. Lecture IV.

For detailed treatment, see

GRAY: Absolute Measurements. Vol. 2, Chapter II., p. 70 *et seq.*

WIEDEMANN: Elektricität. Vol. 3, p. 220, etc.

The original sources are

GAUSS: Poggendorff's Annalen. 28, p. 241. 1833. And various papers by Gauss and by Weber in Resultate des magnetischen Vereins. Göttingen, 1836, 1837, etc.

The results of such an exploration of the magnetic field often show variations of the most unexpected character, as appears in the description of the study of the Harvard laboratory already referred to.

An exploration of this kind was made in 1889 by F. J. Rogers, who took as the scene of his operations the Magnetic Observatory of Cornell University and its surroundings. This building, which had been constructed with scrupulous care, as regards the exclusion of iron and all other magnetic material, was originally located at a distance of several hundred feet from the nearest of the other University buildings. Its neighborhood was afterwards encroached upon by the machine shops and mechanical laboratories of Sibley College, and at the time of the exploration a considerable number of heavy testing machines were already placed at distances varying from 60 to 100 feet to the west of the Observatory. The magnetic influence of these masses of iron had made itself felt, but no accurate knowledge of the character of the modifications of the earth's field brought about by the proximity of the Mechanical Laboratory had been obtained.

The results of Mr. Rogers' investigation are shown in the accompanying map (Fig. 244), upon which the values of H obtained at the various positions selected are noted. It will be seen that there is a distinct diversion of the earth's lines of force, producing a distortion of the field and changes in the direction of the magnetic meridian. The modifications of the values of H were such that the determination made at any given point within the Observatory itself could no longer be taken as the proper values for other points, even where the distance was only a few feet.

In laboratory buildings containing large masses of iron, particularly in buildings heated by steam and containing extensive systems of iron pipes, the character of the field would be found to be much more complicated than in the example just given. In steam-heated buildings during the winter season, the

problem is further complicated by magnetic changes due to fluctuations in the temperature of the steam coils.

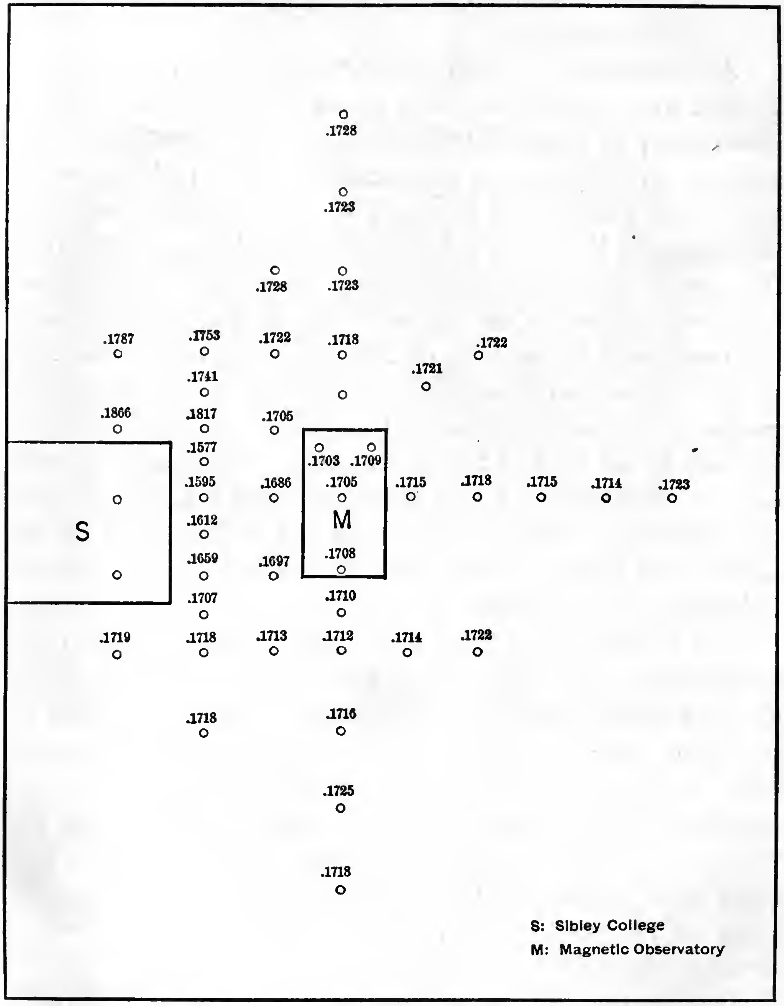


Fig. 244.—Values of H in the neighborhood of the Magnetic Observatory of Cornell University (from observations by F. J. Rogers, 1889).

An exploration of a laboratory room, containing small quantities of iron, viz. steam pipes situated overhead, at a height

of more than 3 m. from the floor, also cast iron supports to certain wall-tables, was undertaken, at the request of the writer, by Messrs. Hewitt and Smith in November, 1893. The roof of this building (the annex to Franklin Hall) is of tinned sheet iron, and the roof trusses are supported by means of a system of iron rods. The building is situated about 10 m. to the north

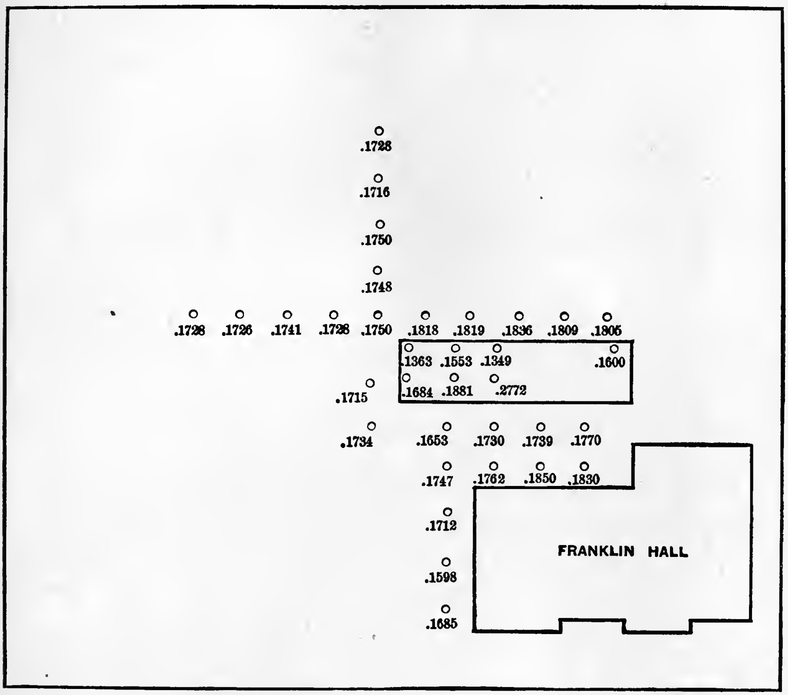


Fig. 245.—Values of *H* in the neighborhood of the annex to Franklin Hall, Cornell University, 1893 (from a survey made by C. E. Hewitt and A. W. Smith).

of Franklin Hall, within the walls of which is a much more extensive system of steam, water, and gas pipes than are contained in the Annex itself. The survey included determinations of *H* at various points between the two buildings. The results are given in Fig. 245.

The comparison of these two diagrams (Figs. 244 and 245) shows that they possess certain features in common. For

example, the value of H within doors is everywhere much below the normal value with the exception of one abnormal value (0.2772) in Hewitt and Smith's map. This high value is that of a station situated very near the massive iron castings of a set of assay furnaces.

Along outside of the north wall of Franklin Hall, of the Annex, and of the Sibley College, on the other hand, H is everywhere greatly in excess of the normal value, whereas near the outer face of eastern or western walls, wherever measurements were made, the value was low.

Finally, these two surveys indicate that in the entire region surrounding buildings containing iron, disturbances of H affecting the fourth place of decimals in the numerical value do not disappear until the distance from the building exceeds 30 m.

II.

Method of the tangent galvanometer.

Instead of using the method of the Kew magnetometer a satisfactory determination of the field may be obtained as follows: A portable tangent galvanometer, the constant of the coils of which is very accurately known, the same having been determined either by measurement or by calibration, is set up at the reference point. A current of known intensity is sent through it, and the deflection noted. The value of H at the reference point can then be obtained by computation in the usual manner. The current may be measured by means of a standard galvanometer placed in the same circuit, provided an instrument, the constant of which is known with sufficient accuracy, is obtainable. This may be another galvanometer in series with the first, or it may consist of an accurate potentiometer of any form, shunted around a known resistance. In the absence of such instruments the current value may be obtained by the use of the copper

voltameter * or by means of a silver voltameter, following the well-known precautions necessary to secure accuracy with that instrument.

Appendix to Chapter VII.

Statement of the method of the Kew magnetometer (from the official sheets furnished by the observatory at Kew), together with tables for facilitating the computation, and certain data concerning Kew magnetometer. No. 47 (Elliott).

Calculation of the value of the horizontal component of the earth's magnetic force from observations of vibration and deflection.

T_0 = Observed time of one vibration of the magnet.

T_1 = Time of vibration, corrected for rate of chronometer and arc of vibration.

T = Time of vibration, corrected for rate of chronometer, arc of vibration, temperature, torsion force of the suspending thread, and induction.

s = Daily rate of chronomoter, + when gaining, − when losing.

a, a' = Semiarc of vibration, at the beginning and end of the observation, expressed in parts of radius.

$\frac{H}{F}$ = Ratio of the force of torsion of the suspending thread to the magnetic directive force. (This is obtained from the formula $\frac{H}{F}=\frac{u}{90^\circ-u}$, where u = the angle through which the magnet is deflected by a twist of 90° in the thread.)

q = The correction for the decrease of the magnetic moment of the magnet produced by an increase of temperature of 1° C. (This correction is not constant at all temperatures, and the correction is more exactly expressed by a formula of the form, — correction to $t^\circ = q\ (t_0-t)+q'\ (t_0-t)^2$, t_0 being the observed temperature, and t an adopted standard temperature.)

* See the Spiral Coil Voltameter, Transactions of the American Institute of E. E., Vol. 6, p. 322. See also this Manual, Vol 1, Exp. R_2.

K = Moment of inertia of the magnet, including its suspending stirrup and other appendages. (This is constant for the same magnet and suspension, but varies slightly with temperature, owing to the expansion of the materials.)

π = Ratio of the circumference to the diameter of the circle $= 3.1415927$.

μ = The increase in the magnetic moment of the magnet produced by the inducing action of a magnetic force equal to unity of the metric system of absolute measurement.

r_0 = Apparent distance between the centers of the deflecting and suspended magnets in the observation of deflection.

r = Distance corrected for error of graduation and temperature.
[$r = r_0(1 + 0.000018\,(t_0 - 0^\circ\text{ C.}))$ + correction for scale error.]

u_0 = Observed angle of deflection.

P = A constant depending upon the distribution of magnetism in the deflecting and suspended magnets. (This is to be determined from several series of observations of deflection at two or more distances. The most convenient distances to be employed for this purpose are 0.3 m. and 0.4 m. The correction is very small and may remain unapplied until the conclusion of the series.)

m = Magnetic moment of the deflected or vibrating magnet.

X = Horizontal component of the earth's magnetic force.

$\dfrac{m_0}{X_0}$ = Approximate value of $\dfrac{m}{X}$.

$\dfrac{m'}{X'}$ = Value of $\dfrac{m}{X}$ before the application of the correction $\left(1 - \dfrac{P}{r_0^2}\right)$.

$$T_1 = T_0\left[1 - \frac{s}{86400} - \frac{aa'}{16}\right]. \qquad T^2 = T_1^2\left[1 + \frac{H}{F} - q(t_0 - t) + \mu\frac{X_0}{m_0}\right].$$

$$mX = \frac{\pi^2 K}{T^2}. \qquad \frac{m_0}{X_0} = \tfrac{1}{2} r^3 \sin u_0.$$

$$\frac{m'}{X'} = \frac{m_0}{X_0}\left[1 + \frac{2\mu}{r_0^3} + q(t_0 - t)\right]. \qquad \frac{m}{X} = \frac{m'}{X'}\left(1 - \frac{P}{r_0^2}\right).$$

Let A = value of $\frac{m'}{X'}$ from deflection at the distance r,

and A' = value of $\frac{m'}{X'}$ from deflection at the distance r';

then
$$P = \frac{A - A'}{\frac{A}{r^2} - \frac{A'}{r'^2}}.$$

The quantity K is obtained by observing the time of vibration of the magnet alternately with its usual mounting, and with its moment of inertia increased by the addition of a gun-metal ring or cylinder of known weight and dimensions.

When a cylinder is employed, the value of K is obtained from the formula $K = W\left(\frac{l^2}{12} + \frac{d^2}{16}\right)\frac{t^2}{t'^2 - t^2}$, where W is the weight of the cylinder in grams, l and d its length and diameter expressed in meters; t' and t being the times of vibration (corrected for torsion, temperature, etc.) of the magnet with and without the additional weight.

$K = W\left(\frac{r_i^2 + r_e^2}{2}\right)\frac{t^2}{t'^2 - t^2}$, r_i and r_e being the internal and external radii of the ring.

TABLES TO FACILITATE THE CALCULATION OF THE OBSERVATIONS.

Table I.

Value of $1 - \frac{s}{86400}$ *for Different Rates of the Chronometer employed.*

Daily Rate.	Chronometer Gaining.	Chronometer Losing.
sec.		
5	0.99994	1.00006
10	.99988	.00012
15	.99983	.00017
20	.99977	.00023
25	.99971	.00029
30	.99965	.00035
35	.99959	.00041
40	.99954	.00046
45	.99948	.00052
50	.99942	.00058

Table II.

Value of $\frac{aa'}{16}$ *for different Initial and Terminal Semiarcs of Vibration.*

Semiarc at Commencement.	Semiarc at End of Observation.					
	80′	70′	60′	50′	40′	30′
′						
100	0.00004	0.00004	0.00003	0.00003	0.00002	0.00002
90	.00004	.00003	.00003	.00002	.00002	.00001
80	.00003	.00003	.00003	.00002	.00002	.00001
70		.00003	.00002	.00002	.00001	.00001
60			.00002	.00002	.00001	.00001
50				.00001	.00001	.00001

TABLE III.

Value of $1 + \frac{H}{F}$ *for Different Values of the Deflection produced in the Magnet by a Twist of* 90° *of the Suspension Thread.*

Effect of 90° of Torsion.	$1+\frac{H}{F}$.	Effect of 90° of Torsion.	$1+\frac{H}{F}$.	Effect of 90° of Torsion.	$1+\frac{H}{F}$.
′		′		′	
1	1.00019	6	1.00111	11	1.00204
2	.00037	7	.00130	12	.00223
3	.00056	8	.00148	13	.00241
4	.00074	9	.00167	14	.00260
5	.00093	10	.00185	15	.00278

TABLE IV.

Value of $1 + \frac{2\mu}{r_0^3}$ *for Different Distances.*

Distance.	$1+\frac{2\mu}{r_0^3}$.
meter.	
0.25	1.00064
.30	.00037
.35	.00023
.375	.00019
.40	.00016
.45	.00011
.50	.00008

With each magnetometer, certain data are given which apply only to the individual instrument. In the case of Kew Magnetometer No. 47 (Elliott), for example, which is in possession of the Department of Physics, Cornell University, the following constants and correction tables are furnished by the makers.

Constants, Coefficients, and Corrections for the Unifilar Magnetometer No. 47, *by Elliott Bros., London.*

Error of graduation of deflection bar,

Mean apparent 0.15 meter = 0.14996 true meter at 0° C.

Mean apparent 0.50 meter = 0.49990 true meter at 0° C.

Deflection apparatus, angular value of one scale division $= 1' \, 30.''5$.

When the scale reading is *above* the middle point of the scale, the correction to the circle reading is *additive*, and when below it, is *subtractive*.

Vibration magnet, angular value of one scale division $= 1'.78$.

The deflecting magnet employed is marked 47 A.

The suspended magnet employed is marked 47 C.

For deflecting magnet:

$$\text{Correction to } 0^\circ \text{ C.} \quad = 0.000296(t_0 - 0^\circ \text{ C.}) + 0.0000011\,(t_0 - 0^\circ \text{ C.})^2.$$

$$\text{Induction coefficient } \mu = 0.0000050.$$

$$\text{Log } \pi^2 K \text{ at } 0^\circ \text{ C.} \quad = 9.45745.$$

Dimensions of inertia cylinder:

Weight $= 63.5441$ grams.
Length $= 0.095374$ meter.
Diameter $= 0.009962$ meter.

TABLE V.

Temperature Corrections for the Magnet (47 *A*).

Temp. t_0.	Correction to 0° C.	Temp. t_0.	Correction to 0° C.	Temp. t_0.	Correction to 0° C.
C.		C.		C.	
− 5°	− 0.00145	10°	+ 0.00307	25°	+ 0.00809
4	116	11	339	26	844
3	87	12	372	27	880
2	58	13	404	28	916
− 1	− 29	14	436	29	951
0	0.00000	15	+ 0.00469	30	+ 0.00987
+ 1	+ 30	16	502	31	1024
2	60	17	536	32	1060
3	90	18	569	33	1097
4	121	19	602	34	1134
5	+ 0.00151	20	+ 0.00636	35	+ 0.01171
6	182	21	670	36	1209
7	213	22	705	37	1246
8	244	23	740	38	1284
9	276	24	774	39	1322
10	+ 0.00307	25	+ 0.00809	40	+ 0.01360

Table VI.

Values of $Log\ \pi^2 K$, and of $Log\ \frac{1}{2} r^3$ for Different Temperatures.

Temp. C.	Log $\pi^2 K$.	Log $\frac{1}{2} r^3$.					
		$r_0 = 0.25\ m.$	$r_0 = 0.30\ m.$	$r_0 = 0.35\ m.$	$r_0 = 0.40\ m.$	$r_0 = 0.45\ m.$	$r_0 = 0.50\ m.$
0°	9.45745	7.89258	8.13007	8.33088	8.50489	8.65832	8.79562
5	.45750	.89270	.13019	.33099	.50501	.65843	.79574
10	.45756	.89281	.13031	.33111	.50512	.65855	.79585
15	.45761	.89293	.13042	.33123	.50524	.65867	.79597
20	.45767	.89305	.13054	.33134	.50536	.65879	.79609
25	.45772	.89317	.13066	.33146	.50547	.65890	.79620
30	.45778	.89328	.13077	.33158	.50559	.65902	.79632
35	.45783	.89340	.13089	.33170	.50571	.65914	.79644
40	9.45789	7.89352	8.13101	8.33181	8.50582	8.65925	8.79655

The following forms will serve to indicate the usual methods of tabulating the readings and results obtained in the measurement of H with the Kew magnetometer.

OBSERVATIONS OF VIBRATION.

18 .

STATION Lat. ° ′ ″ Long. ° ′ ″

Chronometer ( ). Error at Station = Daily Rate (s) =

Magnet ( ) suspended. q = Log μ =

Effect of 90° of Torsion = Div. = ′ . One Div. of Scale = ′ °

	Mean Time at Station.	Semiarc of Vib.	Temp. of Magnet	
At Commencement	h. m.			$t_0' =$
At End				$t_0 =$

Scale Moving Apparently to the Right.					Scale Moving Apparently to the Left.				
No. of Vib.	Time of Center passing Wire.	No. of Vib.	Time of Center passing Wire.	Time of 100 Vib.	No. of Vib.	Time of Center passing Wire.	No. of Vib.	Time of Center passing Wire.	Time of 100 Vib.
	h. m. s.		m. s.	m. s.		h. m. s.		m. s.	m. s.
0		100			5		105		
10		110			15		115		
20		120			25		125		
30		130			35		135		
40		140			45		145		
50		150			55		155		
Diff.					Diff.				
100at			Mean(1) =		105at			Mean(2) =	

$$T_1 = T_0\left\{1 - \frac{s}{86400} - \frac{aa'}{16}\right\}. \qquad T^2 = T_1^2\left\{1 + \frac{H}{F} - q(t_0 - t) + \mu\frac{X_0}{m_0}\right\}. \qquad mX = \frac{\pi^2 K}{T^2}.$$

$1 - \frac{s}{86400} =$

$-\frac{aa'}{16} =$

$1 - \frac{s}{86400} - \frac{aa'}{16} =$

$1 + \frac{H}{F} =$

$-q(t_0 - t) =$

$+\mu \div \frac{m_0}{X_0} =$

$1 + \frac{H}{F} - q(t_0 - t) + \mu \div \frac{m_0}{X_0} =$

μ Log =

$\frac{m_0}{X_0}$ Log =

$\mu \div \frac{m_0}{X_0} =$ Log =

Mean (1) =

" (2) =

$T_0 =$ Log =

- - - - - - - Log =

T_1 Log =

2

T_1^2 Log. =

- - - - - - - Log =

T^2 Log =

$\pi^2 K$ Log =

$mX = \frac{\pi^2 K}{T^2}$ Log =

$\frac{m'}{X'}$ Log =

$mX \div \frac{m'}{X'} = X'^2$ Log =

$X' =$ Log =

m'^2 Log =

$m' =$ Log =

Observation of Torsion.

Circle turned.	Scale.	Mean.	Diff.
0° =			
+ 180° =			
0° =			
− 180° =			
0° =			
		90° =	

Observer.

OBSERVATIONS OF DEFLECTION.

18 .

STATION Lat. ° ' " Long. ° ' "

Mean Time at Station; commencing , ending .

Magnet ( ) deflecting; ( ) suspended. One Div. of Scale = "

DEFLECTING MAGNET.			Readings of Verniers.	Scale Reading.	Correction to Middle of Scale.	Mean of Verniers.	Corrected Circle Reading.	Means and Differences.	
Distance.	N. End.	Temp.							
EAST		°	° ' "	Div.	' "	° ' "	° ' "	° ' "	
m.	E.								
"	W.								
"	E.								Diff. u_0.
"	W.								
WEST									
m.	W.								
"	E.								Diff. u_0'.
"	W.								
"	E.								
Mean	$t_0 =$		Observed Angles of Deflection (u_0') $r_0 = \quad =$; $r_0' = \quad =$						

$$\frac{m_0}{X_0} = \tfrac{1}{2} r^3 \operatorname{Sin} u_0 ; \qquad \frac{m'}{X'} = \frac{m_0}{X_0}\left\{ 1 + \frac{2\mu}{r_0^3} + q(t_0 - t) \right\} ; \qquad \frac{m}{X} = \frac{m'}{X'}\left(1 - \frac{P}{r_0^2}\right).$$

$r =$ $r' =$ $r =$ $r' =$

$1 + \frac{2\mu}{r_0^3} =$

$+ (t_0 - t)q =$

$1 + \frac{2\mu}{r_0^3}(t_0 - t)q =$

$P = \dfrac{A - A'}{\frac{A}{r^2} - \frac{A'}{r'^2}}$. $1 - \frac{P}{r_0^2} =$

A and A' = values of $\frac{m'}{X'}$ for distances r and r' respectively.

Observer.

$\tfrac{1}{2} r^3$ Log =

Sin u_0 Log =

$\frac{m_0}{X_0}$ Log =

Log =

$\frac{m'}{X'}$ Log =

Log =

$\frac{m}{X}$ Log =

mX Log =

$mX \div \frac{m}{X} = X^2$ Log =

$X =$ Log =

APPENDIX A.

LOGARITHM TABLE OF LIGHT RATIOS FOR 1000–PART PHOTOMETER BAR.

With the reading on the photometer bar as an argument, take from the table the logarithm of the light ratio, and to this add the logarithm of the candle-power of the standard; the result being the logarithm of the candle-power of the light which is measured.

A small mark (*) after the last figure of a logarithm indicates that the fifth figure is a five. The differences have been formed taking account of this.

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
50	2.5575	181	95	1.9578	100	140	1.5767	72
51	394	178	96	478	100	41	695	72
52	216	175	97	378	98	42	624	71
53	041	171	98	280	98	43	553	71
54	2.4870	169	99	182	97	44	1.5482	70
55	2.4701	165	100	1.9085	96	145	1.5412	70
56	536	163	1	1.8989	95	46	342	69
57	373	161	2	894	95	47	273	69
58	212	158	3	799	94	48	204	69
59	055	155	4	705	93	49	135	68
60	2.3900	153	105	1.8613	92	150	1.5067	68
61	747	151	6	521	92	51	1.4999	68
62	596	148	7	429	90	52	931	67
63	448	146	8	339	90	53	864	67
64	302	144	9	249	89	54	797	66
65	2.3158	142	110	1.8160	88	155	1.4730	66
66	016	140	11	072	88	56	664	65
67	2.2876	138	12	1.7984	87	57	599	66
68	738	136	13	897	86	58	533	65
69	602	134	14	811	86	59	468	65
70	2.2468	133	115	1.7725	85	160	1.4403	64
71	335	131	16	640	84	61	339	64
72	204	129	17	555	84	62	275	64
73	075	127	18	472	83	63	211	64
74	2.1948	126	19	389	83	64	147	63
75	2.1822	125	120	1.7306	82	165	1.4084	63
76	697	123	21	224	81	66	021	62
77	574	121	22	143	81	67	1.3959	63
78	453	120	23	062	80	68	896	62
79	333	119	24	1.6982	80	69	834	61
80	2.1214	117	125	1.6902	79	170	1.3773	62
81	097	116	26	823	79	71	711	61
82	2.0981	115	27	744	78	72	650	61
83	866	114	28	666	77	73	589	60
84	752	112	29	589	77	74	529	61
85	2.0640	111	130	1.6512	77	175	1.3468	60
86	529	110	31	435	76	76	408	59
87	419	109	32	359	76	77	349	60
88	310	107	33	283	75	78	289	59
89	203	107	34	208	74	79	230	59
90	2.0096	106	135	1.6134	74	180	1.3171	59
91	1.9990	104	36	059	74	81	112	58
92	886	104	37	1.5986	73	82	054	59
93	782	102	38	913	73	83	1.2995	58
94	680	101	39	840	73	84	937	57

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
185	1.2880	58	230	1.0495	49	275	0.8420	43
86	822	57	31	446	49	76	377	44
87	765	57	32	397	48	77	333	43
88	708	57	33	349	49	78	290	43
89	651	56	34	300	48	79	247	44
190	1.2595	57	235	1.0252	48	280	0.8203	43
91	538	56	36	204	48	81	160	43
92	482	56	37	156	48	82	117	43
93	426	55	38	108	48	83	075	43
94	371	56	39	060	48	84	032	43
195	1.2315	55	240	1.0012	48	285	0.7989	42
96	260	55	41	0.9964	47	86	947	43
97	205	55	42	917	47	87	904	42
98	150	54	43	870	47	88	862	43
99	096	55	44	823	47	89	819	42
200	1.2041	54	245	0.9776	47	290	0.7777	42
1	1.1987	54	46	729	47	91	735	42
2	933	54	47	682	47	92	693	42
3	879	53	48	635	46	93	651	42
4	826	54	49	589	46	94	609	42
205	1.1772	53	250	0.9542	46	295	0.7567	41
6	719	53	51	496	46	96	526	42
7	666	53	52	450	46	97	484	42
8	613	52	53	404	46	98	442	41
9	561	53	54	358	46	99	401	41
210	1.1508	52	255	0.9312	45	300	0.7360	42
11	456	52	56	267	46	1	318	41
12	404	52	57	221	45	2	277	41
13	352	52	58	176	46	3	236	41
14	300	51	59	130	45	4	195	41
215	1.1249	52	260	0.9085	45	305	0.7154	41
16	197	51	61	040	45	6	113	41
17	146	51	62	0.8995	45	7	072	41
18	095	51	63	950	45	8	031	41
19	044	51	64	905	44	9	0.6990	40
220	1.0993	50	265	0.8861	45	310	0.6950	41
21	943	50	66	816	44	11	909	40
22	893	51	67	772	44	12	869	41
23	842	50	68	727	44	13	828	40
24	792	50	69	683	44	14	788	40
225	1.0742	49	270	0.8639	44	315	0.6748	41
26	693	50	71	595	44	16	707	40
27	643	49	72	551	44	17	667	40
28	594	50	73	507	43	18	627	40
29	544	49	74	464	44	19	587	40

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
320	0.6547	40	365	0.4810	38	410	0.3161	36
21	507	40	66	772	37	11	125	35
22	467	39	67	735	38	12	090	36
23	428	40	68	697	37	13	054	36
24	388	40	69	660	37	14	018	36
325	0.6348	39	370	0.4623	37	415	0.2982	36
26	309	40	71	586	38	16	946	35
27	269	39	72	548	37	17	911	36
28	230	39	73	511	37	18	875	36
29	191	40	74	474	37	19	839	35
330	0.6151	39	375	0.4437	37	420	0.2804	36
31	112	39	76	400	37	21	768	36
32	073	39	77	363	37	22	732	35
33	034	39	78	326	37	23	697	36
34	0.5995	39	79	289	37	24	661	35
335	0.5956	39	380	0.4252	37	425	0.2626	36
36	917	39	81	215	37	26	590	35
37	878	39	82	178	37	27	555	36
38	839	39	83	142	37	28	519	35
39	800	39	84	105	37	29	484	36
340	0.5761	38	385	0.4068	36	430	0.2448	35
41	723	39	86	032	37	31	413	36
42	684	39	87	0.3995	37	32	377	35
43	645	38	88	958	36	33	342	35
44	607	39	89	922	37	34	307	36
345	0.5568	38	390	0.3885	36	435	0.2271	35
46	530	38	91	849	37	36	236	35
47	492	39	92	812	36	37	201	36
48	453	38	93	776	36	38	165	35
49	415	38	94	740	37	39	130	35
350	0.5377	38	395	0.3703	36	440	0.2095	36
51	339	38	96	667	36	41	059	35
52	301	38	97	631	37	42	024	35
53	263	38	98	594	36	43	0.1989	35
54	225	38	99	558	36	44	954	35
355	0.5187	38	400	0.3522	36	445	0.1919	36
56	149	38	1	486	36	46	883	35
57	111	38	2	449	36	47	848	35
58	073	38	3	413	36	48	813	35
59	035	37	4	377	36	49	778	35
360	0.4998	38	405	0.3341	36	450	0.1743	35
61	960	38	6	305	36	51	708	35
62	922	37	7	269	36	52	673	35
63	885	38	8	233	36	53	638	35
64	847	37	9	197	36	54	603	35

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
455	0.1568	35	**500**	0.0000	35	**545**	9.8432	35
56	533	35	**1**	9.9965	35	**46**	397	35
57	498	35	**2**	930	35	**47**	362	35
58	463	35	**3**	896	35	**48**	327	35
59	428	35	**4**	861	35	**49**	292	35
460	0.1393	35	**505**	9.9826	34	**550**	9.8257	35
61	358	35	**6**	792	35	**51**	222	35
62	323	35	**7**	757	35	**52**	187	35
63	288	35	**8**	722	35	**53**	152	35
64	253	35	**9**	687	35	**54**	116	36
465	0.1218	35	**510**	9.9652	35	**555**	9.8081	35
66	183	35	**11**	618	35	**56**	046	35
67	148	35	**12**	583	35	**57**	011	35
68	113	35	**13**	548	35	**58**	9.7976	35
69	078	34	**14**	513	34	**59**	941	36
470	0.1044	35	**515**	9.9479	35	**560**	9.7905	35
71	009	35	**16**	444	35	**61**	870	35
72	0.0974	35	**17**	409	35	**62**	835	35
73	939	35	**18**	374	34	**63**	800	36
74	904	35	**19**	340	35	**64**	764	35
475	0.0869	35	**520**	9.9305	35	**565**	9.7729	36
76	834	34	**21**	270	35	**66**	693	35
77	800	35	**22**	235	35	**67**	658	35
78	765	35	**23**	200	34	**68**	623	36
79	730	35	**24**	166	35	**69**	587	35
480	0.0695	35	**525**	9.9131	35	**570**	9.7552	36
81	660	34	**26**	096	35	**71**	516	35
82	626	35	**27**	061	35	**72**	481	36
83	591	35	**28**	026	35	**73**	445	35
84	556	35	**29**	9.8991	35	**74**	410	36
485	0.0521	34	**530**	9.8956	34	**575**	9.7374	35
86	487	35	**31**	922	35	**76**	339	36
87	452	35	**32**	887	35	**77**	303	35
88	417	35	**33**	852	35	**78**	268	36
89	382	35	**34**	817	35	**79**	232	36
490	0.0347	34	**535**	9.8782	35	**580**	9.7196	35
91	313	35	**36**	747	35	**81**	161	36
92	278	35	**37**	712	35	**82**	125	36
93	243	35	**38**	677	35	**83**	089	35
94	208	34	**39**	642	35	**84**	054	36
495	0.0174	35	**540**	9.8607	35	**585**	9.7018	36
96	139	35	**41**	572	35	**86**	9.6982	36
97	104	35	**42**	537	35	**87**	946	36
98	069	34	**43**	502	35	**88**	910	35
99	035	35	**44**	467	35	**89**	875	36

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
590	9.6839	36	**635**	9.5190	37	**680**	9.3453	40
91	803	36	**36**	153	38	**81**	413	40
92	767	36	**37**	115	37	**82**	373	40
93	731	36	**38**	078	38	**83**	333	40
94	695	36	**39**	040	38	**84**	293	41
595	9.6659	36	**640**	9.5002	37	**685**	9.3252	40
96	623	36	**41**	9.4965	38	**86**	212	40
97	587	37	**42**	927	38	**87**	172	41
98	550	36	**43**	889	38	**88**	131	40
99	514	36	**44**	851	38	**89**	091	41
600	9.6478	36	**645**	9.4813	38	**690**	9.3050	40
1	442	36	**46**	775	38	**91**	010	41
2	406	37	**47**	737	38	**92**	9.2969	41
3	369	36	**48**	699	38	**93**	928	41
4	333	36	**49**	661	38	**94**	887	41
605	9.6297	37	**650**	9.4623	38	**695**	9.2846	41
6	260	36	**51**	585	38	**96**	805	41
7	224	36	**52**	547	39	**97**	764	41
8	188	37	**53**	508	38	**98**	723	41
9	151	36	**54**	470	38	**99**	682	42
610	9.6115	37	**655**	9.4432	39	**700**	9.2640	41
11	078	36	**56**	393	38	**1**	599	41
12	042	37	**57**	355	39	**2**	558	42
13	005	37	**58**	316	39	**3**	516	42
14	9.5968	36	**59**	277	38	**4**	474	41
615	9.5932	37	**660**	9.4239	39	**705**	9.2433	42
16	895	37	**61**	200	39	**6**	391	42
17	858	37	**62**	161	39	**7**	349	42
18	821	37	**63**	122	39	**8**	307	42
19	785	37	**64**	083	39	**9**	265	42
620	9.5748	37	**665**	9.4044	39	**710**	9.2223	42
21	711	37	**66**	005	39	**11**	181	43
22	674	37	**67**	9.3966	39	**12**	138	42
23	637	37	**68**	927	39	**13**	096	43
24	600	37	**69**	888	39	**14**	053	42
625	9.5563	37	**670**	9.3849	40	**715**	9.2011	43
26	526	37	**71**	809	39	**16**	9.1968	43
27	489	37	**72**	770	39	**17**	925	43
28	452	38	**73**	731	40	**18**	882	42
29	414	37	**74**	691	39	**19**	840	43
630	9.5377	37	**675**	9.3652	40	**720**	9.1796	43
31	340	37	**76**	612	40	**21**	753	43
32	303	38	**77**	572	39	**22**	710	43
33	265	37	**78**	533	40	**23**	667	44
34	228	38	**79**	493	40	**24**	623	43

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
725	9.1580	44	**770**	8.9505	49	**815**	8.7120	57
26	536	43	**71**	456	50	**16**	063	58
27	493	44	**72**	406	49	**17**	005	59
28	449	44	**73**	357	50	**18**	8.6946	58
29	405	44	**74**	307	49	**19**	888	59
730	9.1361	44	**775**	8.9258	50	**820**	8.6829	59
31	317	45	**76**	208	50	**21**	770	59
32	272	44	**77**	158	51	**22**	711	60
33	228	44	**78**	107	50	**23**	651	59
34	184	45	**79**	057	50	**24**	592	60
735	9.1139	44	**780**	8.9007	51	**825**	8.6532	61
36	095	45	**81**	8.8956	51	**26**	471	60
37	050	45	**82**	905	51	**27**	411	61
38	005	45	**83**	854	51	**28**	350	61
39	9.0960	45	**84**	803	52	**29**	289	62
740	9.0915	45	**785**	8.8751	51	**830**	8.6227	61
41	870	46	**86**	700	52	**31**	166	62
42	824	45	**87**	648	52	**32**	104	63
43	779	46	**88**	596	52	**33**	041	62
44	733	45	**89**	544	52	**34**	8.5979	63
745	9.0688	46	**790**	8.8492	53	**835**	8.5916	63
46	642	46	**91**	439	52	**36**	853	64
47	596	46	**92**	387	53	**37**	789	64
48	550	46	**93**	334	53	**38**	725	64
49	504	46	**94**	281	53	**39**	661	64
750	9.0458	47	**795**	8.8228	54	**840**	8.5597	65
51	411	46	**96**	174	53	**41**	532	65
52	365	47	**97**	121	54	**42**	467	66
53	318	47	**98**	067	54	**43**	401	65
54	271	47	**99**	013	54	**44**	336	66
755	9.0224	47	**800**	8.7959	55	**845**	8.5269	67
56	177	47	**1**	904	54	**46**	203	67
57	130	47	**2**	850	55	**47**	136	67
58	083	47	**3**	795	55	**48**	069	68
59	035	47	**4**	740	55	**49**	001	68
760	8.9988	48	**805**	8.7685	56	**850**	8.4933	68
61	940	48	**6**	629	55	**51**	865	69
62	892	48	**7**	574	56	**52**	796	69
63	844	48	**8**	518	56	**53**	727	69
64	796	48	**9**	462	57	**54**	658	70
765	8.9748	48	**810**	8.7405	56	**855**	8.4588	70
66	700	49	**11**	349	57	**56**	518	71
67	651	48	**12**	292	57	**57**	447	71
68	603	49	**13**	235	57	**58**	376	72
69	554	49	**14**	178	58	**59**	304	72

Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.	Scale Reading.	Log. Light Ratio.	Diff.
860	8.4233	73	**890**	8.1840	89	**920**	7.8786	119
61	160	73	**91**	751	90	**21**	667	120
62	087	73	**92**	661	90	**22**	547	121
63	014	74	**93**	571	92	**23**	426	123
64	8.3940	74	**94**	479	92	**24**	303	125
865	8.3866	74	**895**	8.1387	93	**925**	7.8178	126
66	792	75	**96**	294	93	**26**	052	127
67	717	76	**97**	201	95	**27**	7.7925	129
68	641	76	**98**	106	95	**28**	796	131
69	565	77	**99**	011	96	**29**	665	133
870	8.3488	77	**900**	8.0915	97	**930**	7.7532	134
71	411	77	**1**	818	98	**31**	398	136
72	334	78	**2**	720	98	**32**	262	138
73	256	79	**3**	622	100	**33**	124	140
74	177	79	**4**	522	101	**34**	7.6984	142
875	8.3098	80	**905**	8.0421	101	**935**	7.6842	144
76	018	80	**6**	320	102	**36**	698	146
77	8.2938	81	**7**	217	103	**37**	552	148
78	857	81	**8**	114	104	**38**	404	151
79	776	82	**9**	010	106	**39**	253	153
880	8.2694	83	**910**	7.9904	107	**940**	7.6100	155
81	611	83	**11**	797	107	**41**	7.5945	157
82	528	84	**12**	690	109	**42**	788	161
83	444	84	**13**	581	110	**43**	627	163
84	360	85	**14**	471	111	**44**	464	165
885	8.2275	86	**915**	7.9360	112	**945**	7.5299	169
86	189	86	**16**	248	114	**46**	130	171
87	103	87	**17**	134	115	**47**	7.4959	175
88	016	88	**18**	091	116	**48**	784	178
89	8.1928	88	**19**	7.8903	117	**49**	606	181

APPENDIX B.

CURVES OF LIGHT RATIOS FOR 1000–PART PHOTOMETER BAR.

MULTIPLY the ordinate corresponding to any bar reading by the candle-power of the standard light source.

INDEX TO VOLUME II.

WORKS ON PHYSICS, ETC.

PUBLISHED BY

MACMILLAN & CO.

AIRY. Works by Sir G. B. AIRY, K.C.B., formerly Astronomer-Royal.
On Sound and Atmospheric Vibrations. With the Mathematical Elements of Music. 12mo. $2.50.
Gravitation. An Elementary Explanation of the Principal Perturbations in the Solar System. 12mo. $1.90.

ALDIS: Geometrical Optics. An Elementary Treatise. By W. STEADMAN ALDIS, M.A. Third Edition, Revised. 12mo. $1.00.

DANIELL: A Text-Book of the Principles of Physics. By ALFRED DANIELL, D.Sc. Illustrated. 8vo. $3.50. *Revised edition in press.*

DAUBENY'S Introduction to the Atomic Theory. 16mo. $1.50.

DONKIN (W. F.): **Acoustics.** Second Edition. 12mo. $1.90.

EVERETT: Units and Physical Constants. By J. D. EVERETT, F.R.S. 16mo. $1.25.

FERRERS: Spherical Harmonics and Subjects Connected with them. By Rev. N. M. FERRERS, D.D., F.R.S. 12mo. $1.90.

FISHER: Physics of the Earth's Crust. By OSMOND FISHER. 8vo. $3.50.

FOURIER: The Analytical Theory of Heat. By JOSEPH FOURIER. Translated with Notes, by A. FREEMAN, M.A. 8vo. $4.50.

GALLATLY: Examples in Elementary Physics. Comprising Statics, Dynamics, Hydrostatics, Heat, Light, Chemistry, and Electricity. With Examination Papers. By W. GALLATLY, M.A. $1.00.

GARNETT: An Elementary Treatise on Heat. By W. GARNETT, M.A., D.C.L. Fifth Edition, Revised and Enlarged. $1.10.

GLAZEBROOK: Heat and Light. By R. T. GLAZEBROOK, M.A., F.R.S. *Cambridge Natural Science Manuals.* Cr. 8vo. $1.40. Bound separately. **Heat,** $1.00. **Light,** $1.00.

HEATH: Treatise on Geometrical Optics. By R. S. HEATH. 8vo. $3.50.
An Elementary Treatise on Geometrical Optics. 12mo. $1.25.

HOGG'S (JABEZ) **Elements of Experimental and Natural Philosophy.** With Index and upwards of 400 Wood-cuts. $1.50.

IBBETSON: The Mathematical Theory of Perfectly Elastic Solids. By W. J. IBBETSON. 8vo. $5.00.

JELLETT (JOHN H. B. D.): **A Treatise on the Theory of Friction.** 8vo. $2.25.

JONES: Examples in Physics. By D. E. JONES, B.Sc. 16mo. 90 cents.
Sound, Light, and Heat. 16mo. 70 cents.
Lessons in Heat and Light. 16mo. $1.00.

LOEWY (B.): **Experimental Physics.** Questions and Examples in Physics, Sound, Light, Heat, Electricity, and Magnetism. 16mo. 50 cents.
A Graduated Course of Natural Science, Experimental and Theoretical, for Schools and Colleges. 16mo. PART I. 60 cents. PART II. 60 cents.

LOVE: Treatise on the Mathematical Theory of Elasticity. 8vo. Vol. I. $3.00. Vol. II. $3.00.

LUPTON: **Numerical Tables and Constants in Elementary Science.** By SYDNEY LUPTON. 16mo. 70 cents.

MACFARLANE: **Physical Arithmetic.** By ALEXANDER MACFARLANE, Professor of Physics, University of Texas. 12mo. $1.90.

MAXWELL: **The Scientific Papers of James Clerk Maxwell, M.A., LL.D., B.Sc.**, etc., etc. Edited by W. D. NIVEN, M.A., F.R.S. With Steel Portraits and Page Plates. 2 vols. 4to. $25.00.

McAULAY (A.): **Utility of Quaternions in Physics.** 8vo. $1.60.

MOLLOY: **Gleanings in Science.** Popular Lectures on Scientific Subjects. By the Rev. GERARD MOLLOY, D.D., D.Sc. 8vo. $2.25.

NEWTON'S Principia. Edited by Professor Sir W. THOMSON and Professor BLACKBURN. (Latin Text.) 4to. $12.00.

This volume does not contain an English Translation.

First Book of Newton's Principia. Sections I., II., III. With Notes and Problems. By P. FROST, M.A. Third Edition. 8vo. $3.00.

The First Three Sections of Newton's Principia, with an Appendix; and the Ninth and Eleventh Sections. By J. H. EVANS, M.A. The Fifth Edition edited by P. T. MAIN. $1.00.

PARKER: **A Treatise on Thermodynamics.** By T. PARKER, M.A., Fellow of St. John's College, Cambridge. $2.25.

PARKINSON: **A Treatise on Optics.** By S. PARKINSON, D.D., F.R.S. Fourth Edition, Revised and Enlarged. 12mo. $2.50.

PEARSON. See TODHUNTER.

PEARSON: **A History of the Theory of Elasticity.** By ISAAC TODHUNTER. Edited by Professor KARL PEARSON. Vol. I. Galilei to Saint-Venant, 1639–1850. 8vo. $6.00. Vol. II. Saint-Venant to Lord Kelvin (Sir William Thomson). In Two Parts. $7.50.

PERRY: **An Elementary Treatise on Steam.** By JOHN PERRY. With Woodcuts, Numerical Examples, and Exercises. 18mo. $1.10.

PRESTON (T.): **The Theory of Light.** 8vo. $5.00.

The Theory of Heat. 8vo. $5.50.

RAYLEIGH: **The Theory of Sound.** By LORD RAYLEIGH, M.A., F.R.S. 8vo. *New edition in two volumes in the press.* VOL. I. (*Out of print.*) VOL. II. $3.25. VOL. III. (*In the press.*)

SAINT-VENANT: **The Elastic Researches of.** Edited for the Syndics of the Cambridge University Press by KARL PEARSON, M.A. 8vo. $2.75.

SHANN: **An Elementary Treatise on Heat in Relation to Steam and the Steam-Engine.** By G. SHANN, M.A. 12mo. $1.10.

SHAW: **Practical Work at the Cavendish Laboratory.** Edited by W. N. SHAW. **Heat.** 8vo. 90 cents.

SPOTTISWOODE: **Polarization of Light.** By W. SPOTTISWOODE, LL.D. Illustrated. 12mo. $1.25.

STEWART. Works by BALFOUR STEWART, F.R.S.

Lessons in Elementary Physics. 16mo. $1.10.

Questions on the Same for Schools. By T. H. CORE. 40 cents.

A Treatise on Heat. Fourth Edition. 16mo. $1.90.

STEWART and GEE: **Lessons on Elementary Practical Physics.** By BALFOUR STEWART, M.A., LL.D., F.R.S., and W. W. HALDANE GEE.

VOL. I. **General Physical Processes.** 12mo. $1.50.
VOL. II. **Electricity and Magnetism.** $2.25.
VOL. III. **Optics, Heat, and Sound.** (*In the press.*)

Practical Physics for Schools and the Junior Students of Colleges.

VOL. I. **Electricity and Magnetism.** 16mo. 60 cents.
VOL. II. **Optics, Heat, and Sound.** (*In the press.*)

STOKES (G. G.): **On Light.** On the Nature of Light. On Light as a Means of Investigation. On the Beneficial Effects of Light. 12mo. $2.00.
Mathematical and Physical Papers. 8vo.
VOL. I. $3.75. VOL. II. $3.75. VOL. III. (*In the press.*)

STONE: **Elementary Lessons on Sound.** By W. H. STONE, M.B. With Illustrations. 16mo. 90 cents.

TAIT. Works by P. G. TAIT, M.A., SEC. R.S.E.
Lectures on Some Recent Advances in Physical Science. With Illustrations. Third Edition, Revised and Enlarged, with the Lecture on Force delivered before the British Association. 12mo. $2.50.
Heat. With Numerous Illustrations. 12mo. $2.00.
Light. An Elementary Treatise. With Illustrations. 12mo. $2.00.
Properties of Matter. Second Edition, Enlarged. 12mo. $2.25.

TAYLOR: **Sound and Music.** By SEDLEY TAYLOR, M.A. Illustrated. Second Edition. 12mo. $2.50.
Theories of Sound. Paper. 10 cents.

THOMSON. Works of J. J. THOMSON, Professor of Experimental Physics in the University of Cambridge.
A Treatise on the Motion of Vortex Rings. 8vo. $1.75.
Application of Dynamics to Physics and Chemistry. 12mo. $1.90.

THOMSON. Works of Sir W. THOMSON, F.R.S., Professor of Natural Philosophy in the University of Glasgow.
Mathematical and Physical Papers.
VOL. I. 8vo. $5.00. VOL. II. 8vo. $4.50. VOL. III. 8vo. $5.50.
Popular Lectures and Addresses.
VOL. I. **Constitution of Matter.** 12mo. $2.00.
VOL. II. **Geology and General Physics.** 12mo. $2.00.
VOL. III. **Navigational Affairs.** With Illustrations. 12mo. $2.00.
On Elasticity. 4to. $1.25.
On Heat. 4to. $1.25.

TODHUNTER: **A History of the Theory of Elasticity.** By ISAAC TODHUNTER. Edited by Professor KARL PEARSON.
VOL. I. Galilei to Saint-Venant, 1639–1850. 8vo. $6.00.
VOL. II. Saint-Venant to Lord Kelvin (Sir William Thomson). In Two Parts. $7.50.

TURNER: **A Collection of Examples on Heat and Electricity.** By H. H. TURNER, B.A. 12mo. 75 cents.

WALKER: **The Theory and Use of a Physical Balance.** By JAMES WALKER, M.A. 8vo. 90 cents.

WATSON and **BURBURY**: **A Treatise on the Application of Generalized Co-ordinates to the Kinetics of a Material System.** By H. W. WATSON, D.Sc., and S. H. BURBURY, M.A. 8vo. $1.50.

WOOD: **Light.** By Sir H. TRUMAN WOOD. 16mo. 60 cents.

WOOLCOMBE: **Practical Work in Heat.** By W. G. WOOLCOMBE, M.A., B.Sc. Crown 8vo. $1.00.

WRIGHT: **Light.** A Course of Experimental Optics, Chiefly with the Lantern. By LEWIS WRIGHT. 12mo. $2.50.

ELECTRICITY AND MAGNETISM.

ALLSOP (F. C.): **Practical Electric Light Fitting.** $1.50.

BENNETT: **The Telephoning of Great Cities.** Paper. 35 cents.

BLAKESLEY: **Alternating Currents of Electricity.** Third Edition, Enlarged. (*In the press.*)

BONNEY (G. E.): **Induction Coils.** $1.00.
Electrical Experiments. 12mo. 75 cents.

BOTTONE (S. R.): **Electricity and Magnetism.** 16mo. 90 cents.
How to manage the Dynamo. A Handbook for Ship Engineers, Electric Light Engineers, etc. 16mo. 60 cents.
A Guide to Electric Lighting. 16mo. 75 cents.

CAVENDISH: **Electrical Researches of the Honourable Henry Cavendish, F.R.S.** Written between 1771 and 1781. Edited from original manuscripts of the late J. CLERK MAXWELL, F.R.S. 8vo. $5.00.

CUMMING: **An Introduction to the Theory of Electricity.** By LINNÆUS CUMMING, M.A. With Illustrations. 12mo. $2.25.

DAY: **Electric Light Arithmetic.** By R. E. DAY, M.A. 18mo. 40 cents.

EMTAGE: **An Introduction to the Mathematical Theory of Electricity and Magnetism.** By W. T. A. EMTAGE, M.A. 12mo. $1.90.

GRAY: **The Theory and Practice of Absolute Measurements in Electricity and Magnetism.** By ANDREW GRAY, M.A., F.R.S.E. In two volumes. Vol. I., 12mo. $3.25. Vol. II. (in two parts). $6.25.
Absolute Measurements in Electricity and Magnetism for Beginners. By ANDREW GRAY, M.A., F.R.S.E. Students' Edition. 16mo. $1.25.

GUILLEMIN: **Electricity and Magnetism.** By AMÉDÉE GUILLEMIN. Translated and edited, with Additions and Notes, by Professor SILVANUS P. THOMPSON. Super royal 8vo. $8.00.

HAWKINS and **WALLIS**: **The Dynamo.** Its Theory, Design, and Manufacture. By C. C. HAWKINS and F. WALLIS. $3.00.

HEAVISIDE (OLIVER): **Electrical Papers.** 2 vols. 8vo. $10.00.

HERTZ (H.): **Researches in the Propagation of Electrical Forces.** Authorized Translation by D. E. JONES, B.Sc. Illustrated. 8vo. $3.00.

JACKSON (D. C.): **A Text-Book on Electro-Magnetism** and the Construction of Dynamos. By DUGALD C. JACKSON, B.S., C.E., Professor of Electrical Engineering, University of Wisconsin. 12mo. $2.25.

LODGE (OLIVER J.): **Modern Views of Electricity.** By OLIVER J. LODGE, LL.D., D.Sc., F.R.S. Illustrated. $2.00.
Lightning Conductors and Lightning Guards. With numerous Illustrations. $4.00.

MAXWELL: **An Elementary Treatise on Electricity.** By JAMES CLERK MAXWELL, M.A. Edited by WILLIAM GARNETT, M.A. 8vo. $1.90.
A Treatise on Electricity and Magnetism. By JAMES CLERK MAXWELL, M.A. 2 vols. 8vo. Second Edition. $8.00.
Supplementary volume, by J. J. THOMSON, M.A. $4.50.

MAYCOCK: **A First Book of Electricity and Magnetism.** By W. PERREN MAYCOCK, M.I.E.E. Crown 8vo. 60 cents.
Electric Lighting and Power Distribution. Illustrated. Complete in three parts. 16mo. Paper. 75 cents each.

MURDOCK: **Notes on Electricity and Magnetism.** A companion to Thompson's "Elementary Lessons in Electricity and Magnetism." By J. B. MURDOCK, Lieut. U.S.N. 18mo. 60 cents.

POOLE: **The Practical Telephone Handbook.** By JOSEPH POOLE. $1.00.

PREECE and **STUBBS**: **A Manual of Telephony.** By WILLIAM HENRY PREECE and ARTHUR J. STUBBS. $4.50.

RUSSELL: **Electric Light Cables and the Distribution of Electricity.** By STUART A. RUSSELL, A.M., I.C.E. 12mo. $2.25.

STEWART and **GEE**: **Practical Physics for Schools and Junior Students of Colleges.** By BALFOUR STEWART, M.A., LL.D., F.R.S., and W. W. HALDANE GEE, B.Sc.
VOL. I. Electricity and Magnetism. 16mo. 60 cents.

Lessons in Elementary Practical Physics. By BALFOUR STEWART, M.A., LL.D., F.R.S., and W. W. HALDANE GEE, B.Sc.
VOL. II. Electricity and Magnetism. 12mo. $2.25.

THOMSON: Notes on Recent Researches in Electricity and Magnetism. Intended as a Sequel to Professor CLERK MAXWELL'S "Treatise on Electricity and Magnetism." By J. J. THOMSON. 8vo. $4.50.

THOMSON: Reprints of Papers of Electrostatics and Magnetism. By Sir WILLIAM THOMSON, D.C.L., LL.D., F.R.S., F.R.S.E. 8vo. $5.00.

THOMPSON: Elementary Lessons in Electricity and Magnetism. By SILVANUS P. THOMPSON, D.Sc., B.A., F.R.A.S. New Edition. With Illustrations. 16mo. $1.25.
NOTES TO THE SAME, by J. B. MURDOCK. 60 cents.

WALKER: How to Light a Colliery by Electricity. 4to. Limp. 75 cents.
Town Lighting by Electricity. (*In the press.*)

WATSON and BURBURY: The Mathematical Theory of Electricity and Magnetism. By H. W. WATSON, D.Sc., F.R.S., and S. H. BURBURY, M.A.
VOL. I. Electrostatics. 8vo. $2.75.
VOL. II. Magnetism and Electrodynamics. 8vo. $2.60.

MECHANICS.

ALDIS: Rigid Dynamics, An Introductory Treatise on. By W. STEADMAN ALDIS, M.A. $1.00.

ALEXANDER and THOMPSON: Elementary Applied Mechanics. PART II. Transverse Stress. $2.75.

BALL: Experimental Mechanics. A Course of Lectures delivered to the Royal College of Science for Ireland. By Sir R. S. BALL, LL.D., F.R.S. Second Edition. With Illustrations. 12mo. $1.50.

BASSET: A Treatise on Hydrodynamics. 2 vols. 8vo. $9.00.
An Elementary Treatise on Hydrodynamics and Sound. 8vo. $3.00.
A Treatise on Physical Optics. 8vo. $6.00.

BAYNES: Lessons on Thermodynamics. By R. E. BAYNES, M.A. 12mo. $1.90.

BESANT: A Treatise on Hydromechanics. Fifth Edition, Revised. PART I. Hydrostatics. 12mo. $1.25.
A Treatise on Dynamics. $1.75.
Elementary Hydrostatics. 16mo. $1.00.
SOLUTIONS TO THE EXAMPLES. (*In the press.*)

CLIFFORD: Works by W. KINGDON CLIFFORD, F.R.S.
Elements of Dynamic. An Introduction to the Study of Motion and Rest in Solid and Fluid Bodies.
PART I. Books I.–III. 12mo. $1.90.
PART II. Book IV. and Appendix. 12mo. $1.75.

COTTERILL: Applied Mechanics. An Elementary General Introduction to the Theory of Structures and Machines. By JAMES H. COTTERILL, F.R.S. 8vo. $5.00.

COTTERILL and SLADE: Elementary Manual of Applied Mechanics. By Prof. J. H. COTTERILL, F.R.S., and J. H. SLADE. 12mo. $1.25.

CREMONA (LUIGI): **Graphical Statics.** Two Treatises on the Graphical Calculus and Reciprocal Figures in Graphical Calculus. Authorized English Translation by T. HUDSON BEARE. 8vo. $2.25.

GARNETT: Elementary Dynamics, A Treatise on. For the Use of Colleges and Schools. By WILLIAM GARNETT, M.A., D.C.L. Fifth Edition, Revised. $1.50.

GOODWIN: Elementary Statics. By H. GOODWIN, D.D. 75 cents.

GREAVES: Works by JOHN GREAVES, M.A.
A Treatise on Elementary Statics. 12mo. $1.90.
Statics for Beginners. 16mo. 90 cents.
Treatise on Elementary Hydrostatics. 12mo. $1.10.

GREENHILL: **Hydrostatics.** By A. G. GREENHILL. 12mo. $1.90.

GUILLEMIN (A.): **The Applications of Physical Forces.** Translated and Edited by J. NORMAN LOCKYER, F.R.S. With Colored Plates and Illustrations. Royal 8vo. $6.50.

HICKS: **Elementary Dynamics of Particles and Solids.** By W. M. HICKS. 12mo. $1.60.

HOROBIN: **Elementary Mechanics.** By J. C. HOROBIN, B.A. With Numerous Illustrations. 12mo. Cloth. Stages I. and II., 50 cents each. Stage III. (*In preparation.*)
Theoretical Mechanics. Division I. (*In the press.*)

HOSKINS: **The Elements of Graphic Statics.** A Text-book for Students of Engineering. By L. M. HOSKINS, C.E., M.S. 8vo. $2.25.

JELLETT: **A Treatise on the Theory of Friction.** By JOHN H. JELLETT, B.D., late Provost of Trinity College, Dublin. 8vo. $2.25.

JESSOP: **The Elements of Applied Mathematics,** including Kinetics, Statics, and Hydrostatics. By C. M. JESSOP. $1.25.

KENNEDY: **The Mechanics of Machinery.** By ALEXANDER B. W. KENNEDY, F.R.S. With Illustrations. 12mo. $3.50.

LAMB: **Hydrodynamics.** A Treatise on the Mathematical Theory of Fluid Motion. By H. LAMB. 8vo. $3.00.

LOCK: Works by the REV. J. B. LOCK, M.A.
Dynamics for Beginners. 16mo. $1.00.
Elementary Statics. 16mo. $1.10. KEY. 12mo. $2.25.
Mechanics for Beginners. PART I. 90 cents. Mechanics of Solids.
Elementary Hydrostatics. (*In preparation.*)
Mechanics of Solids. 16mo. (*In the press.*)
Mechanics of Fluids. 16mo. (*In the press.*)

LONEY: **A Treatise on Elementary Dynamics.** New and Enlarged Edition. By S. L. LONEY, M.A. 12mo. $1.90.
SOLUTIONS OF THE EXAMPLES CONTAINED IN THE ABOVE. 12mo. $1.90.
The Elements of Statics and Dynamics.
PART I. Elements of Statics. $1.25.
PART II. Elements of Dynamics. $1.00.
Complete in one volume. 12mo. $1.90. KEY. 12mo. $1.90.
Mechanics and Hydrostatics for Beginners. 16mo. $1.25.

MACGREGOR: **An Elementary Treatise on Kinematics and Dynamics.** By JAMES GORDON MACGREGOR, M.A., D.SC., Munro Professor of Physics, Dalhousie College, Halifax. 12mo. $2.60.

MINCHIN: Works by G. M. MINCHIN, M.A.
A Treatise on Statics. Third Edition, Corrected and Enlarged.
VOL. I. **Equilibrium of Coplanar Forces.** 8vo. $2.25.
VOL. II. **Statics.** 8vo. $4.00.
Uniplanar Kinematics of Solids and Fluids. 12mo. $1.90.
Hydrostatics and Elementary Hydrokinetics. $2.60.

PARKINSON (R. M.): **Structural Mechanics.** $1.10.

PARKINSON: **A Treatise on Elementary Mechanics.** For the Use of the Junior Classes at the University and the Higher Classes in Schools. With a collection of Examples by S. PARKINSON, F.R.S. Sixth Edition. 12mo. $2.25.

PIRIE: **Lessons on Rigid Dynamics.** By the Rev. G. PIRIE, M.A. 12mo. $1.50.

RAWLINSON: Elementary Statics. By G. RAWLINSON, M.A. Edited by E. STURGES. 8vo. $1.10.

ROUTH: Works by E. J. ROUTH, LL.D., F.R.S.

A Treatise on the Dynamics of a System of Rigid Bodies. With Examples. New Edition, Revised and Enlarged. 8vo. In Two Parts.

PART I. Elementary. Fifth Edition, Revised and Enlarged. $3.75.

PART II. Advanced. $3.75.

Stability of a Given State of Motion, Particularly Steady Motion. 8vo. $2.25.

A Treatise on Analytical Statics. With Numerous Examples. Vol. I. 8vo. $3.75.

SANDERSON: Hydrostatics for Beginners. By F. W. SANDERSON, M.A. 16mo. $1.10.

SELBY: Elementary Mechanics of Solids and Fluids. $1.90.

SYLLABUS OF ELEMENTARY DYNAMICS.

PART. I. Linear Dynamics. With an Appendix on the Meanings of the Symbols in Physical Equations. Prepared by the Association for the Improvement of Geometrical Teaching. 4to. 30 cents.

TAIT and STEELE: A Treatise on Dynamics of a Particle. By Professor TAIT, M.A., and W. J. STEELE. Sixth Edition, Revised. 12mo. $3.00.

TAYLOR: Resistance of Ships, and Screw Propulsion. By D. W. TAYLOR. $3.75.

TODHUNTER. Works by ISAAC TODHUNTER, F.R.S.

Mechanics for Beginners. With Numerous Examples. New Edition. 18mo. $1.10. KEY. $1.75.

A Treatise on Analytical Statics. Fifth Edition. Edited by Professor J. D. EVERETT, F.R.S. 12mo. $2.60.

WALTON: Mechanics, A Collection of Problems in Elementary. By W. WALTON, M.A. Second Edition. $1.50.

Problems in Theoretical Mechanics. Third Edition Revised. With the addition of many fresh Problems. By W. WALTON, M.A. 8vo. $4.00.

WEISBACH and HERRMANN: The Mechanics of Hoisting Machinery, including Accumulators, Excavators, and Pile-Drivers. A Text-book for Technical Schools, and a Guide for Practical Engineers. By Dr. JULIUS WEISBACH and Professor GUSTAV HERRMANN. Authorized Translation from the Second German Edition. By KARL P. DAHLSTROM, M.E., Instructor of Mechanical Engineering in the Lehigh University. With 177 Illustrations. $3.75.

Mechanics of Pumping Machinery. (*In the press.*)

ZIWET: An Elementary Treatise on Theoretical Mechanics. In Three Parts: Kinematics, Statics, and Dynamics. By ALEXANDER ZIWET, University of Michigan.

PART I. $2.25. PART II. $2.25. PART III. (*In preparation.*)

MACMILLAN & CO.,

66 FIFTH AVENUE, NEW YORK.